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And lastly but most importantly, I thank you, the students who are using these books to prepare for putting your trust in us. I assure, we at Crack Every Test will do everything in our power to give you the best guidance and will work hard to build your future.

- Jigar Parekh
Founder, Crack Every Test
JBIMS,22

QA TECH | CONTENTS

NUMBER SYSTEM

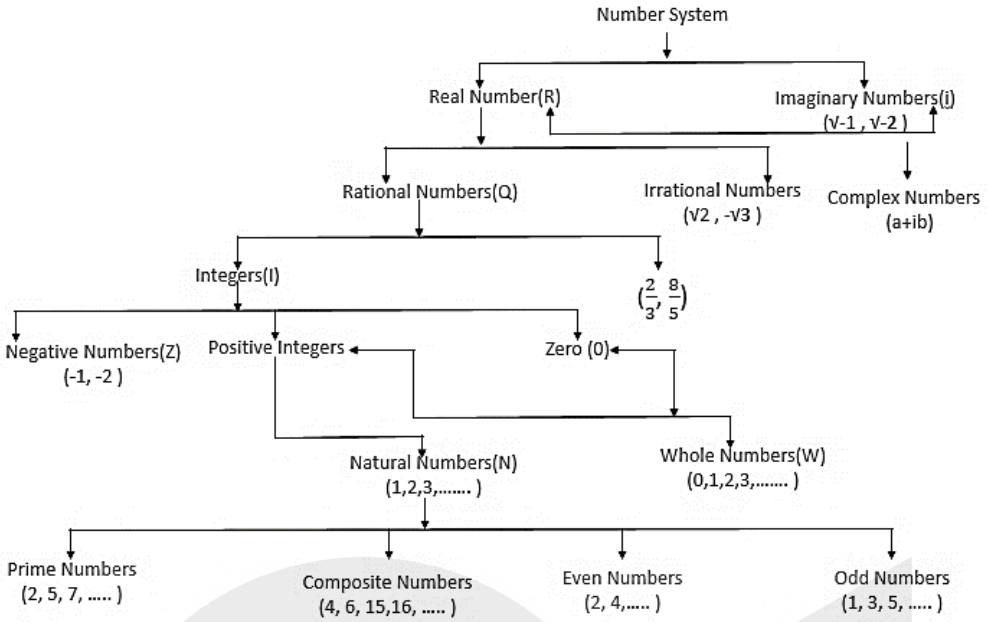
Number system is a writing system for presenting number on the number line. A number system is a system of writing or expressing numbers.

There are generally two types of Number

- **Whole Number**
- **Natural Number.**

Basics of Number System:

- **Natural Numbers**
  - All positive integers are called natural numbers. All counting numbers from 1 to infinity are natural numbers.
    N = {1, 2, 3, 4, 5, 6, ⋯, ∞}
- **Whole Numbers**
  - The set of numbers that includes all natural numbers and the number zero are called whole numbers. They are also called as non-negative integers.
    W = { 0,1,2,3,4,5,6,7,8, ⋯, ∞}
- **Integers**
  - All numbers that do not have the decimal places in them are called integers.
    Z = {∞⋯-3, -2, -1, 0, 1, 2, 3, ⋯, ∞}
  - a. Positive Integers: 1, 2, 3, 4, ... is the set of all positive integers.
  - b. Negative Integers: −1, −2, −3, ... is the set of all negative integers.
  - c. Non-Positive and Non-Negative Integers: 0 is neither positive nor negative.
- **Real Numbers**
  - All numbers that can be represented on the number line are called real numbers.
- **Rational Numbers**
  - A rational number is defined as a number of the form a/b where 'a' and 'b' are integers and b ≠ 0. The rational numbers that are not integers will have decimal values. These values can be of two types
  - a. Terminating decimal fractions: For example: 0.625, 14.253, 9.1253
  - b. non-Terminating decimal fractions: For example: 2.03333⋯ , $3.\overline{142}$
- **Irrational Numbers**
  - It is a number that cannot be written as a ratio $\frac{x}{y}$ form (or fraction). Irrational numbers are non-terminating and non-periodic fractions. For example: $\sqrt{2} = 1.414$
- **Complex Numbers**
  - The complex numbers are the set {a+bi}, where, a and b are real numbers and 'I' is the imaginary unit.
- **Imaginary Numbers**
  - A number does not exist on the number line is called imaginary number. For example, square root of negative numbers are imaginary numbers. It is denoted by 'i'.
  - Imaginary numbers have the form bi and can also be written as complex numbers by setting a=0
- **Even Numbers**
  - A number divisible by 2 is called an even number.
  - For example: 2, 6, 8, 14, 18, 246, etc.
- **Odd Numbers**
  - A number not divisible by 2 is called an odd number.
  - For example: 3, 7, 9, 15, 17, 373, etc.
- **Prime numbers**
  - A number greater than 1 is called a prime number, if it has exactly two factors, namely 1 and the number itself.
  - For example: 2, 3, 5, 7, 11, 13, 17, etc.
- **Composite numbers**
  - Numbers greater than 1 which are not prime, are known as composite numbers. For example: 4, 6, 8, 10, etc.



Decimals:

The decimals have two types, and those are:

- Non recurring decimal or terminating decimal
  - Non-recurring signifies the number which do not reappear or reiterate that is they terminate or come to an end such as, 0.5, 0.25, as well as 0.125 or more.
  - For the mater of converting these fractions, only the numerator has to be divided with the denominator.
  - For instance, if you have to convert 1/4 into a decimal then you need to divide 1 by 4 to get the answer as 0.25.
- Recurring decimal or non-terminating decimal
  - The number which does not reappear or reiterate that is they do not terminate or come to an end, such as. 0.333⋯, 0.545454⋯. and more.
  - For converting these fractions, all we have to do is to divide the numerator with the denominator.
  - For instance, if you want to convert 1/3 in a decimal, then divide 1 by 3 to get the result as 0.333⋯
  - In a recurring decimal, if a single figure is repeated, then it is expressed by putting a dot on it. If a set of figures is repeated, it is expressed by putting a bar on the set.
  - Example: $\frac{1}{3} = 0.\overline{33}$, $\frac{22}{7} = 3.\overline{142857}$
- Pure Recurring Decimal:
  - A decimal fraction, in which all the figures after the decimal point are repeated, is called a pure recurring decimal.
- Converting a Pure Recurring Decimal into Vulgar Fraction
  - Write the repeated figures only once in the numerator and take as many nines in the denominator as is the number of repeating figures.
  - For example: $0.\overline{55} = \frac{5}{9}$, $0.\overline{53} = \frac{53}{99}$
- Mixed Recurring Decimal
  - A decimal fraction in which some figures do not repeat and some of them are repeated, is called a mixed recurring decimal.
  - For example: $0.17555 = 0.17\overline{5}$

Fractions:

- The value of fraction (x/y) = 1, if numerator = denominator
- The value of fraction is zero, if numerator=0 and denominator ≠0
- The value of fraction is infinity, if the denominator=0
- The value of fraction remains unchanged, even if the numerator and denominator are multiplied or divided by same number.

- **VBODMAS Rule**

V – Vinculum (bar)	e.g., $5 \times \overline{3+5} = 5 \times 8 = 40$	Here, Vinculum can be used to give priority to an operation over other operations. Here, the part of the expression having a bar over it is solved first.
B – Bracket (), {}, []	e.g., [5 × {20 ÷ (7 – 3)}] = [5 × {20 ÷ 4}] = [5 × 5] = 25	If there are brackets inside brackets, the innermost bracket is solved first.
O – Of	e.g., $\frac{2}{5}$ of $35 = \frac{2}{5} \times 35 = 14$	Of means multiplication
D – Division M – Multiplication	e.g., $15 \times 2 \div 3$ $= 30 \div 3$ $= 10$	If divisions and multiplications are the consecutive operations, then perform the operations in the order of appearance.
A – Addition S – Subtraction	e.g., $200 - 58 + 36$ $= 142 + 36$ $= 178$	If additions and subtractions are the consecutive operations, then perform the operations in the order of appearance.

It gives the order in which the various operations are to be performed.

- **Rules of Signs**

Number 1	Sign	Number 2	=	Answer
+	×	+	=	+
+	×	–	=	–
+	÷	–	=	–
–	÷	+	=	–
–	÷	–	=	+

- **Operations on Odd and Even numbers**

ODD					EVEN					Odds & Evens				
Odd	×	Odd	=	Odd	Even	×	Even	=	Even	Odd	×	Even	=	Even
Odd	+	Odd	=	Even	Even	+	Even	=	Even	Odd	+	Even	=	Odd
Odd	–	Odd	=	Even	Even	–	Even	=	Even	Odd	–	Even	=	Odd
Odd	÷	Odd	=	Odd	Even	÷	Even	=	Even or Odd	Even	÷	Odd	=	Even
										Odd	÷	Even	=	Not divisible

- **DIVISIBILITY**

A number x is said to be divisible by another number 'y' if it is completely divisible by Y (i.e., it should leave no remainder).

In general, it can be said that any integer I, when divided by a natural number N, there exist a unique pair of numbers Q and R which are called the quotient and Remainder respectively.

Thus, I = QN + R.

Divisibility Rules

Divisibility by 2: A number is divisible by 2 if the last digit is divisible by 2.

Divisibility by 3 (or 9): All such numbers the sum of whose digits are divisible by 3 (or 9) are divisible by 3 (or 9).

Divisibility by 4: A number is divisible by 4 if the last 2 digits are divisible by 4.

Divisibility by 5: A number is divisible by 5 if the last digit is divisible by 5 or 0.

Divisibility by 6: A number is divisible by 6 if it is simultaneously divisible by 2 and 3.

Divisibility by 8: A number is divisible by 8 if the last 3 digits of the number are divisible by 8.

Divisibility by 10: A number is divisible by 10 if the last digit is 0.

Divisibility by 11: A number is divisible by 11 if the difference of the sum of the digits in the odd places and the sum of the digits in the even places is zero or is divisible by 11.

Divisibility by 12: All numbers divisible by 3 and 4 are divisible by 12.

Divisibility by 7, 11 or 13: The integer n is divisible by 7, 11 or 13 if and only if the difference of the number of its thousands and the remainder of its division by 1000 is divisible by 7, 11 or 13.
For Example: 473312 is divisible by 7 since the difference between 473 – 312 = 161 is divisible by 7.

Divisibility by 16: last 4 digits divisible by 16

Divisibility by 17: multiply last digit by 5. Subtract it from the remaining no. The result must be divisible by 17.
E.g., 136
13 – (5*6) = 13 – 30 = 17
which is divisible by 17. So, 136 is divisible by 17.

Divisibility by 19: multiply the last digit with 2 and add it to the remaining no. The result must be divisible by 19.
E.g., 285
28 + (5*2) = 28 + 10 = 38
Which is divisible by 19. So, 285 is divisible by 19.

Theorems of Divisibility

(a) If a is divisible by b, then ac is also divisible by b.
(b) If a is divisible by b and b is divisible by c then a is divisible by c.
(c) If a and b are natural numbers such that a is divisible by b and b is divisible by a then a = b.
(d) If n is divisible by d and m is divisible by d then (m + n) and (m – n) are both divisible by d.
This has an important implication. Suppose 28 and 742 are both divisible by 7. Then (742 + 28) as well as (742 – 28) are divisible by 7. (And in fact, so is + 28 – 742).
(e) If a is divisible by b and c is divisible by d then ac is divisible by bd.
(f) The highest power of a prime number p, which divides n! exactly is given by
[n/p] + [n/p2] + [n/p3] + ⋯
where [x] denotes the greatest integer less than or equal to x.

Important concepts

1. Using the standard form of a number to find the sum and the number of factors of the number:

(a) Suppose we have to find the sum of factors and the number of factors of 240.
$240 = 2^4 \times 3^1 \times 5^1$
The sum of factors will be given by:
$(2^0 + 2^1 + 2^2 + 2^3 + 2^4)(3^0 + 3^1)(5^0 + 5^1)$
$= 31 \times 4 \times 6 = 744$
Note: This is a standard process, wherein you create the same number of brackets as the number of distinct prime factors the number contains and then each bracket is filled with the sum of all the powers of the respective prime number starting from 0 to the highest power of that prime number contained in the standard form.

(b) Number of factors of the number:
Let us explore the sum of factors of 40 in a different context.
$(2^0 + 2^1 + 2^2 + 2^3)(5^0 + 5^1)$
$= 2^0 \times 5^0 + 2^0 \times 5^1 + 2^1 \times 5^0 + 2^1 \times 5^1 + 2^2 \times 5^0 + 2^2 \times 5^1 + 2^3 \times 5^0 + 2^3 \times 5^1$
$= 1 + 5 + 2 + 10 + 4 + 20 + 8 + 40 = 90$
A clear look at the numbers above will make you realize that it is nothing but the addition of
the factors of 40
Hence, we realise that the number of terms in the expansion of $(2^0 + 2^1 + 2^2 + 2^3)(5^0 + 5^1)$

will give us the number of factors of 40. Hence, 40 has 4 × 2 = 8 factors.

Note: The moment you realise that $40 = 2^3 \times 5^1$ the answer for the number of factors can be got by (3 + 1) (1 + 1) = 8

2. Sum and Number of even and odd factors of a number.

Suppose you are trying to find out the number of factors of a number represented in the standard form by: $2^3 \times 3^4 \times 5^2 \times 7^3$
As you are already aware the answer to the question is (3 + 1) (4 + 1) (2 + 1) (3 + 1) and is based on the logic that the number of terms will be the same as the number of terms in the expansion: $(2^0 + 2^1 + 2^2 + 2^3)$ $(3^0 + 3^1 + 3^2 + 3^3 + 3^4)(5^0 + 5^1 + 5^2)$ $(7^0 + 7^1 + 7^2 + 7^3)$.
Now, suppose you have to find out the sum of the even factors of this number. The only change you need to do in this respect will be evident below. The answer will be given by:
$(2^1 + 2^2 + 2^3)(3^0 + 3^1 + 3^2 + 3^3 + 3^4)$ $(5^0 + 5^1 + 5^2)$ $(7^0 + 7^1 + 7^2 + 7^3)$
Note: We have eliminated 20 from the original answer. By eliminating 20 from the expression for the sum of all factors you are ensuring that you have only even numbers in the expansion of the expression.
Consequently, the number of even factors will be given by: (3) (4 + 1) (2 + 1) (3 + 1)

i.e., Since 20 is eliminated, we do not add 1 in the bracket corresponding to 2.
Let us now try to expand our thinking to try to think about the number of odd factors for a number.
In this case, we just have to do the opposite of what we did for even numbers. The following step will make it clear:
Odd factors of the number whose standard form is: $2^3 \times 3^4 \times 5^2 \times 7^3$
Sum of odd factors = (2^0) $(3^0 + 3^1 + 3^2 + 3^3 + 3^4)$ $(5^0 + 5^1 + 5^2)$ $(7^0 + 7^1 + 7^2 + 7^3)$
i.e.: Ignore all powers of 2. The result of the expansion of the above expression will be the complete set of odd factors of the number. Consequently, the number of odd factors for the number will be given by the number of terms in the expansion of the above expression.
Thus, the number of odd factors for the number $2^3 \times 3^4 \times 5^2 \times 7^3 = 1 \times (4 + 1) (2 + 1) (3 + 1)$.

3. Sum and number of factors satisfying other conditions for any composite number

These are best explained through examples:
(i) Find the sum and the number of factors of 1200 such that the factors are divisible by 15.
Solution: $1200 = 2^4 \times 5^2 \times 3^1$.
For a factor to be divisible by 15 it should compulsorily have 3^1 and 5^1 in it. Thus, sum of factors divisible by 15 = $(2^0 + 2^1 + 2^2 + 2^3 + 2^4) \times (5^1 + 5^2)$ (3^1) and consequently the number of factors will be given by 5 × 2 × 1 = 10.
(What we have done is ensure that in every individual term of the expansion, there is a minimum of $3^1 \times 5^1$. This is done by removing powers of 3 and 5 which are below 1.

NUMBER OF ZEROES IN AN EXPRESSION

Suppose you have to find the number of zeroes in a product:
24 × 32 × 17 × 23 × 19 = $(2^3 \times 3^1) \times (2^5) \times 17^1 \times 23 \times 19$.
As you can notice, this product will have no zeroes because it has no 5 in it.
However, if you have an expression like: 8 × 15 × 23 × 17 × 25 × 22
The above expression can be rewritten in the standard form as: $2^3 \times 3^1 \times 5^1 \times 23 \times 17 \times 5^2 \times 2^1 \times 11^1$
Zeroes are formed by a combination of 2 × 5. Hence, the number of zeroes will depend on the number of pairs of 2's and 5's that can be formed. In the above product, there are four twos and three fives. Hence, we shall be able to form only three pairs of (2 × 5). Hence, there will be 3 zeroes in the product.

Finding the Number of Zeroes in a Factorial Value

Suppose you had to find the number of zeroes in 6!
6! = 6 × 5 × 4 × 3 × 2 × 1 = (3 × 2) × (5) × (2 × 2) × (3) × (2) × (1).
The above expression will have only one pair of 5 × 2, since there is only one 5 and an abundance of 2's. It is clear that in any factorial value, the number of 5's will always be lesser than the number of 2's. Hence, all we need to do is to count the number of 5's.
For e.g., 58! Find the zeros in given factorial.
= 47/5, Quotient 9.

= 9/5, Quotient = 1.

= 9 + 1 = 10 zeroes.

Remainders

Dividend = quotient × divisor + remainder

The basic framework of remainder are as follows:

1. If N is a number divisible by 7, it can be written as 7K = N, where K is the quotient.

2. When N is divided by 7, remainder obtained is 3. Therefore, it can be written as 7K + 3 = N, where K is the quotient.

3. When N is divided by 7, remainder obtained is 3 and it is equivalent of saying remainder obtained is (–4) when divided by 7. It can be understood that when N is divided by 7, remainder obtained is 3 = N is 3 more than a multiple of 7 => Therefore, N is 4 short of another multiple of 7. Therefore, remainder obtained = –4.

4. When N is divided by 8, different remainders can be obtained. They are 0, 1, 2, 3, 4, 5, 6, 7 (8 different remainders). Similarly, when it is divided by 5, remainders 0, 1, 2, 3, 4 (5 different remainders) are obtained.

Basics of Remainder

1. If any positive number A is divided by any other positive number B and if B > A, then the remainder will be A itself. In other words, if the numerator is smaller than the denominator, then the numerator is the remainder. For example,

Remainder of 5/12 = 5

Remainder of 21/45 = 21

2. Remainder should always be calculated in its actual form, that is, we cannot reduce the fraction to its lower ratio. For example,

Remainder of ½ = 1

Remainder of 2/4 = 2

E.g., What is the remainder when 5×10^5 is divided by 6×10^6?

Solution As we know that we cannot reduce the fractions to its lower terms and numerator is less than denominator, the remainder obtained will be equal to 5×105.

3. The concept of negative remainder—as obvious from the name, this remainder implies that something has been left or something remains there. Therefore, remainder can simply never be negative. Its minimum value can be zero only and non-negative. For example,

What is the remainder when –50 is divided by 7?

Solution is -50/7 = -56+6/7 ; this gives a remainder of 6

Now, there are two methods to find the remainder of any expression:

1. Cyclicity method

for every expression of the remainder, there comes attached a specific cyclicity of remainders.

For e.g.

1. What is the remainder when 4^{1000} is divided by 7?

   Solution

   To find the cyclicity, we keep finding the remainders until some remainder repeats itself. It can be understood with the following example:

Number/7	4^1	4^2	4^3	4^4	4^5	4^6	4^7	4^8
Remainder	4	2	1	4	2	1	4	2

   Now, 4^4 gives us the same remainder as 4^1; therefore, the cyclicity is of 3 (this is because remainders start repeating themselves after 4^3).

   Thus, any power of 3 or a multiple of 3 will give a remainder of 1, and hence, 4^{999} will give 1 as the remainder. Final remainder = 4.

2. What is the remainder when 4^{96} is divided by 6?

   Solution

   Let us find the cyclicity.

Number/6	4^1	4^2	4^3	4^4	4^5	4^6	4^7	4^8
Remainder	4	4	4	4	4	4	4	4

   In all cases, the remainder is 4, and therefore, the final remainder will be 4. Actually, it is not required to find remainders till 4^8 or even 4^3. 4^2 itself gives us a remainder of 4 when divided by 6, which is same as the remainder obtained when 4^1 is divided by 6. Therefore, the length of cycle = 1.

   Hence, final remainder = 4.

2. Remainder Theorem Method

The product of any two or more than two natural numbers has the same remainder when divided by any natural number as the product of their remainders.

Let us understand this through an example:

Remainder 12x13 / 7 = Remainder 156/7 = 2

Solution

The conventional way of doing this is Product → Remainder

Using the theorem method, we get

Remainder → Product → Remainder

Therefore, first, we will find the remainders of each individual number, and then, we will multiply these individual remainders to find the final remainder.

Remainder 12/7 = 5

Remainder 13/7 = 6

Remainder 12x13 / 7 = (5 × 6)/7 = Remainder 30/7 = 2

E.g., What is the remainder obtained when (1421× 1423 × 1425) is divided 12?

Solution:

Remainder of 1421/12 = 5

Remainder of 1423/12 = 7

Remainder of 1425/12 = 9

Remainder (1421 × 1423 × 1425)/12 = Remainder (5 × 7 × 9)/12 = Remainder (5 × 63)/12 = Remainder (5 × 3)/12 = 3

Fermat's Remainder Theorem

Let P be a prime number and N be a number non-divisible by P. Then, remainder obtained when AP–1 is divided by P is 1.

(The remainder obtained when A^{P-1} / P = 1 , if HCF (A, P) = 1.)

For e.g. What is the remainder when 2100 is divided by 101?

Solution Since it satisfies the Fermat's theorem format, remainder = 1.

Derivations:

1. $(A+1)^N$ / A will always give 1 as the remainder (for all natural values of A and N).

E.g., What is the remainder when 9100 is divided by 8?

Solution

For A = 8, it satisfies the abovementioned condition.

Therefore, remainder = 1.

Alternatively, we can apply either of cyclicity or theorem method to find the remainder.

2. $(A)^N$/A+1 when N is even, remainder is 1, and when N is odd, then remainder is A (for all natural values of A and N).

E.g., What is the remainder when 210 is divided by 3?

Solution

Since N is even, remainder = 1

3. $(a^n + b^n)$ is divisible by (a + b), if n is odd. The extension of the abovementioned formula

$(a^n + b^n + c^n)$ is divisible by (a + b + c), if n is odd and a, b, and c are in arithmetic progression.

E.g. . What is the remainder obtained when $7^7 + 10^7 + 13^7 + 16^7$ / 46

Solution

It can be seen that 7, 10, 13, and 16 are in arithmetic progression and power n is odd. Further, denominator = 7 + 10 + 13 + 16 = 46. Hence, it will be divisible.

Therefore, remainder obtained = 0.

Similarly, the abovementioned situation can be extended for any number of terms.

$(a^n – b^n)$ is divisible by (a + b), if n is even.

$(a^n – b^n)$ is divisible by (a – b), if n is even

Units Digit

As discussed earlier, cyclicity exists for units digit of the numbers also. (However, it is necessary to remember that there is no relation between the cyclicity of remainders and the units digit.)

Let us consider a simple example: $–2^5 = 32$. Here, we know that units digit of 2^5 is 2. However, problem occurs when we start taking large numbers like $25{,}678^{2345}$, and so on.

To find the units digit of these numbers, we have some standard results, which we use as formula.

(Any even number)4^n = ⋯6

It means that any even number raised to any power, which is a multiple of 4, will give 6 as the units digit.

(Any odd number)4^n = ⋯1

It means that any odd number raised to any power, which is a multiple of 4, will give 1 as the units digit.

Exception: 0, 1, 5, 6 [These are independent of power, and units digit will be the same.]

E.g. . What is the units digit of 32^{32}

Solution

32 is an even number that is having a power of the form 4^n.

Therefore, it will give 6 as the units digit·

Tens Digit

(i) (Any even number)^{20N} will give 76 as its last two digits (where N is any natural number).

However, if units digit = 0, then it will give '00' as the last two digits.

(ii) (Any odd number)^{20N} will give 01 as its last two digits (where N is any natural number). However, if units digit = 5, then it will give '25' as the last two digits.

E.g., What is the tens place digit of 12^{42}?

Solution

Using generalization (i), we get 12^{20} = ⋯76 (76 as last two digits)

$12^{20} \times 12^{20} = 12^{40}$

= (⋯76) × (⋯76)

= (⋯76)

$12^{42} = 12^{40} \times 12^2 = (\cdots 76) \times (144)$

Since we are required to calculate the last two digits, we will focus only on the last two digits of both the numbers. $(\cdots 76) \times (44) = 3344$. Hence, 44 is the last two digits of 1242.

Note: we are not certain if 3 is at 100s place of this number.

E.g., 2. Find the tens place digit of 784^{1000}.

Solution

Tens place digit of 784^{1000} = Tens place digit of 841,000

As discussed earlier, $(\text{any even number})^{20N}$ will give 76 as the last two digits.

$84^{1000} = (84)^{20 \times 50} = (84)^{20N}$. This will have 76 as the last two digits.

Palindrome Numbers:

Numbers that read the same backwards and forwards.

The number "17371" is a Palindromic Number.

But "1234" is NOT, because backwards it is "4321" (not the same).

Pascal's Triangle:

Pascal's triangle, is a triangular arrangement of numbers that gives the coefficients in the expansion of any binomial expression, such as $(x + y)^n$.

Pascal's Triangle is the triangular arrangement of numbers that gives the coefficients in the expansion of any binomial expression. The numbers are so arranged that they reflect as a triangle.

To construct the triangle:

start at row zero and write only the number one.

From there, to obtain the numbers in the following rows, add the number directly above and to the left of the number with the number above and to the right of it.

If there are no numbers on the left or right side, replace a zero for that missing number and proceed with the addition.

```
                1
              1   1
            1   2   1
          1   3   3   1
        1   4   6   4   1
      1   5   10  10  5   1
```

As you go down the triangle, the sum of the rows is equal to increasing powers of 2.

For example, the sum of the numbers in the first row is 1, or 2^0. The second row is $2 = 2^1$, the third row is $4 = 2^2$, and so on.

Fibonacci Sequence:

The Fibonacci Sequence is the series of numbers:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ...

The next number is found by adding up the two numbers before it:

the 2 is found by adding the two numbers before it (1+1),

the 3 is found by adding the two numbers before it (1+2),

the 5 is (2+3),

and so on!

Example: the next number in the sequence above is 21+34 = 55

The Fibonacci Sequence can be written as a "Rule":

$x_n = x_{n-1} + x_{n-2}$

where:

x_n is term number "n".

x_{n-1} is the previous term (n–1)

x_{n-2} is the term before that (n–2)

n =	0	1	2	3	4	5	6	7	8	9	10	11	12	13	⋯
X_n =	0	1	1	2	3	5	8	13	21	34	55	89	144	233	⋯

Solved Examples

1. Find the Units digit in $2^2 \times 4^4 \times 6^6 \times 8^8$

   Solution

   The units digit would be given by the units digit of the multiplication of 4 x 6 x 6 x6 = 4

2. Find the number of zeroes in the factorial of the number 18.

   Solution

   18! contains 15 and 5, which combined with one even number give zeroes. Also, 10 is also contained in 18! which will give an additional zero. Hence, 18! contains 3 zeroes and the last digit will always be zero.

3. Find the number of zeroes in 137!

   Solution

   [137/5] + [137/52] + [137/53] = 27 + 5 + 1 = 33 zeroes (since the restriction on the number of zeroes is due to the number of fives.)

4. Find the number of zeroes in the following multiplication:

   5 × 10 × 15 × 20 × 25 × 30 × 35 × 40 × 45 × 50.

   Solution

   The number of zeroes depends on the number of fives and the number of twos. Here, close scrutiny shows that the number of twos is the constraint. The expression can be written as

   5 × (5 × 2) × (5 × 3) × (5 × 2 × 2) × (5 × 5) × (5 × 2 × 3) × (5 × 7) × (5 × 2 × 2 × 2) × (5 × 3 × 3) × (5 ×5 × 2)

   Number of 5s – 12, Number of 2s – 8.

   Hence: 8 zeroes.

5. If 381A is divisible by 9, find the value of smallest natural number A.

   (a) 5 (b) 5

   (c) 7 (d) 6

   Solution

   For the number A381 to be divisible by 11, the sum of the even placed digits and the odds placed digits should be either 0 or a multiple of 11. This means that (A + 8) – (3 + 1) should be a multiple of 11 – as it is not possible to make it zero. Thus, the smallest value that A can take (and in fact the only value it can take) is 7. Option (c) is correct.

6. What is the greatest number of 4 digits that when divided by any of the numbers 6, 9, 12, 17 leaves a remainder of 1?

   (a) 9997 (b) 9793

   (c) 9895 (d) 9487

   Solution

   The LCM of the 4 numbers is 612. The highest 4-digit number which would be a common multiple of all these 4 numbers is 9792. Hence, the correct answer is 9793.

7. Find the remainder of 2^{1000} when divided by 3.

   (a) 1 (b) 2

   (c) 4 (d) 6

   Solution

   2^{1000} /3 is of the form $(a)^{\text{EVEN POWER}}$/ (a + 1). The remainder = 1 in this case as the power is even. Option (a) is correct.

8. Find two natural numbers whose difference is 66 and the least common multiple is 360.

   (a) 120 and 54 (b) 90 and 24

   (c) 180 and 114 (d) 130 and 64

   Solution

   Option (b) can be verified to be true as the LCM of 90 and 24 is indeed 360.

9. Find the number of numbers between 200 and 300, both included, which are not divisible by 2, 3, 4 and 5.

   (a) 27 (b) 26

   (c) 25 (d) 28

   Solution

   Start with 101 numbers (i.e., all numbers between 200 and 300 both included) and subtract the number of numbers which are divisible by 2 (viz. [(300 – 200)/2] + 1 = 51 numbers), the number of numbers which are divisible by 3 but not by 2 (Note: This would be given by the number of terms in the series 201, 207, ⋯297. This series has 17 terms) and the number of numbers which are divisible by 5 but not by 2 and 3. (The numbers are 205, 215, 235, 245, 265, 275, 295. A total of 7 numbers). Thus, the required answer is given by 101 – 51 – 17 – 7 = 26. Option (b) is correct.

10. If a number is multiplied by 22 and the same number is added to it, then we get a number that is half the square of that number. Find the number.

    (a) 45 (b) 46

    (c) 47 (d) data insufficient

    Solution

If the number is n, we will get that 22n + n = 23n is half the square of the number n. Thus, we have n^2 = 46n ➔ n = 46

Exercise – 1

1. P is a whole number which when divided by 4 gives 3 as remainder. What will be the remainder when 2P is divided by 4?
   A. 3 B. 2
   C. 1 D. 0
   E. 7
2. How many prime numbers exist in $6^7 \times 35^3 \times 11^{10}$?
   A. 30 B. 29
   C. 27 D. 31
   E. 37
3. The difference between two numbers is 2395. When the larger number is divided by the smaller one, the quotient is 6 and the remainder is 15. The smaller number is
   A. 120 B. 239
   C. 476 D. 523
   E. 560
4. If x is a whole number, then $x^2(x^2 - 1)$ is always divisible by
   A. 12^x B. Multiple of 12
   C. 12 D. 14
   E. 16
5. How many terms are there in 2, 4, 8, 16, ⋯ , 1024?
   A. 14 B. 11
   C. 12 D. 10
   E. 16
6. The digit in unit's place of the product 71 × 72 × ⋯ × 79 is
   A. 2 B. 0
   C. 6 D. 8
   E. 4
7. How many numbers between 190 and 580 are divisible by 4,5 and 6?
   A. 6 B. 7
   C. 8 D. 9
   E. 5
8. The largest natural number which exactly divides the product of any four consecutive natural numbers is:
   A. 6 B. 12
   C. 24 D. 36
   E. 8
9. Which of the following number should be added to 11158 to make it exactly divisible by 77?
   A. 6 B. 5
   C. 8 D. 7
   E. 4
10. What least number must be subtracted from 427398 so that the remaining number is divisible by 15?
    A. 3 B. 1
    C. 16 D. 11
    E. 4
11. The smallest value of n, for which 2n+1 is not a prime number, is
    A. 3 B. 4
    C. 5 D. none of these
    E. 7
12. Find the number of factors of 9321.
    A. 3 B. 6
    C. 8 D. 16
    E. 19
13. What is the rightmost integer of the expression $65776^{759} + 54697^{467}$
    A. 3 B. 5
    C. 7 D. 9
    E. 6
14. Three times the first of three consecutive odd integers is 3 more than twice the third. The third integer is:
    A. 9 B. 11
    C. 13 D. 15
    E. 17
15. The difference between a two-digit number and the number obtained by interchanging the positions of its digits is 36. What is the difference between the two digits of that number?
    A. 3 B. 4
    C. 9 D. Cannot be determined
    E. None of these
16. The difference between a two-digit number and the number obtained by interchanging the digits is 36. What is the difference between the sum and the difference of the digits of the number if the ratio between the digits of the number is 1 : 2 ?
    A. 4 B. 8
    C. 16 D. None of these
    E. 14

17. The sum of the digits of a two-digit number is 15 and the difference between the digits is 3. What is the two-digit number?
    A. 69 B. 78
    C. 96 D. Cannot be determined
    E. None of these
18. A number consists of two digits. If the digits interchange places and the new number is added to the original number, then the resulting number will be divisible by:
    A. 3 B. 5
    C. 9 D. 11
    E. 15
19. A number consists of 3 digits whose sum is 10. The middle digit is equal to the sum of the other two and the number will be increased by 99 if its digits are reversed. The number is:
    A. 145 B. 253
    C. 370 D. 352
    E. 275
20. A number is as much greater than 36 as is less than 86. Find the number:
    A. 38 B. 43
    C. 61 D. 73
    E. 57
21. Find a number such that when 15 is subtracted from 7 times the number, the result is 10 more than twice the number.
    A. 5 B. 10
    C. 15 D. 20
    E. 25
22. The sum of two numbers is 15 and the sum of their squares is 113. find the numbers.
    A. 3 and 4 B. 4 and 6
    C. 7 and 8 D. 8 and 10
    E. 8 and 12
23. The average of four consecutive even numbers is 27. find the largest of these numbers.
    A. 22 B. 24
    C. 27 D. 30
    E. 32
24. The sum of the squares of three consecutive odd numbers is 2531. Find the numbers.
    A. 21, 23 and 25 B. 15, 17 and 20
    C. 27, 29 and 31 D. 29, 31 and 33
    E. 31, 33, and 37
25. Out of two numbers, 4 times the smaller one is less than 3 times the larger one by 5, If the sum of the numbers is larger than 6 times their difference by 6, find the two numbers.
    A. 55 and 58 B. 23 and 28
    C. 59 and 43 D. 65 and 67
    E. 67 and 59
26. A number exceeds its two fifth by 75. The number is:
    A. 112 B. 150
    C. 125 D. 100
    E. 127
27. Out of two numbers, 5 times the smaller one is less than 4 times the larger one by 4, If the sum of the numbers is larger than 8 times their difference by 4, find the two numbers.
    A. 44 and 48 B. 43 and 48
    C. 44 and 56 D. 45 and 57
    E. 48 and 49
28. If three numbers are added in pairs, the sums equal 10, 19 and 21. Find the numbers?
    A. 2, 3 and 5 B. 6, 8 and 10
    C. 6, 4 and 15 D. 12, 9 and 15
    E. 4, 6 and 17
29. Find out whether 359 and 323 are prime numbers.
    A. 359 is a prime number
    B. 323 is a prime number
    C. both 359 and 323 are prime numbers
    D. neither 359 nor 323 are prime numbers
    E. none of these
30. Find the greatest number which when dividing 126 and 149 leaves a remainder 6 and 5 respectively.
    A. 21 B. 19
    C. 33 D.24
    E. 27
31. If 259 gives a remainder of 7 when divided by a certain natural number n, what is the remainder when 547 is divided by 2n assuming that the quotient in both cases is the same?
    A. 53 B. 43
    C. 35 D. 55
    E. 45
32. When a certain number is divided by 13, the remainder left is 1. However, when the same number

is divided by 17, the remainder left is 11. Find the first such number

A. 94 B. 83
C. 75 D. 81
E. 79

33. Three numbers which are co-prime to each other are such that the product of the first two is 851 and that of the last two is 1517. What is the sum of the three numbers?

A. 95 B. 100
C. 101 D. 115
E. 97

34. A certain number when divided by 14 leaves a remainder of 3. What will be the remainder when the square of that number is divided by 14?

A. 5 B. 9
C. 12 D. 7
E. 10

35. A number 247x is divisible by 7: What is the value of x if the number is also divisible by 3?

A. 1 B. 2
C. 7 D. 8
E. 5

36. If n is a prime number and $123(n^3 + 32n)$ is exactly divisible by 24, what is the value of n?

A. 8 B. 4
C. 0 D. 2
E. 5

37. How many of the following numbers are divisible by 143?

429, 396, 462, 792, 858, 2178, 3432, 7150

A. 1 B. 5
C. 4 D. 0
E. 2

38. What is the unit digit in $\{(8654)^{1793} \times (245)^{317} \times (531)^{491}\}$?

A. 0 B. 7
C. 5 D. 8
E. 2

39. The difference of two numbers is 1535. On dividing the larger number by the smaller, we get 5 as quotient and the 95 as remainder. What is the smaller number?

A. 634 B. 325
C. 447 D. 360
E. 763

40. The difference between a positive proper fraction and its reciprocal is 11/30. The fraction is:

A. $\frac{6}{7}$ B. $\frac{4}{5}$
C. $\frac{3}{4}$ D. $\frac{2}{5}$
E. $\frac{5}{6}$

41. What is the unit digit in the product $(3^{69} \times 6^{75} \times 7^{59})$?

A. 3 B. 2
C. 9 D. 5
E. none of these

42. The sum of all two-digit numbers divisible by 4 is:

A. 954 B. 1188
C. 1535 D. 964
E. none of these

43. The sum of even numbers between 1 and 51 is:

A. 450 B. 650
C. 564 D. 865
E. 870

44. $2 + 2^2 + 2^3 + \ldots + 2^{10} = ?$

A. 2048 B. 2046
C. 2148 D. 2430
E. 1932

45. The difference of the squares of two consecutive even integers is divisible by which of the following integers?

A. 2 B. 5
C. 7 D. 4
E. 6

46. What will be remainder when 21^{300} is divided by 22?

A. 5 B. 0
C. 1 D. 3
E. none of these

47. $(1^2 + 2^2 + 3^2 + \ldots + 10^2) = ?$

A. 464 B. 374
C. 346 D. 385
E. 467

48. $\left(1 - \frac{1}{n}\right) + \left(1 - \frac{2}{n}\right) + \left(1 - \frac{3}{n}\right) + \ldots$ up to n terms=

A. $\frac{1}{2}$ (n -1) B. $\frac{1}{2}$ n
C. $\frac{2}{3}$ (n-1) D. $\frac{1}{2}$ n(n -1)
E. none of these

49. Which of the following numbers will completely divide $(81^{17} - 1)$?

A. 9 B. 10
C. 15 D. 12

E. 8

50. How many 3-digit numbers are divisible by 6 in all?

A. 125 B. 200

C. 150 D. 175

E. 100

ANSWER KEY:

1) B	11) B	21) A	31) B	41) E
2) A	12) C	22) C	32) E	42) B
3) C	13) D	23) D	33) C	43) B
4) C	14) D	24) C	34) B	44) B
5) D	15) B	25) C	35) D	45) D
6) B	16) B	26) C	36) D	46)C
7) A	17) D	27) C	37) C	47) D
8) C	18) D	28) C	38) A	48) D
9) D	19) B	29) A	39) D	49)B
10) A	20) C	30) D	40) E	50) C

SURDS & INDICES

Surds

- Surds are the figures left in 'square root or cube form.'
- They are consequently irrational numbers.
- The number left in the form of square root or cube root form because in decimal form such value goes on forever.
- Furthermore, the square roots of figures, which do not have precise square roots, are known as Surds.
- Numbers which can be expressed in the form √p+√q, where p and q are natural numbers and not perfect squares.
- Irrational numbers which contain the radical sign $\sqrt[n]{\_}$ are called surds.
- Hence, the numbers in the form of $\sqrt{3}, 3\sqrt{2}, \cdots, \sqrt[n]{x}$ are called surds.
- For example: √3 , can't be simplified.
- √4 can be simplified, so it is not a surd.
- Laws of Surds:
  - $\sqrt[b]{p} = p^{\frac{1}{b}}$
  - $\sqrt[b]{pq} = \sqrt[b]{p} \times \sqrt[b]{q}$
  - $\sqrt[b]{\frac{p}{q}} = \frac{\sqrt[b]{p}}{\sqrt[b]{q}}$
  - $(\sqrt[b]{p})^b = p$
  - $\sqrt[a]{\sqrt[b]{p}} = \sqrt[ab]{p}$
  - $(\sqrt[b]{p})^a = \sqrt[b]{p^a}$

Indices

- Indices denote the power at which a particular figure is raised.
- The index of a number shows the number of times a figure is used in multiplication, or it shows that a figure is recurrently multiplied by itself.
- It is shown as small number to the right-hand side and directly above the base number.
- Indices refers to the power to which a number is raised.
- For example, 3^2
- Laws of Indices:
  - $(p^a)^b = p^{ab}$
  - $(pq)^b = p^b \times q^b$
  - $\frac{p^a}{p^b} = p^{a-b}$
  - $\left(\frac{p}{q}\right)^b = \frac{p^b}{q^b}$
  - $p^{-1} = \frac{1}{p}$

Formula or Rule for solving Surds and Indices:

- A number that is raised to the power zero will always be equals to one. For instance, $a^0 = 1$
- Surd $\sqrt[b]{p}$ can be solved further only if the factor of p is a perfect square.
- If surds are included in the denominator of a fraction, then it is required to rationalize the denominator through multiplying both denominator and numerator by an associated surd.
- In solving the equations related to surd, it is essential to understand that every surd is an irrational number.
- Every irrational number is not a surd.
- It is necessary to rationalize the denominator and to eliminate the surd in order to simplify different expressions.
- In indices, the multiplication rule for solving the question with same base, the formula used is $x^a \times x^b = x^{a+b}$.
- For division with same base formula used: $\frac{x^a}{x^b} = x^{a-b}$
- Multiplication rule for same indices: $x^a \times y^a = (x \times y)^a$
- Division rule for same indices: $\frac{x^a}{y^a} = \left(\frac{x}{y}\right)^a$
- The square root or cube root of a positive real number is known as a surd only if its value is not exactly determined.
- Two simple quadratic surd's sum and difference are known as complementary surds to one another.
- If m and n are both rational numbers and $\sqrt{p}$ and $\sqrt{q}$ are both surds and $m + \sqrt{p} = n + \sqrt{q}$ then m = n and p = q.
- If $m - \sqrt{p} = n - \sqrt{q}$ then m = n and p = q.
- If $m + \sqrt{p} = 0$, then m = 0 and p = 0.
- If $m - \sqrt{p} = 0$, then m = 0 and p = 0.

Types of Surds:

- Pure Surds: Those surds which do not have factors other than 1.
  For example: $2\sqrt{3}, 3\sqrt{7}$
- Mixed Surds: Those surds which do not have a factor of 1.
  For example: $\sqrt{27} = 3\sqrt{3}$, $\sqrt{50} = 5\sqrt{2}$
- Similar Surds: When the radicands of two surds are the same.
  For example: $5\sqrt{2}, 7\sqrt{2}$
- Unlike Surds: When the radicands are different.
  For example: $\sqrt{2}, 2\sqrt{5}$

Surds and Indices Basic Formula

- $(a+b)(a-b) = a^2 - b^2$
- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a-b)^2 = a^2 - 2ab + b^2$
- $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$
- $(a+b-c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$
- $(a-b+c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$
- $(-a+b+c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
- $(a-b-c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
- $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
- $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
- $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
- $a^3 + b^3 + c^3 = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ac)$
- When $a+b+c = 0$, then $a^3 + b^3 + c^3 = 3abc$

Surds & Indices Rule:

Rule Name	Surds Rule	Indices Rule
Multiplication Rule	$\sqrt[n]{a} \times \sqrt[n]{b} = (a \times b)^{\frac{1}{n}}$ For e.g.: $3^2 \times 2^2 = (3 \times 2)^2 = 6^2 = 36$	$a^m \times a^n = a^{m+n}$ For e.g.: $2^3 \times 2^2 = 2^{3+2} = 2^5 = 32$
Division Rule	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ For e.g.: $\sqrt[3]{\frac{64}{8}} = \frac{\sqrt[3]{64}}{\sqrt[3]{8}} = 2$	$\frac{a^m}{a^n} = a^{m-n}$ For e.g.: $\frac{3^5}{3^2} = 3^{5-2} = 3^3 = 27$
Power Rule	$\sqrt[n]{a} = a^{\left(\frac{1}{n}\right)}$ For e.g.: $\sqrt[3]{27} = 27^{\left(\frac{1}{3}\right)} = 3$	$a^{(n^m)} = a^{n^m}$ For e.g.: $2^{(3^2)} = 2^{3^2} = 2^9 = 512$ $(a^n)^m = (a)^{nm}$ For e.g.: $(2^3)^2 = (2)^{3\times2} = (2)^6 = 64$ $a^{-n} = \frac{1}{a^n}$ For e.g.: $2^{-1} = \frac{1}{2^1} = \frac{1}{2} = 0.5$

Solved Examples:

Type 1: Simplify the expression.

Question 1:

Find the value: $(1728)^{\frac{-2}{3}}$

A. $\frac{1}{144}$ B. 144

C. $-\frac{1}{144}$ D. $\frac{1}{12}$

Solution

$\sqrt[3]{1728} = 12$

$\therefore (12)^{-3^{(\frac{2}{3})}} = (12)^{-3\times\frac{2}{3}} = 12^{-2}$

$\therefore a^{-n} = \frac{1}{a^n}$

$\therefore 12^{-2} = \frac{1}{12^2} = \frac{1}{144}$

Correct Option: A

Question 2:

Find the value of: $7^{-25} - 7^{-26}$

A. 6×7^{-26} B. 6×7^{-25}

C. 7×7^{-25} D. 7×7^{-26}

Solution

$7^{-25} - 7^{-26} = \frac{1}{7^{25}} - \frac{1}{7^{26}} = \frac{7-1}{7^{26}} = 6 \times 7^{-26}$

Correct Option: A

Question 3:

Simplify: $(256)^{\frac{3}{4}}$

A. 16 B. 12

C. 256 D. 64

Solution

$(256)^{\frac{3}{4}} = (4^4)^{\frac{3}{4}} = 4^3 = 64$

Correct Option: D

Question 4:

Find the value of: $8^{112} \div 8^{110}$

A. 72 B. 64

C. 81 D. 49

Solution

$\frac{a^m}{a^n} = a^{m-n}$

$\therefore \frac{8^{112}}{8^{110}} = 8^{112-110} = 8^2 = 64$

Correct Option: B

Type 2: Find the value of x.

Question 5:

Find the value of x if:

$\left(\frac{p}{q}\right)^{x-1} = \left(\frac{q}{p}\right)^{x-3}$

A. 3 B. 2

C. 1 D. -2

Solution

$\left(\frac{p}{q}\right)^{x-1} = \left(\frac{q}{p}\right)^{x-3}$

$\left(\frac{p}{q}\right)^{x-1} = \left(\frac{p}{q}\right)^{-(x-3)} = \left(\frac{p}{q}\right)^{(3-x)}$

$\therefore x - 1 = 3 - x \rightarrow x = 2$

Correct Option: B

Question 6:

If $4^x + 4^{x+1} = 80$, then the value of x^x is:

A. 16 B. 9

C. 25 D. 4

Solution

$4^x + 4^{x+1} = 80 \rightarrow \therefore 4^x(1+4) = 80$

$\therefore 4^x \times 5 = 80 \rightarrow \therefore 4^x = 16$

$\therefore 4^x = 4^2 \rightarrow x = 2 \rightarrow \therefore x^x = 2^2 = 4$

Correct Option: D

Question 7:

If $2^a = \sqrt[3]{32}$, then a is equal to:

A. $\frac{1}{3}$ B. 4

C. $\frac{5}{3}$ D. $\frac{1}{2}$

Solution

$2^a = \sqrt[3]{32}$

$2^a = \sqrt[3]{32} = (32)^{\frac{1}{3}} = (2^5)^{\frac{1}{3}} = (2)^{\frac{5}{3}} \rightarrow \therefore a = \frac{5}{3}$

Correct Option: C

Question 8:

If x and y are whole numbers such that $x^y = 169$, then find the value of $(x-1)^{y+1}$

A. 1331 B. 2744

C. 1728 D. 729

Solution

$169 = 13^2$

$\therefore x = 13, y = 2$

$\therefore (x-1)^{y+1} = (13-1)^{(2+1)} = (12)^{(3)} = 1728$

Correct Option: C

Exercise – 1

1. $\left[\frac{\left(\frac{3}{14}\right)^{-6}}{\left(\frac{3}{14}\right)^{3}}\right]^{3} \times \left(\frac{3}{14}\right)^{(-9)} = ?$
   A. $(3/14)^{-36}$ B. $(3/14)^{-30}$
   C. $(3/14)^{-32}$ D. $(3/14)^{-27}$
   E. None of these
2. Find the value of n when, $5^{-8} \times 5^{3n+4} = \frac{5^{11}}{5^3}$
   A. 8 B. 4
   C. 6 D. 2
   E. None of these
3. Find the value of n when, $7^{8} \times 7^{6n+3} = \frac{7^{25}}{7^n}$
   A. 8 B. 2
   C. 6 D. 5
   E. None of these
4. If $16^2 \times 64^{6n+5} - 4^m = 0$, then m equal to?
   A. 18n+120 B. 18n+19
   C. 18n+16 D. 16n+19
   E. None of these
5. If $27^3 \times 81^{2n+3} - 9^m = 0$, then 2m equal to?
   A. 6n+22 B. 8n+23
   C. 8n+21 D. 5n+21
   E. None of these
6. If $(3 + 4\sqrt{3})^2 = a + b\sqrt{3}$, then what is the value of (a, b)?
   A. (57, 24) B. (24, 57)
   C. (57, 25) D. (52, 24)
   E. None of these
7. If $\left(\frac{4}{5}\right)^{x} = \frac{256}{625}$, then what is the value of x^x ?
   A. 4 B. 64
   C. 256 D. 16
   E. None of these
8. If $\left(\frac{11}{15}\right)^{x} = \frac{1331}{3375}$, then what is the value of x^{-x} ?
   A. $\frac{1}{3}$ B. $\frac{1}{81}$
   C. $\frac{1}{27}$ D. $\frac{1}{9}$
   E. None of these
9. Simplify.
   $625^{0.03} \times 625^{0.22} = 5^a \times 25^3$ Find a?
   A. 5 B. -5
   C. 4 D. -4
   E. None of these
10. By how much does $(3\sqrt{5} + 6\sqrt{2})$ exceed $(\sqrt{20} - \sqrt{18})$?
    A. $5 + 9\sqrt{2}$ B. $\sqrt{5} + 9\sqrt{3}$
    C. $\sqrt{5} + 6\sqrt{2}$ D. $\sqrt{5} + 9\sqrt{2}$
    E. None of these
11. $\frac{\left(\frac{1}{343}\right)^{-\left(\frac{5}{3}\right)}}{\left(\frac{1}{49}\right)^{\frac{3}{2}}} = ?$
    A. 7^8 B. 7^5
    C. 7^6 D. 7^7
    E. None of these
12. $\frac{100000^5}{1000^7} + 1 = ?$
    A. $10^6 + 1$ B. $10^3 + 1$
    C. $10^4 + 1$ D. $10^5 + 1$
    E. None of these
13. $7^{-35} - 7^{-36} = ?$
    A. $7^{-37} \times 6$ B. $7^{-36} \times 6$
    C. $7^{-35} \times 6$ D. $7^{-36} \times 7$
    E. None of these
14. $\left(\frac{35}{21}\right)^{1.8} \times 45^{0.9} = 5^x$. Find x?
    A. 2.7 B. 0.9
    C. 3.6 D. 1.8
    E. None of these
15. If $3^{4n+3} = \frac{1}{9^{n-2}}$, then the value of n is:
    A. $\frac{1}{9}$ B. $\frac{1}{4}$
    C. $\frac{1}{6}$ D. $\frac{1}{3}$
    E. None of these
16. If $5^{n+4} - 5^{n+2} = 24$, then the value of n is:
    A. 3 B. -3
    C. 2 D. -2
    E. None of these
17. Simplify: $\left(\frac{256}{576}\right)^{\frac{1}{4}} \times \left(\frac{64}{27}\right)^{-\frac{1}{3}} \times \left(\frac{216}{8}\right)^{-1}$
    A. $\frac{1}{18\sqrt{6}}$ B. $\frac{1}{18\sqrt{3}}$
    C. $\frac{1}{18\sqrt{2}}$ D. $\frac{1}{16\sqrt{6}}$
    E. None of these
18. $\sqrt{\sqrt[3]{0.000729}}$ is equal to :
    A. 0.0003 B. 0.3
    C. 0.03 D. 0.00003
    E. None of these
19. Simplified form of $\left[\sqrt{x^{\left(-\frac{2}{5}\right)}}\right]^{25}$ as x=2
    A. $\frac{1}{8}$ B. $\frac{1}{16}$
    C. $\frac{1}{32}$ D. $\frac{1}{64}$
    E. None of these
20. $\frac{\left(0.0036^{\frac{1}{2}} + 0.000027^{\frac{1}{3}} + 1\right)}{\left(0.0081^{\frac{1}{2}} - 0.0064^{\frac{1}{2}}\right)} = ?$
    A. 108 B. 109

C. 118 D. 190
E. None of these

21. Given √2 = 1.414 then the value of $\sqrt{8} + 2\sqrt{32} - 3\sqrt{128} + 4\sqrt{50}$ is
A. 8.484 B. 8.526
C. 8.426 D. 8.876
E. None of these

22. When simplified equal to $(1331)^{\left(\frac{8}{3}\right)\left(\frac{1}{4}\right)}$?
A. 131 B. 11
C. 121 D. 1331
E. None of these

23. $729^{0.3} \times 27^{-\frac{0.1}{3}} \times (81)^{0.2}$ = ?
A. $3^{2.5}$ B. $3^{2.2}$
C. 3^2 D. $3^{2.1}$
E. None of these

24. $2^{-2^{-2}}$ = ?
A. $\sqrt{2}$ B. $\sqrt{8}$
C. $\frac{1}{\sqrt[4]{2}}$ D. 2
E. None of these

25. $(0.04)^{-1.5}$ = ?
A. 15 B. 25
C. 5 D. 125
E. None of these

26. $(0.000064)^{(0.25)\left(\frac{4}{3}\right)}$ =?
A. 0.0004 B. 0.004
C. 0.04 D. 0.4
E. None of these

27. $8^{11.36} \times 8^a = 8^{15.85}$, then a =?
A. 4.29 B. 4.69
C. 4.49 D. 4.59
E. None of these

28. If $\left(\frac{a}{b}\right)^{2y+3} = \left(\frac{b}{a}\right)^{y-2}$, then the value of y?
A. $-\frac{2}{3}$ B. $-\frac{1}{3}$
C. $\frac{1}{3}$ D. $\frac{2}{3}$
E. None of these

29. Given that $5^{10.2}$ = x, $5^{1.5}$ = y and $x^z = y^3$, then the value of z is close to:
A. 0.49 B. 0.44
C. 0.33 D. 0.54
E. None of these

30. If 2^a = 4096, then the value of 2^{a-5} is:
A. 1024 B. 64
C. 512 D. 256
E. None of these

31. If $5^{(x+y)}$ = 125 and $5^{(x-y)}$ = 15625, then x is equal to:
A. 2.5 B. 3.5
C. 4.5 D. 4.05
E. None of these

32. $3^{x-1} + 3^{x+3} + 3^{x-2} = 741$
A. 1 B. 3
C. 4 D. 5
E. 6

33. What is the square root of $\left(15 + 2\sqrt{56}\right)$?
A. $\sqrt{7} + \sqrt{6}$ B. $\sqrt{5} + \sqrt{8}$
C. $\sqrt{6} + \sqrt{8}$ D. $\sqrt{7} + \sqrt{8}$
E. None of these

34. What is the square root of $(12+2\sqrt{35})$?
A. $\sqrt{7} + \sqrt{6}$ B. $\sqrt{5} + \sqrt{8}$
C. $\sqrt{6} + \sqrt{8}$ D. $\sqrt{7} + \sqrt{5}$
E. None of these

35. Find the value of x?
$2^3 \times 5^6 \times \frac{2240}{28} = 10^x$
A. 5 B. 6
C. 7 D. 8
E. None of these

36. Find the value of x?
$2^5 \times 3^3 \times \frac{2700}{50} = 6^x$
A. 4 B. 6
C. 3 D. 5
E. None of these

37. If $\left(4 + 6\sqrt{5}\right)^2$ = 196 + K$\sqrt{5}$, then what is the value of K?
A. 36 B. 48
C. 20 D. 15
E. None of these

38. Simplify. $\sqrt{13 - 2\sqrt{36}}$
A. 4 B. 1
C. 6 D. 8
E. None of these

39. Simplify. $\sqrt{23 - 4\sqrt{33}}$
A. $\sqrt{13} - \sqrt{10}$ B. $\sqrt{14} - \sqrt{9}$
C. $\sqrt{8} - \sqrt{15}$ D. $\sqrt{12} - \sqrt{11}$
E. None of these

40. Simplify.
$256^{1.20} \times 16^{2.60} = 4^{2a} \times 16^2$ Find a?
A. 4 B. 5
C. -3 D. 3
E. None of these

41. If $2^x = 16^y = 32^z$ and 1/x + 1/8y + 1/12z = 46, then what is the value of x?
A. $\frac{1}{20}$ B. $\frac{1}{24}$
C. $\frac{1}{27}$ D. $\frac{1}{30}$
E. None of these

42. If $3^{2x} = 81^y = 27^{3z}$ and $\frac{4}{x} + \frac{1}{12y} + \frac{5}{18z} = 13$, then what is the value of 1/x?
A. $\frac{1}{20}$ B. $\frac{12}{5}$
C. $\frac{12}{7}$ D. $\frac{5}{12}$
E. None of these

43. If $25^x = 125^{2y} = 5^{3z}$ and $\frac{3}{8x} + \frac{1}{3y} + \frac{4}{18z} = \frac{82}{36}$, then what is the value of y?
A. $\frac{1}{4}$ B. $\frac{1}{5}$
C. $-\frac{1}{6}$ D. $\frac{1}{8}$
E. None of these

44. $216^{-\frac{1}{3}} + 64^{\frac{2}{3}} + 32^{-\frac{1}{5}} = ?$
A. 99 B. 105
C. 100 D. 102
E. None of these

45. $\sqrt{3} + \sqrt{8 - 2\sqrt{15}}$ is equal to
A. $\sqrt{5}$ B. $\sqrt{4}$
C. $\sqrt{3}$ D. $\sqrt{2}$
E. None of these

46. The value of $\frac{216\times216\times216-125\times125\times125}{216\times216+216\times125+125\times125}$ = ?
A. 96 B. 99
C. 91 D. 100
E. None of these

47. A The square root of $(35 + \sqrt{16})(18 - \sqrt{25})$?
A. $15\sqrt{3}$ B. $13\sqrt{3}$
C. $13\sqrt{2}$ D. $15\sqrt{2}$
E. None of these

48. The square root of $(58 + \sqrt{36})(11 - \sqrt{81})$?
A. $15\sqrt{3}$ B. $13\sqrt{3}$
C. $8\sqrt{2}$ D. $15\sqrt{2}$
E. None of these

49. Which of the following statements is/are true?
I. $4\sqrt{3} > 6\sqrt{2}$ II. $2\sqrt{5} < 3\sqrt{4}$
A. Only I B. Only II
C. Both I and II D. Neither I or II
E. Either I or II

50. Which of the following statements is/are true?
I. $8\sqrt{3} > 7\sqrt{2}$ II. $6\sqrt{2} < 5\sqrt{3}$
A. Only I B. Only II
C. Both I and II D. Neither I or II
E. Either I or II

ANSWER KEY:

1) A	11) A	21) A	31) C	41) B
2) B	12) C	22) C	32) B	42) B
3) B	13) B	23) A	33) D	43) A
4) B	14) A	24) C	34) D	44) C
5) C	15) C	25) D	35) C	45) A
6) A	16) D	26) C	36) B	46) C
7) C	17) A	27) C	37) B	47) B
8) C	18) B	28) B	38) B	48) C
9) B	19) C	29) B	39) D	49) B
10) D	20) B	30) E	40) D	50) C

LINEAR EQUATIONS

Formulas & Definitions for Linear Equations

- A linear equation is an algebraic equation in which each term has an exponent of one and the graphing of the equation results in a straight line.
- Standard form of linear equation is y = **mx + b,** Where, x is the variable and y, m, and b are the constants.

Types of equations

1) **Linear equations in one variable**

- A Linear Equation in one variable is defined as **ax + b = 0**
- Where, a and b are constant, **a ≠ 0**, and x is an unknown variable
- The solution of the equation ax + b = 0 is x = -b/a . We can also say that –b/a is the root of the linear equation ax + b = 0.

2) Linear equations in two variables

- A Linear Equation in two variables is defined as **ax + by + c = 0**
- Where a, b, and c are constants and also, both a and b ≠ 0

3) Linear equations in three variables

- A Linear Equation in three variables is defined as **ax + by + cz = d**
- Where a, b, c, and d are constants and also, a, b and c ≠ 0

Methods to solve Linear equations

1) Substitution Method

Step 1: Solve one of the equations either for x or y.

Step 2: Substitute the solution from step 1 into the other equation.

Step 3: Now solve this equation for the second variable.

2) Elimination Method

Step 1: Multiply both the equations with such numbers to make the coefficients of one of the two unknowns numerically same.

Step 2: Subtract the second equation from the first equation.

Step 3: In either of the two equations, substitute the value of the unknown variable. So, by solving the equation, the value of the other unknown variable is obtained.

3) Cross-Multiplication Method

Suppose there are two equations,

$p_1x + q_1y = r_1$ ⋯(1)

$p_2x + q_2y = r_2$ ⋯(2)

Multiply Equation (1) with p_2

Multiply Equation (2) with p_1

$p_1p_2x + q_1p_2y = r_1p_2$

$p_1p_2x + q_2p_1 y = r_2p_1$

Subtracting,

$q_1p_2y - q_2p_1y = r_1p_2 - r_2p_1$

or, $y(q_1p_2 - q_2p_1) = r_1p_2 - r_2p_1$

Therefore, $y = r_1p_2 - r_2p_1 / q_1p_2 - q_2p_1$

where $(q_1p_2 - q_2p_1) \neq 0$

Multiply Equation (1) with q_2

Multiply Equation (2) with q_1

$p_1q_2x + q_1q_2y = r_1q_2$

$p_2q_1x + q_1q_2y = r_2q_1$

Subtracting,

$p_1q_2x - p_2q_1x = r_1q_2 - r_2q_1$

or, $x(p_1q_2 - p_2q_1) = r_1q_2 - r_2q_1$

Therefore, $x = r_1q_2 - r_2q_1 / p_1q_2 - p_2q_1$

where $p_1q_2 - p_2q_1 \neq 0$

Therefore,

$x = r_1q_2 - r_2q_1 / p_1q_2 - p_2q_1$

$y = r_1p_2 - r_2p_1 / q_1p_2 - q_2p_1$

Important Formulas of Linear Equation & key points to Remember

- Suppose there are two linear equations: $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$

Now,

(A) If **$a_1/a_2 = b_1/b_2$**, then there will be **one solution**, and the graphs will have **intersecting lines**.

(B) If **$a_1/a_2 = b_1/b_2 = c_1/c_2$**, then there will be **numerous solutions (infinitely many solutions)**, and the graphs will have **coincident lines**.

(C) If **$a_1/a_2 = b_1/b_2 \neq c_1/c_2$**, then there will be **no solution**, and the graphs will have **parallel lines**.

Solved Examples:

1) If 2x + 3y = 16 and 2x – 3y= 36, the value of x is:

A. 13 B. 23

C. 33 D. 43

Solution

2x + 3y = 16 ⋯(1)

2x – 3y= 36 ⋯(2)

adding both eq we get

4x = 52 → x = 13

Correct option: A

2) If 3a + 7b = 75 and 5a – 5b = 25, what is the value of a + b?

A. 11 B. 6

C. 5 D. 17

Solution

3a + 7b = 75 ⋯(1)

5a – 5b = 25 (divide the equation by 5)

we get, a – b = 5 ⋯.(2)

Now multiplying eq. (2) by 7

and add to eq. (1), we get

3a + 7b = 75

7a – 7b = 35

On solving

10a = 110

a = 110/10 = 11

Now put the value of a in eq (2)

11 – b = 5

b = 11 – 5 = 6

Therefore, a = 11 and b = 6

The value of a + b = 6 + 11 = 17

Correct option: D

3) If $2^{x+y} = 16$ and $16^{x-y} = 2$, then find the value of x?

A. 1/4 B. 17/4

C. 17/8 D. 4

Solution

Given, $2^{x+y} = 16$

$2^{x+y} = 2^4$

x + y = 4 ⋯(1)

Now, $16^{x-y} = 2$

$(2^4)^{x-y} = 2^1$

$2^{4x-4y} = 2^1$

∴ 4x – 4y = 1

∴ x – y = 1/4 ⋯(2)

On solving equation 1 and 2

We get,

2x = 17/4

x = 17 / (4 × 2) = 17/8

Correct option: C

4) The system of equations 3a + 5b = 6 and 6a + 10y = 6 has

A. No Solution B. One Solution

C. Two Solution D. Infinite Solution

Solution

$a_1/a_2 = 3/6 = 1/2$

$b_1/b_2 = 5/10 = 1/2$

$c_1/c_2 = 6/6 = 1$

$a_1/a_2 = b_1/b_2 \neq c_1/c_2$

Therefore, there is no Solution

Correct option: A

5) The difference between the two numbers is 45. The ratio of the two numbers is 8:3. Find the two numbers?

A. 72 and 27 B. 90 and 45

C. 81 and 36 D. 60 and 15

Solution

Let the first number be 8x

Let the second number be 3x

Now, the difference between the two numbers is 45

Therefore, 8x – 3x = 45

5x = 45

x = 45/5 = 9

Now, put the value of x in

8x = 8 × 9 = 72

3x = 3 × 9 = 27

Correct option: A

6) The breadth of a rectangle is twice its length. If the perimeter of the rectangle is 84m. Then, calculate the length and breadth of the rectangle?

A. L=12 and B= 24 B. L = 14 and B = 28

C. L = 28 and B = 14 D. L = 24 and B = 12

Solution

Perimeter of rectangle = 2 (l+b)

Length of the rectangle = x

Breadth of the rectangle = 2x

Perimeter of the rectangle = 84

2 (x + 2x) = 84

2 (3x) = 84

6x = 84

x = 84/6 = 14

Therefore, the Length of the rectangle = 14m

And Breadth of the rectangle = 14 × 2 = 28m

Correct option: B

7) Ajay bought 5 tickets for two concerts A and B and 10 tickets for concert A and C. He paid Rs. 350. Now the total of a ticket for concert A and B and ticket of A and C is Rs. 42, then what is the ticket price for concert A and B?

A. Rs. 10 B. Rs. 42
C. Rs. 14 D. Rs. 28

Solution

Let the ticket price of concert A and B = a
Let the ticket price of concert A and C = b
According to the question, a + b = 42…… (1)
Ticket bought by Ajay = 5a + 10b = 350
= a + 2b = 70 …(2)
Now solve equation 1 and 2
a + b = 42
a + 2b = 70
b = 70 – 42 = 28
Now put the value of b in equation 1
a+ 28 = 42
a = 42 – 28 = 14
Hence, the ticket price for concert A and B = Rs. 14

Correct option: C

8) The cost of 5 blankets and 6 bedsheets is Rs.1500. The cost of 6 blankets and 5 bedsheets is Rs.1300. Find out the total cost of one blanket and one bedsheet.

A. Rs. 255 B. Rs. 250
C. Rs. 81.81 D. Rs. 254.545

Solution

Let the cost of blankets be x and the cost of bedsheets be y.
According to the question:
5x+ 6y= 1500 …(1)
6x+ 5y=1300 …(2)
Multiply Eqn. 1 by 5 and Eqn. 2 by 6,
we get.
25x+30y = 7500…(3)
36x+30y = 7800…(4)
Subtract equation (3) from equation (4)
11x = 300
x = 300/11
5 × 300/11+6y =1500
6y = 1500 – 1500/11
6y = 1500(1- 1/11)
6y = 1500 × 10/11
y = 2500/11
Total cost = x + y
= 300/11+2500/11
= 2800/11 = 254.545

Correct option: D

Exercise – 1

1. Total Rs.1400 are divided among 3 persons (P, Q, and R). P gets half of Q and Q gets half of R. How much money does R has?
   A. Rs 800 B. Rs 900
   C. Rs 1000 D. Rs 850
   E. None of these
2. Value of 'x' in 5x – 4 = 2x + 17 should be:
   A. 4 B. 5
   C. 6 D. 7
   E. 8
3. If the list price of a book is reduced by Rs 5 then a person can buy 2 more books for Rs 200. What is the original cost of the book?
   A. Rs 30 B. Rs 25
   C. Rs 20 D. Rs 35
   E. None of these
4. The cost of two pencil, three pens and six erasers are Rs.22 while the cost of six pencils, five pens and two erasers are Rs.42. How much will two pencils, two pens and two erasers cost?
   A. Rs 20 B. Rs 18
   C. Rs 15 D. Rs 16
   E. None of these
5. If a lemon and apple together cost Rs. 16, a tomato and lemon cost Rs.6 and an apple costs Rs.4 more than a tomato or a lemon, then which of the following can be the price of an apple?
   A. 12 B. 10
   C. 15 D. 20
   E. None of these
6. The cost of 2 sarees and 5 shirts is Rs. 2600 while cost of one saree is equal to cost of 4 shirts. The cost of 8 shirts is?
   A. Rs. 1500 B. Rs. 2000
   C. Rs. 1600 D. Rs. 1800
   E. None of these
7. If 4 chairs and 3 table cost Rs. 1200 and 7 chairs and 6 tables cost Rs. 1800 then the cost of 3 chairs and 3 tables is?
   A. Rs 800 B. Rs 600
   C. Rs 700 D. Rs 500
   E. None of these
8. The length of a rectangle is 6cm more than its breadth. If the perimeter of the rectangle is 72 cm, find its length and breadth respectively.
   A. (21, 15) B. (22, 16)
   C. (23, 17) D. (18, 12)
   E. None of these
9. If 6 added to thrice of a number it becomes 93, then the number is
   A. 22 B. 25
   C. 28 D. 29
   E. 31
10. The sum of the two numbers is 13 and their product is 42, then the numbers are
    A. –7, 6 B. 7, 6
    C. –7, –6 D. –6, 7
    E. None of these
11. Sum of two numbers is 441 and their difference is 35, then the greatest number is
    A. 238 B. 244
    C. 258 D. 264
    E. None of these
12. Sum of two numbers is 53 and their difference is 13, then the greatest number is
    A. 35 B. 38
    C. 36 D. 33
    E. None of these
13. A horse and three cows together cost Rs.520. If a horse costs Rs.60 more than a cow, then the cost of horse is
    A. Rs 175 B. Rs 170
    C. Rs 150 D. Rs 155
    E. None of these
14. A table and two chair together cost Rs.380. If a table costs Rs.110 more than a chair, then the cost of table is
    A. Rs 175 B. Rs 200
    C. Rs 150 D. Rs 155
    E. None of these
15. A shirt and a tie together cost Rs. 900. If the cost of the shirt is Rs. 560 more than the cost of the tie, then the cost of the shirt is:
    A. Rs. 730 B. Rs. 750
    C. Rs. 780 D. Rs. 760
    E. None of these

16. The sum of two consecutive numbers is 241. Find the numbers.
A. (122, 123) B. (121, 122)
C. (119, 120) D. (120, 121)
E. None of these

17. If 6x - 10y = 10 and $\frac{x}{x+y} = \frac{5}{7}$, then (x-y)=?
A. 7 B. 3
C. 6 D. 4
E. None of these

18. Ten chairs and six tables together cost ₹ 6200, three chairs and two tables together cost ₹ 1900, The cost of 4 chairs and 5 tables is(in rupees) ?
A. 2000 B. 3000
C. 2800 D. 4000
E. None of these

19. The ratio of incomes of two persons is 8 : 5 and the ratio of their expenditure is 2 : 1. If each of them manages to save ₹ 1000 per month, find the difference of their monthly income.
A. ₹ 700 B. ₹ 850
C. ₹ 1500 D. ₹ 1200
E. None of these

20. A fraction becomes $\frac{7}{8}$, if 5 is added to both the numerator and the denominator. If 3 is added to both the numerator and the denominator, it becomes $\frac{6}{7}$. Find the fraction.
A. $\frac{10}{11}$ B. $\frac{8}{11}$
C. $\frac{9}{11}$ D. $\frac{12}{11}$
E. None of these

21. The difference between a two- digit number and the number obtained by interchanging the position of its digits is 36. What is the difference between the two digits of that number?
A. 4 B. 5
C. 9 D. 11
E. None of these

22. In a two-digit number, the digit in the unit's place is three times the digit in tenth's place. The sum of the digits is equal to 8. What is the number?
A. 12 B. 56
C. 26 D. 42
E. None of these

23. A number when subtracted by 1/7 of itself gives the same value as the sum of all the angles of a triangle. What is the number?
A. 120 B. 240
C. 210 D. 140
E. None of these

24. A number of two digits has 3 for its unit's place and the sum of digits is $\frac{1}{7}$ of the number itself, the number is
A. 53 B. 63
C. 73 D. 43
E. None of these

25. 54 is to be divided into two parts such that the sum of 10 times the first and 22 times the second is 780. The bigger part is :
A. 32 B. 30
C. 36 D. 34
E. None of these

26. The present ages of two persons are 36 and 50 years respectively, if after n years the ratio of their ages will be 3:4, then the value of n is
A. 3 B. 5
C. 7 D. 6
E. None of these

27. 11 friends went to a resort and decided to pay the bill amount equally, but 10 of them could pay ₹ 60 each, as a result 11th has to pay ₹ 50 extra than his share . Find the amount paid by him?
A. ₹ 125 B. ₹ 115
C. ₹ 105 D. ₹ 100
E. None of these

28. Rohan has some pigeons and some sheep. If the total number of animal heads is 90 and the total number of animal feet is 248, what is the total number of sheep Rohan has?
A. 46 B. 45
C. 35 D. 47
E. None of these

29. The numerator of a fraction is 6x + 1 and the denominator is 7 - 4x, x can have any value between -2 and 2, both included. The values of x for which the numerator is greater
A. $\frac{2}{5} < x \leq 2$ B. $\frac{3}{5} < x \leq 4$
C. $\frac{3}{5} < x \leq 2$ D. $\frac{2}{5} < x \leq 4$
E. None of these

30. The cost of 21 plates and 9 clays is ₹ 819. What is the total cost of 7 plates and 3 clays together (in rupees)?
A. 273 B. 250

C. 249 D. 268
E. None of these

31. A man has some hens and cows. If the number of heads be 32 and number of feet equals 90, the number of hens will be:
A. 19 B. 13
C. 25 D. 7
E. None of these

32. A man has some peacocks and horses. If the number of heads be 52 and number of feet equals 140, the number of horses will be:
A. 38 B. 22
C. 34 D. 18
E. None of these

33. A man has some peacock and horses. If the number of heads be 84 and number of feet equals 200, the number of peacocks will be:
A. 16 B. 68
C. 64 D. 12
E. None of these

34. If from twice the greater of the two numbers 30 is subtracted, the result is the other number. If 10 is added to the smaller number, the result is the first number. The largest number is:
A. 10 B. 20
C. 15 D. 30
E. 25

35. If from thrice the greater of the two numbers 16 is subtracted, the result is the other number. If from twice the smaller number 3 is subtracted, the result is the first number. The largest number is:
A. 7 B. 5
C. 8 D. 10
E. 2

36. Divide 513 in three points, so that half of I part, $\frac{1}{3}^{rd}$ of II part and $\frac{1}{4}^{th}$ of the III parts shall be equal. Then the IIIrd part is :
A. 145 B. 228
C. 200 D. 230
E. 215

37. Divide 816 in three points, so that $\frac{1}{4}^{th}$ of I part, $\frac{1}{5}^{th}$ of II part and $\frac{1}{3}^{rd}$ of the III parts shall be equal. Then the IInd part is :
A. 350 B. 360
C. 340 D. 345
E. 215

38. If a two-digit number is added to a number obtained by reversing the digits of the given number, then the sum is always divisible by which one of the following numbers?
A. 9 B. 8
C. 11 D. 12
E. 10

39. What is the sum of two numbers whose differences is 35 and the greater number is five more than the two times the smaller number?
A. 90 B. 95
C. 80 D. 100
E. None of these

40. A person plans to go for a trip so he decides to buy tickets. He bought 4 tickets from a India to a Dubai and 5 tickets from the Dubai to a New York. He paid Rs. 443000. If the sum of a ticket from India to Dubai and a ticket from Dubai to New York is Rs. 96000, then what is the fare from Dubai to New York?
A. ₹ 60k B. ₹ 78k
C. ₹ 59k D. ₹ 62k
E. None of these

41. If $8x - 3y = 18$, $7x + 4y = 29$ then $5x + 8y = ?$
A. 30 B. 31
C. 33 D. 32
E. 35

42. If $9x - 5y = 11$, $4x + 2y = 26$ then $6x + 10y = ?$
A. 70 B. 71
C. 73 D. 74
E. 75

43. Shruti could get equal number of Rs. 35, Rs. 65 and Rs. 85 tickets for a movie. She spends Rs. 3145 for all the tickets. How many of each did she buy?
A. 16 B. 15
C. 18 D. 17
E. 19

44. Veer could get an equal number of Rs. 101, Rs. 108 and Rs. 120 shares of different company. He spends Rs. 8225 for all the shares. How many of each did he buy?
A. 26 B. 25
C. 28 D. 27
E. 19

45. In a test, a candidate secured 540 marks out of maximum marks 'A". Had the maximum marks 'A' converted to 800, he would have secured 360 marks. What was the maximum marks of the test?
A. 1250 B. 1200
C. 1100 D. 1000
E. None of these

46. In an air force training test, a candidate secured 455 marks out of maximum marks 'A". Had the maximum marks 'A' converted to 480, he would have secured 312 marks. What was the maximum marks of the test?
A. 500 B. 800
C. 700 D. 900
E. None of these

47. Four – fifth of the third number is equal to thirty percent of the second number. The first number is equal to two-third of the third number. The value of the first number is 420. What is 25% of the second number?
A. 420 B. 422.5
C. 425 D. 422.75
E. None of these

48. Shrusti enters a stationary shop to buy pencils, erasers and scales. She has to buy at least 9 units of each. She buys more erasers than pencils and more scales than erasers. She picks up a total of 32 items. How many maximum erasers does she buy?
A. 10 B. 11
C. 12 D. 9
E. None of these

49. Ranjan enters a shop to buy ice-creams, cookies and pastries. She has to buy at least 11 units of each. He buys more cookies than ice-creams and more pastries than cookies. He picks up a total of 38 items. How many minimum cookies does he can buy?
A. 10 B. 11
C. 12 D. 9
E. None of these

50. The present age of Surya is one fourth of that of her father. After 6 years the father's age will be twice the age of Aryan. If Aryan celebrated his fifth birthday 8 years ago, what is Surya's present age?
A. 8 B. 9
C. 11 D. 7
E. 10

ANSWER KEY:

1) A	11) A	21) A	31) A	41) B
2) D	12) D	22) C	32) D	42) D
3) B	13) A	23) C	33) B	43) D
4) D	14) B	24) B	34) B	44) B
5) B	15) A	25) D	35) A	45) B
6) C	16) D	26) D	36) B	46) C
7) B	17) B	27) B	37) C	47) A
8) A	18) B	28) E	38) C	48) B
9) D	19) C	29) C	39) B	49) C
10) B	20) C	30) A	40) C	50) A

PROBLEMS ON AGES

Introduction

Problems on Ages Questions are a type of word problem based on algebra, which deals with the current, past, and future age of people. It can be further categorized into three forms, i.e., Questions for determining the present age, Questions for calculating the age of person after particular years, and questions that calculate the age of a person before given years. Moreover, these types of questions cover cases of different types with various combinations of ratios and fractions.

Basic Formulas on Ages:

1. If the present age is x, then n times the age is nx.
2. If the present age is x, then age n years later/hence = x + n.
3. If the present age is x, then age n years ago = x – n.
4. The ages in a ratio a: b will be ax and bx.
5. If the present age is x, then 1/ n of age = x/n

Important things to remember on How to Solve Ages Questions are given below: –

- Decide which age – either it is present or past or future.
- After deciding the age, Consider it as X.
- In most cases, we take the present age as 'X', i.e., the base year works just fine.
- Past will express as (x-5) years.
- The future can be expressed as (x+5).
- But sometimes, 'present age' is not directly given in words. Then, *take 'x' to be the age you are going to find.*
- Sometimes when nothing works then just look at the options and solve it through back calculations! It also works fine.

Types of questions:

Type 1: Based on Present Age

Question 1.

In the next 6 years, C will be double the age of D 6 years back. If at present C is 5 years elder to D, then find D's present age.

A. 32 B. 25

C. 23 D. 20

Solution

Let D's present age be x

Given, (x + 5) = C's age

Given, (x + 5) + 6 = 2(x – 6)

= x + 11 = 2x – 12

x = 23

Correct Answer: C

Type 2: Based on Age Before K Years

Question 2.

Nayan said to his daughter, "I was as old as you are today when you were born." If Nayan's age is 45 today, then his daughter's age 3 years ago was?

A. 22 B. 19.5

C. 23.5 D. 21.5

Solution

let Nayan's daughter age = z.

Given, 45- z = z.

So, 2z = 45, z = 22.5.

So, 3 yrs ago, daughter age will be 22.5 – 3 = 19.5.

Correct Answer: B

Type 3: Based on Age After K Years

Question 3.

Lalita is four times the age of Om. If after 3 years, she would be three times of Om, then additional after 3 years, how many time would be her age of Om?

A. 2.5 B. 3.5

C. 2 D. 3

Solution

Let us assume Om's age as s and Lalita's age as 4s

Given, 4s + 3 = 3(s + 3)

= 4s + 3 = 3s + 9

s = 6

So, Om's age is 6 and Lalita's age is 24.

Therefore, after 6 years Lalita's will be 30 and Om will be 12.

Lalita will be 2.5 times elder.

Correct Answer: A

Solved Examples:

1. A boy asks his father, "What is the age of grandfather?". Father replied, "He is x years old in x^2 years", and said, "we are talking about the 20th century." What is the year of birth of grandfather?

   A. 1995 B. 1892
   C. 1895 D. 1893

   Solution

   20th century means 1900 to 2000 and as he is x years old in x^2 years.
   only square year in the 20th century is 44^2=1936
   so, Grandfather is 44 years old in 1936
   & year of birth of grandfather is =1936-44=1892.

   Correct option: B

2. Seven years ago, the combined ages of Amir and Akshay was twice that of Saif. If the sum of the ages of Akshay and Amir, fifteen years hence, is 98 years, what is the present age of Saif?

   A. 27 yrs B. 35 yrs
   C. 34 yrs D. 49 yrs

   Solution

   Let Amir = M, Akshay = K, Saif = S
   (M -7)+(K-7) =2(S-7)
   M + K – 14 = 2S – 14
   M + K = 2S
   M+K+30= 98
   M+K=68
   S=68/2 =34 yrs.

   Correct option: C

3. Fathers age is 5 times his son's age. 4 years back the father was 9 times older than son. Find the fathers' present age.

   A. 36 yrs B. 40 yrs
   C. 50 yrs D. 43 yrs

   Solution

   Let son age = x, fathers age = 5x
   4 years back, so (x-4) = (5x-4)
   then father was 9 times older than son so, 9(x-4)=(5x-4)…..(1)
   solving eqn (1), x=8
   and fathers age =8*5=40 years

   Correct option: B

4. In a shopping mall with a staff of 5 members, the average age is 45 years. After 5 years a person joined them and the average age is again 45 years. What is the age of 6th person?

   A. 35 B. 23
   C. 20 D. 21

   Solution

   The Average age of 5 person after 5 yrs will be 50 yrs.
   so total age of 5 person = 50*5=250
   and total age of 6 person = 45*6=270
   so age of 6th person= 270-250= 20 yrs

   Correct option: C

5. A few people are standing in a queue with different age group. After two years their average age will be 43 and the seventh person joined them. Hence the current average age has become 45. Find the age of the seventh person?

   A. 69 B. 70
   C. 40 D. 45

   Solution

   Let the age of 6 persons is 'x'
   After two years age will be =(x+12)
   average age=43
   therefore x+12/6 =43 $\Longrightarrow$ x = 246
   (x+7th person age)/7 =
   (246+7th person age)/7 = 45(average age)
   (by solving) the 7th person age is 69

   Correct Option: A

6. Ross's age is 2 years more than his friend Sam's age. 12 years hence, his age will be 16 years less than twice the age of Sam at that time. Determine the age of Ross after 12 years.

   A. 20 B. 18
   C. 22 D. 16

   Solution

   Let's assume Sam's current age as s.
   Then, Ross's will be r = (s + 2)
   r + 12 = 2(s + 12) - 16
   s + 2 + 12 = 2s + 24 - 16

s = 6

r = s + 2

r = 8

Age of Ross after 12 years = 12 + 8 = 20

Hence option A is the correct one.

Correct option: A

7. Ashley and Dennis's present age is in the proportion of 5:3. If the proportion of Ashley's age 3 years back and Dennis's age 3 years later would be 1:1. Calculate the proportion of Ashley's age after 3 years and Dennis's age three years before.

A. 2:1 B. 22:11

C. 3:1 D. 13:6

Solution

The ratio of their present age is 5:3.

So, their present age can be 5x and 3x

Ashley's age 3 years ago was 5x – 3

Dennis's age 3 years hence will be 3x + 3

The ratio of Ashley's age 3 years before and Dennis's age 3 years hence is 1:1

(5x - 3) / (3x + 3) = 1/1

2x = 6

x = 3

Now, we have to find the ratio among Ashley's age 3 years later and Dennis's age 3 years before.

Ashley's age: (5x + 3)

Dennis's age: (3x – 3)

On applying x's value, we get:

(5x + 3) / (3x - 3) = 3:1

Therefore, option C is the correct one.

Correct option: C

8. Saif is one-fifth years old than his mother. After three years, his mother's age will be four times that of Saif. How old would Saif's mother be 3 years hence?

A. 22 B. 32

C. 33 D. 48

Solution

Let Saif's age be a, hence his mother's age= 5a

Given, 3 years hence mother's age= 4 times of Saif's

= (5a+3) = 4(a+3)

= 5a+3 = 4a+12

a = 9

If Saif's age is 9, then mother's age= 9*5 = 45 years

Further after 3 years mother's age will become 45+3 = 48

Therefore Option D is the correct one.

Correct option: D

9. Kim got married 6 years ago. Her present age is 1¼ Times of what her age was when she got hitched. Her daughter is 1/10 times her age. Calculate her daughter's current age?

A. 2 B. 7

C. 6 D. 3

Solution

Kim's daughter's age be a then Kim's age = 10a

Kim's age at the time of marriage= (10a-6)

Therefore, 10a= 54(10a–6)

Or 40= 50a-30 or a= 3

Hence, option D is the correct one.

Correct option: D

Exercise – 1

1. If the product of the ages of Ramu and Sonia is 864 years and the sum of the ages of Ramu and Rinu is 75 years. Rinu was 16 years old when Sonia was born. If the average of the ages of Ramu, Rinu and Sonia is 34 years, then what is the present age of Ramu?
   A. 35 B. 32
   C. 39 D. 36
   E. Cannot be determined
2. Five years ago, the ratio of the ages of A to B was 2:1 and the present age of A is 10 years more than B. If C's age is 20% more than that of the present age of A and B together, then what is C's age after 8 years?
   A. 56 B. 32
   C. 48 D. 40
   E. Cannot be determined
3. 5 years ago, the ratio of the age of A to B was 7:2. If the present age of A is two times more than the present age of B, then find the age of B 10 years hence.
   A. 55 B. 30
   C. 45 D. 25
   E. 35
4. Lagan's present age is 1/4th of his mother's age, after 4 years. After 6 years, Lagan's mother's age will be two times of Rajiv's age. Find the present age of Lagan, if 3 years back, Rajiv celebrated his 22nd birthday?
   A. 15 B. 30
   C. 18 D. 25
   E. 12
5. Ratio of present age of Rajesh and Pravin is 7: 6. Sum of the present ages of Shweta and Rajesh is 54. Anil is 4 years younger than Pravin. Average of present ages of Pravin, Anil and Shweta is 23 1/3 years. Find the difference between Shweta and Anil's age.
   A. 15 B. 6
   C. 18 D. 10
   E. 12
6. Ratio of the present ages of A to C is 7:9 and the present age of B is 3 years more than the 1/4th of the present ages of A and C. B is 12 years younger than C. Find the present age of B.
   A. 18 B. 6
   C. 15 D. 24
   E. 12
7. Three years hence, the age of Olaf is 3/2 of the age of Moana. The present age of Luna is 1/3rd of the present age of Olaf and the sum of the present age of Olaf, Moana and Luna together is 77 years, then find the age of Moana 5 years ago?
   A. 31 B. 22
   C. 25 D. 16
   E. 20
8. A's age 6 years ago is 4/5 of his age 4 years hence from his present age. Find age of A after 12 years.
   A. 58 B. 42
   C. 48 D. 36
   E. 32
9. Vinit's present age is six times of Sohail's age and 3/5th of Rudra's age. If the age difference between Rudra and Sohail is 45 years. Find the age of Vinit after 6 years.
   A. 51 B. 54
   C. 48 D. 36
   E. None of these
10. 3 years ago, the sum of the ages of A and C was 48 years. The present age of A is 20% more than that of B and the present age of C is 3/2 of the present age of B. Find the age of B, 11 years hence?
    A. 21 B. 54
    C. 48 D. 36
    E. 31
11. The ratio of ages of A, B and C is 1: 4: 11. If the difference between the ages B and C is 56, find the average age of B and C.
    A. 30 B. 60
    C. 45 D. 54
    E. 36
12. Age of A after 7 years is equal to the age of B after 2 years. Ratio of present age of A and C is 5:7. If Age of C after 5 years is 40 years, then find the average age of A, B and C?
    A. 25 B. 45

C. 34 D. 30
E. 59

13. Average age of A, B and C is 26 years. The sum of the present ages of B and C is 65 years. The difference between the present age of A and C is 30 years. Find the average age of A and C together?
A. 40 B. 28
C. 35 D. 42
E. None of these

14. The average age of 7 midfield players is 27. If the average of six midfield players is 25 years, then find the age of the 7th midfield player.
A. 18 B. 28
C. 36 D. 42
E. Cannot be determined

15. If the ratio of the age of A, 4 years hence and the age of B, 3 years ago is 2:5 respectively and the age of B is 60% more than that of the age of C. If the present age of A and B is in the ratio of 7/24, then find the average age of B and C together?
A. 31 B. 39
C. 36 D. 22
E. 30.6

16. There are 10 students in the class. The average age of the first five students in the class is 17 years and then the average of the next two students in the class is 20 years. If the remaining three student's average age is 14 years, then what is the average age of the class?
A. 16.7 B. 17.2
C. 15.6 D. 15
E. 17.7

17. Average present ages of A, B and C is 32 years. The ratio of the present ages of B and C is 3:4. After 6 years, the ratio will be B and C is 4:5, then what is the present age of A?
A. 34 B. 26
C. 42 D. 28
E. 21

18. If the ratio of the ages of A and B is x:y and the ratio of A's age after 4 years to B's age after 2 years is 7:9. If the sum of the ages of A and B is 106 years, then find the value of x:y?
A. 17:36 B. 1:1
C. 39:67 D. 45:61
E. 41:63

19. A father said to his son, "I was as old as you are at the present at the time of your birth". If the father age is 40 now, the son's age 5 years back was:
A. 14 B. 19
C. 21 D. 12
E. 15

20. In 10 years, A will be twice as old as B was 10 years ago. If A is now 8 years older than B, the present age of B is :
A. 18 B. 24
C. 38 D. 49
E. 36

21. The age of a man is 3 times that of his son. Five years ago, the man was 8 times as old as his son was at that time. The present age of man is?
A. 15 B. 35
C. 30 D. 45
E. 20

22. The sum of the present ages of a mother and her daughter is 74 years. Four years ago, the mother's age was five times the age of the daughter. So now the daughter's age will be:
A. 15 B. 10
C. 25 D. 20
E. 5

23. The average age of a group of 12 students is 18 years. When 8 more students join the group, the average age increases by 1 year. The average age of the new students is?
A. 28 B. 12
C. 15 D. 21
E. 18

24. A person's present age is 5/12 of the age of his father. After 8 years, he will be one-half of the age of his father. How old is the father at present?
A. 36 yrs B. 38 yrs
C. 40 yrs D. 42 yrs

E. 48 yrs

25. The ages of Krishnan and Venkat are in the proportion of 3 : 5. After 9 years, the proportion of their ages will be 3 : 4. Then the current age of Venkat is:
A. 10 B. 15
C. 13 D. 18
E. 20

26. The present average age of a family of five members is 30 years. If the present age of the youngest member in the family is ten years, then what was the average age of the family at the time of the birth of the youngest member? (Assume no death occurred in the family since the birth of the youngest)
A. 19 years B. 16 years
C. 18 years D. 20 years
E. 25 years

27. The ages of X and Y are in the proportion of 5:8 and the total of their ages is 104 years. The proportion of their ages after 8 years will be
A. 3:6 B. 6:3
C. 8:7 D. 9:5
E. 2:3

28. 15 years ago A was half of B in age. If the ratio of their present ages is 4:5, what will be the total of their present ages?
A. 15 B. 25
C. 35 D. 45
E. 50

29. The sum of the ages of 5 children born at the intervals of 3 years each is 150 years. What is the age of the 3rd child ?
A. 42 B. 36
C. 30 D. 24
E. 32

30. When Tom was born, his father was 33 years older than his brother and his mother was 27 years older than his sister. If Tom's brother is 6 years older than him and his mother is 3 years younger than his father, how old was Tom's sister when Tom was born ?
A. 6 years B. 8 years
C. 10 years D. 12 years
E. 9 years

31. Today is Varun's birthday. One year from today he will be twice more as old as he was 11 years ago. How old is Varun today?
A. 16 years B. 18 years
C. 20 years D.17 years
E. 22 years

32. The average age of a couple is 21 years when they were married seven years ago, but now the average age of the husband, wife and child is 20 years(the child was born during the interval). What is the present age of the child ?
A. 10 yrs B. 11 yrs
C. 6 yrs D. 5 yrs
E. 4 yrs

33. 5 years ago, the ratio of ages of A and B is 2 : 3. After 5 years, the ratio of ages of A and B is 3: 4. Find the sum of the present ages of A and B?
A. 72 years B. 48 years
C. 62 years D. 50 years
E. 60 years

34. The ratio of the present age of A and B is 2:5 and after 16 years the ratio of the ages of A to B becomes 4:7. What is the ratio of the ages of A to B 4 years ago?
A. 1:5 B. 1:4
C. 2:3 D. 1:3
E. 2:5

35. The average age of A, B and C, 4 years ago was 42 years and the sum of age of A and B after 6 years is 102. Find the present age of C.
A. 30 B. 48
C. 28 D. 42
E. 36

36. The ratio of the ages of A to B is 2: 1 and 12 years hence, the ratio of their ages is 8:5. What is the ratio of the ages of A to B 10 years ago?
A. 3:1 B. 5:2
C. 15:4 D. 4:7
E. 11:4

37. Ravi's present age is three times the age of his son and half of the age of his father. Present age of his son is 12 years. Find Ravi's father's age after 5 years?

A. 72 B. 77
C. 66 D. 41
E. 56

38. C is 10 years younger than A and 4 years younger than B. Ratio of the present age of A and B is 6:5 respectively, Find the sum of ages of all three persons after 10 years.
A 100 B. 144
C. 12 D. 122
E. 136

39. The ratio of the present ages of Disha to that of Tara is 9 : 11. Shital is 2 years older than Tara. Shital's age after 9 years will be 44 years. What is the present age of Disha's father, who is 26 years older than Disha?
A. 48 B. 59
C. 61 D. 42
E. 53

40. The present age of Kapil and Pratik is in the ratio of 3 : 7. After 6 years, the age of Kapil and 8 years ago, the age of Pratik is in the ratio of 7 : 9. Find the age of Pratik, 10 years hence.
A. 45 B. 52
C. 64 D. 42
E. 56

41. Anshu's present age is 10 years more than that of Ranjan's age after five years. Ranjan's present age is seven years more than that of Renu's age was 4 years ago. Renu's present age is 20 years. What will be Anshu's age after 5 years ?
A. 39 yrs B. 36 yrs
C. 39 yrs D. 41 yrs
E. 43 yrs

42. Aisha's age is 1/4th of her mother's age. Her mother's age will be twice of Kushal's age after 10 years. If Kushal's 15th birthday was celebrated two years ago, then what is Aisha's present age?
A. 11 years B. 9 years
C. 7 years D. 10 years
E. 11 years

43. Age of A after 5 years is twice the age of B before 4 years and the present age of B is 7 years less than the present age of C. If the present age of C is 32 years, then find the average of the present age of A and B.
A. 26 years B. 36 years
C. 28 years D. 36 years
E. 31 years

44. 6 years hence, the age of P is equal to the age of Q, 10 years ago. The age of Q, 2 years hence is thrice the age of P, 10 years ago. Find the present age of Q?
A. 48 years B. 36 years
C. 28 years D. 51 years
E. 40 years

45. 9 years hence, Aruna will be thrice as old as Manish's age of 9 years ago. If Aruna is 4 years older than Manish, what is the present age of Manish?
A. 48 years B. 36 years
C. 28 years D. 20 years
E. 40 years

46. 4 years ago the age of Vithal was twice more than the present age of Raj. If the present age of Raj is 9 years less than that of Lakshmi and the ratio of their ages after 6 years is 10:13, then find the present age of Vithal?
A. 52 years B. 36 years
C. 28 years D. 20 years
E. 40 years

47. The ratio of the present age of Harish and Bharani is 13:9. After 6 years, the respective ratio of their ages will be 4:3. Find the ratio of their ages after 10 years.
A. 7:5 B. 9:5
C. 11:8 D. 5:3
E. 9:7

48. The ratio of the present age of Akhil and Vanathi is 3:1 and after 11 years the age of Vanathi is half of the age of Akhil at that time. What is the ratio of the ages of Vanathi to Akhil 5 years ago?
A. 7:5 B. 3:14
C. 11:8 D. 4:9
E. 14:3

49. The ratio of the present age of A and B is 5:6. 5 years ago, the age of B was 25% more than the age of A at that time. Find the average age of A and B after 4 years?
A. 40.7 B. 30.2

C. 31.5 D. 28

E 36.75

50. 5 years ago, Somya was 5 times as old as her daughter. 5 years hence her age will be 8 less than three times the corresponding age of her daughter. Find their ages?

A. 24 and 13 years B. 48 and 24 years

C. 39 and 16 years D. 33 and 15 years

E. 35 and 11 years

ANSWER KEY:

1) B	11) B	21) B	31) D	41) E
2) A	12) D	22) A	32) E	42) A
3) E	13) B	23) D	33) E	43) E
4) A	14) A	24) E	34) D	44) E
5) B	15) B	25) B	35) B	45) D
6) C	16) A	26) D	36) C	46) A
7) E	17) B	27) E	37) B	47) E
8) A	18) D	28) D	38) D	48) B
9) D	19) E	29) C	39) E	49) C
10) E	20) C	30) E	40) A	50) E

QUADRATIC EQUATIONS

Introduction

A Quadratic Equation is the equation that can be rearranged in standard form $ax^2 + bx + c = 0$ as where x is a variable and a, b, and c represent constants, where $a \neq 0$. If a = 0, then the equation is linear, not quadratic, as there is no term, and x is the unknown variable.

- If $ax^2+bx+c = 0$ is a quadratic equation, then the value of x is given by the following formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Consider the quadratic equation: $ax^2 + bx + c = 0$

Sign Of b	Sign of c	Sign of roots (big no, small no)
+	+	(+, +)
+	-	(+, -)
-	+	(-, -)
-	-	(-, +)

Factorization

It is very simple method to to solve quadratic equations.

Factorization give 2 linear equations

For example: $x^2 + 3x - 4 = 0$

Here, a = 1, b = 3 and c = -4

Now, find two numbers whose product is – 4 and sum is 3.

So, the numbers are 4 and -1 which are the two factors.

Nature of roots

Nature of roots determine whether the given roots of the equation are real, imaginary, rational or irrational. The basic formula is $b^2 - 4ac$.

This formula is also called discriminant or D. The nature of the roots depends on the value of D. Conditions to determine the nature of the roots are:

Value of Discriminant (D)	Nature of Roots	
D > 0	Real, Distinct	
	D is a perfect square	Rational Roots
	D is not a perfect square	Irrational Roots
D = 0	Real, Equal	
D < 0	Complex, Distinct (A pair of complex conjugates)	

If roots have same magnitude, different signs, then sum $(\alpha + \beta) = 0$

$\therefore \alpha + \beta = \frac{-b}{a} = 0 \rightarrow \therefore -b = 0 \rightarrow \therefore b = 0$

If roots are reciprocals, i.e., product of roots = 1, then:

$\alpha\beta = \frac{c}{a} = 1$

2. Sum and product of the roots

For any given equation the sum of the roots will always be $\frac{-b}{a}$, and the product of the roots will be $\frac{c}{a}$

Thus, the standard equation can be written as

$x^2 - (Sum\ of\ roots)x + (Product\ of\ roots) = 0$

3. Forming a quadratic equation

The equation can be formed when the roots of the equation are given or the product and sum of the roots are given.

- If $p+\sqrt{q}$ is a root of a quadratic equation, then its other root is $p-\sqrt{q}$
- When $D \geq 0$, then $rx^2+ dx + z$ can be expressed as a product of two linear factors.
- If α and β are the roots of $rx^2+ dx + z$, then we can write it as: $x^2 - (\alpha + \beta)x + \alpha\beta = 0$
- In a quadratic equation: $ax^2+bx+c=0$

  When:

  a is positive: we get minimum value.

  a is negative: we get maximum value.

  We get minimum / maximum value at the point $x = \frac{-b}{2a}$

4. Higher Order Equations

Any equation that has a degree greater than 2 is called a higher order equation. The number of roots of such an equation is equal to the highest degree.

For higher order equations, one way to try and find the roots is trial and error. Substitute various values of x and see which value gives zero. In some cases, you can save time by checking for x= 1. If the sum of coefficients of an equation is zero, x = 1 is one of the roots of the equation.

Sometimes, you might be given two higher order equations for comparison. Here, you should try and identify number of positive and negative roots for each.

The general formula is:

Number of sign changes = number of positive roots.

For instance, for $x^5+ 4x^4 - 8x^3 - 13x^2 + 10x - 5 = 0$, the sign for each co-efficient is: (+, +, -, -, -). Thus, the sign changes three times: '+' to '-' to '+' and again '+' to '-'. Hence, the equation has three positive roots and two negative roots.

If a certain power of x is missing, its coefficient is considered a 0 and the sign is considered the same as the previous sign.

For instance, $x^5-8x^3+10x-5 = 0$ can also be written as $x^5+0x^4-8x^3-0x^2+10x-5 = 0$. Now the signs can be checked.

Solved Examples:

Question 1:

Solve for x: $x^2-3x-10 = 0$

Solution

Let us express -3x as a sum of -5x and +2x.

$x^2-5x+2x-10 = 0$ ➔ $x(x-5)+2(x-5) = 0$

$(x-5)(x+2) = 0$ ➔ $x-5 = 0$ or $x+2 = 0$

$x = 5$ or $x = -2$

Question 2:

Solve for the equations $17x^2 + 48x - 9 = 0$ and $13y^2 - 32y + 12 = 0$

A. x < y B. x > y

C. x ≤ y D. x ≥ y

E. Cannot be determined

Solution

$17x^2 + 48x - 9 = 0$···. (1)

$17x^2 + 51x - 3x - 9 = 0$

$(x+3)(17x - 3) = 0$

Therefore, Roots of first equation are -3 and $\frac{3}{17}$

We know that, if sign given in the equation is + and – then their sign of roots is – and + respectively.

Therefore, the roots of the equation are –3 and $\frac{-3}{17}$

Now, $13y^2 - 32y + 12 = 0$. (2)

$13y^2 - 26y - 6y + 12 = 0$ ➔ $(y-2)(13y - 6) = 0$

Therefore, Roots of second equation are 2 and $\frac{6}{3}$

We know that, If sign given in the equation is – and – then their sign of roots is + and + respectively.

Therefore, the roots of the equation are +2 and $+\frac{6}{13}$

Now, compare the roots $-x_1$, $+x_2$, $+y_1$, and $+y_2$

It means y>x

Correct option: A

Question 3:

Solve for equations $9x^2 - 36x + 35 = 0$ and $2y^2 - 15y - 17 = 0$

A. x < y B. x > y

C. x ≤ y D. x ≥ y

E. Cannot be determined

Solution

$9x^2 - 36x + 35 = 0$ ··· (1)

$9x^2 - 21x - 15x + 35 = 0$ ➔ $(3x - 7)(3x - 5) = 0$

Therefore, Roots of first equation are $\frac{7}{3}$ and $\frac{5}{3}$.

We know that, If sign given in the equation is – and – then their sign of roots is + and + respectively

Therefore, the roots of the equation are +1.66 and +2.33

Now,

$2y^2 - 15y - 17 = 0$ ····· (2)

$2y^2 - 17y + 2y - 17 = 0$ ➔ $(y + 1)(2y - 17) = 0$

Therefore, Roots of second equation are 8.5 and -1

Now, compare the roots $+x_1$, $+x_2$, $+y_1$, and $-y_2$

It means, we cannot find any relation between x and y.

Correct option: E

Question 4:

In a quadratic equation problem, one student made a mistake in copying the coefficient of *x* and got roots of 3 and -2. Another student made a mistake in copying the constant term and got the roots of 3 and 2. What are the correct roots?

Solution:

Let the correct equation be:

$ax^2+bx+c= 0$

For the first student, *b* is wrong but *a* and *c* are correct. Thus, the product of roots is the same as that of the correct equation.

$x_1 \times x_2 = \frac{c}{a}$

$3 \times (-2) = \frac{c}{a}$

$\therefore \frac{c}{a} = -6$

For the second student, *c* is wrong but *a* and *b* are correct. Thus, the sum of roots is the same as that of the correct equation.

$x_1 + x_2 = \frac{-b}{a}$

$3 + 2 = \frac{-b}{a}$

$\frac{b}{a} = -5$

$\therefore$ the correct equation = $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$\therefore x^2 - 5x - 6 = 0$ is the correct equation.

Solving for the correct roots

$x^2 - 5x - 6 = 0$ ➔ $(x - 6)(x + 1) = 0$

$\therefore x = 6\ and\ -1$

Question 5:

Solve $7x^4 -28x^2+4 = 0$

Solution

$\because$ this equation has a degree 4, it will have 4 roots.

Let $x^2 = a$

$\therefore 7a^2 - 11a + 4 = 0$

$\therefore (7a-4)(a-1) = 0$

$a = x^2 = 4/7, 1$

$x = \frac{2}{\sqrt{7}}, \frac{-2}{\sqrt{7}}, 1, -1$

Question 6:

What are the roots of $x^3 - 10x^2+27x-18 = 0$?

Solution

Sum of coefficients = 1+27-10-18 = 28-28 = 0

Hence, try to express the given equation in terms of (x-1), so that we can take it out as a common term.

$\therefore x^3 - 10x^2 + 27x - 18 = (x^3 - 2x^2 + x) - 8x^2 + 26x - 18$

$= x(x^2 - 2x + 1) - (8x^2 - 16x + 8) + (10x - 10)$

$= x(x-1)^2 - 8(x^2 - 2x + 1) + 10(x - 1)$

$= (x-1)[x(x - 1) - 8(x - 1) + 10]$

$= (x - 1)(x^2 - 9x + 18)$

$= (x - 1)(x - 3)(x - 6)$

Thus, 1, 3, 6 are the roots of this equation.

Question 7:

Compare: $2x^5 - 3x^4 + x^3 - 5x^2 + 8x - 7 = 0$ and $4y^5 + 2y^4 + 9y^3 + 3y^2 + 7y + 4 = 0$

Solution

The first equation has 5 sign changes i.e., number of sign changes = degree of equation.

In such a case, all roots are positive i.e., $x>0$. This is also a general rule. The second equation has 0 sign changes. In such a case, all roots are negative. i.e., $y < 0$. This is also a general rule.

Exercise - 1

1. $x^2 + 5x + 6 = 30$. Find the roots
   A. +7, -2 B. -8, +4
   C. +3, -8 D. - 4, -7
   E. +3, +8
2. Find the roots of the equation $5x^2 - 14x + 8 = 0$
   A. $+\frac{4}{5}$, +2 B. $-6, -\frac{4}{3}$
   C. $+\frac{5}{2}$, +4 D. $-2, +\frac{4}{5}$
   E. none of these
3. Find the roots of the equation $2x^2 + x - 400 = 0$
   A. 14, - 11 B. 12, 16
   C. -12.5, -16 D. -16, 12.5
   E. none of these
4. Find the roots of the equation, $2x^2 - 19x + 44 = 0$.
   A. + 2, $+\frac{4}{5}$ B. + 6 , $-\frac{5}{2}$
   C. $-4, +\frac{11}{2}$ D. $-4, -\frac{11}{2}$
   E. none of these
5. Compare; $3x^2 - 44x + 96 = 0$ and $y^2 + 14y + 48 = 0$
   A. if x > y B. if x ≤ y
   C. if x ≥ y D. if x < y
   E. if x = y or relationship between x and y can't be established
6. If one of the roots of the equation $x^2 + 4x + a = 0$ is 5, what is the value of a?
   A. 35 B. -45
   C. -30 D. -40
   E. 45
7. The equation $4x^2 - 16x + c = 0$ has equal roots. What is the value of c?
   A. 16 B. 24
   C. - 26 D. -16
   E. 12
8. If the equation $x^2 + ax + 36 = 0$ has equal roots, what is the value of a, if a > 0?
   A. 8 B. 12
   C. -18 D. 15
   E. -12
9. What are the roots of the equation (x - 6) (x-8) = 3?
   A. - 2, 7 B. - 4, -3
   C. 9, 5 D. 4, 5
   E. 8, 3
10. The roots of the equation $23x^2 - 41x + p = 0$ are reciprocal of each other. What is the value of p?
    A. 23 B. 40
    C. -44 D. -33
    E. -15
11. I. $5x^2 + 13x - 28 = 0$
    II. $7y^2 - 16y - 23 = 0$
    A. if x > y B. if x ≤ y
    C. if x ≥ y D. if x < y
    E. if x = y or relationship between x and y can't be established
12. I. $27x^2 + 43x = 14$
    II. $5y^2 + 16y = 7$
    A. if x > y B. if x ≤ y
    C. if x ≥ y D. if x < y
    E. if x = y or relationship between x and y can't be established
13. I. $x^2 - 25x + 156=0$
    II. $y^2 - 19y + 78 = 0$
    A. if x > y B. if x ≤ y
    C. if x ≥ y D. if x < y
    E. if x = y or relationship between x and y can't be established
14. I. $x^2 + 28x + 195= 0$
    II. $y^2 - 23x + 124 = 0$
    A. if x > y B. if x ≤ y
    C. if x ≥ y D. if x < y
    E. if x = y or relationship between x and y can't be established
15. I. $x^2 + 14x - 13 = 0$
    II. $y^2 + 10y - 56 = 0$
    A. if x > y B. if x ≤ y
    C. if x ≥ y D. if x < y
    E. if x = y or relationship between x and y can't be established
16. I. $x^2 + 45x + 296= 0$
    II. $y^2 - 39y + 224 = 0$
    A. if x > y B. if x ≤ y
    C. if x ≥ y D. if x < y
    E. if x = y or relationship between x and y can't be established
17. I. $x^2 - 13x + 12 = 0$
    II. $y^2 + 4y - 5 = 0$
    A. if x > y B. if x ≤ y
    C. if x ≥ y D. if x < y
    E. if x = y or relationship between x and y can't be established
18. I. $12x^2 + 27x - 27 = 0$

II. $8y^2 + 54y = 29$

A. if x > y B. if x ≤ y

C. if x ≥ y D. if x < y

E. if x = y or relationship between x and y can't be established

19. I. $11x^2 - 2 = 3(10x - 6)$

II. $44y + 21 = -6y^2 + 21y$

A. if x > y B. if x ≤ y

C. if x ≥ y D. if x < y

E. if x = y or relationship between x and y can't be established

20. I. $17x^2 + 60x - 32 = 0$

II. $6y^2 + 5y - 14 = 0$

A. if x > y B. if x ≤ y

C. if x ≥ y D. if x < y

E. if x = y or relationship between x and y can't be established

21. I. $4x^2 - 16x + 12 = 0$

II. $15y^2 - 14y + 3 = 0$

A. if x > y B. if x ≤ y

C. if x ≥ y D. if x < y

E. if x = y or relationship between x and y can't be established

22. I. $\frac{19}{\sqrt{x}} + \frac{12}{\sqrt{x}} = 5\sqrt{x}$

II. $2y^2 - 11y + 14 = 0$

A. if x > y B. if x ≤ y

C. if x ≥ y D. if x < y

E. if x = y or relationship between x and y can't be established

23. I: $x^2 = 16$

II: $y^2 = 25$

A. if x > y B. if x ≤ y

C. if x ≥ y D. if x < y

E. if x = y or relationship between x and y can't be established

24. I. $2x^2 - 11x + 12 = 0$

II. $8y^2 - 18y + 9 = 0$

A. if x > y B. if x ≤ y

C. if x ≥ y D. if x < y

E. if x = y or relationship between x and y can't be established

25. I: $4x^2 + 8x + 3 = 0$

II. $2y^2 + 7y + 6 = 0$

A. if x > y B. if x ≤ y

C. if x ≥ y D. if x < y

E. if x = y or relationship between x and y can't be established

26. I. $4x^2 + 13x = 105$

II. $y^2 + 13y - 14 = 0$

A. if x > y B. if x ≤ y

C. if x ≥ y D. if x < y

E. if x = y or relationship between x and y can't be established

27. I. $x^2 + 32x + 255 = 0$

II. $y^2 + 22y + 117 = 0$

A. if x > y B. if x ≤ y

C. if x ≥ y D. if x < y

E. if x = y or relationship between x and y can't be established

28. I. $15x^2 - 63x + 434 = 14x^2 - 24x + 66$

II. $39y + 8 = -22y^2 - 6$

A. if x > y B. if x ≤ y

C. if x ≥ y D. if x < y

E. if x = y or relationship between x and y can't be established

29. I. $x^2 + 105x + 1424 = 0$

II. $y^2 - 67y + 252 = 0$

A. if x > y B. if x ≤ y

C. if x ≥ y D. if x < y

E. if x = y or relationship between x and y can't be established

30. I. $2x^2 + 46x + 63 = 17x - 6$

II. $7y^2 - 42y + 158 = 35y - 52$

A. if x > y B. if x ≤ y

C. if x ≥ y D. if x < y

E. if x = y or relationship between x and y can't be established

31. I. $4x^2 + 48x + 108 = 0$

II. $3y^2 - 61y + 240 = 0$

A. if x > y B. if x ≤ y

C. if x ≥ y D. if x < y

E. if x = y or relationship between x and y can't be established

32. I. $x^2 + 15x = 216$

II. $y^2 + 16y = 336$

A. if x > y B. if x ≤ y

C. if x ≥ y D. if x < y

E. if x = y or relationship between x and y can't be established

33. I. $5x^2 - 17x - 78 = 0$
II. $y^2 - y - 72 = 0$
A. if x > y
B. if x ≤ y
C. if x ≥ y
D. if x < y
E. if x = y or relationship between x and y can't be established

34. I. $8x^2 + 28x - 16 = 0$
II. $y^2 + 8y - 9 = 0$
A. if x > y
B. if x ≤ y
C. if x ≥ y
D. if x < y
E. if x = y or relationship between x and y can't be established

35. I. $2x^2 + 42x = -76$
II. $5y^2 + 32 = 44y$
A. if x > y
B. if x ≤ y
C. if x ≥ y
D. if x < y
E. if x = y or relationship between x and y can't be established

36. Find the value of x if, $6x + \frac{6}{x} = 15$
A. 3/2, -1/2
B. -2, ½
C. 2, ½
D. -4, 5
E. none of these

37. Compare: $x^2+2x-35 = 0$ and $4y^2 - 19y+ 28 = 0$
A. if x > y
B. if x ≤ y
C. if x ≥ y
D. if x < y
E. if x = y or relationship between x and y can't be established

38. Compare;
$3x^5 - 6x^4+3x^3 - 9x^2 +2x - 5 = 0$ and
$4y^5 + 5y^4 +7y^3 + 8y^2 +4y +12 = 0$
A. if x > y
B. if x ≤ y
C. if x ≥ y
D. if x < y
E. if x = y or relationship between x and y can't be established

39. The roots of the equation $x^2 + 12x + 4 = 0$ are α and β. Find the equation with roots α/2 and β/2.
A. $x^2 - 3x - 2 = 0$
B. $x^2 - 6x + 1 = 0$
C. $4x^2 + 2x + 3 = 0$
D. $x^2 + 6x + 1 = 0$
E. none of these

40. The difference between two numbers is 4 and the sum of their squares is 80. What is the product of those two numbers?
A. 24
B. 18
C. 32
D. 12
E. 42

41. What are the roots of the equation (x-7) (x - 5) = (x - 5) (5 - x)?
A. – 5, - 6
B. -7, 5
C. 9, 5
D. 5, 6
E. -8, 3

42. If α and β are the roots of the equation $x^2 - 9x+14 = 0$, what is the value of $\alpha^2 + \beta^2$?
A. 40
B. 50
C. -44
D. 53
E. -35

43. How many positive roots does the equation $9x^7 + 2x^5 - 10x^4 + x^3 -2x -14 = 0$ have?
A. 1
B. 3
C. 5
D. 0
E. 2

44. If $ax^2 + 8x + 4 = 0$ has real roots, what is the maximum value of a?
A. -2
B. 8
C. 4
D. 0
E. -4

45. If one of the roots of the quadratic equation $x^2 + bx + 36 = 0$ is 6 while the equation $x^2 + bx+ c = 0$ has equal roots, what is the value of c?
A. 76
B. -88
C. -46
D. -58
E. 36

46. What is the nature of the roots of the equation $x^2 + 8x + a = 0$ if $a < 4$?
A. Both the roots must be real.
B. Both the roots may be real.
C. Both the roots must be complex.
D. Both the roots may be complex.
E. More than one of the above

47. Which of these expressions is equivalent to $(x^4 - 19x^2 + 60)$?
A. $(x + 2) (x + 2) (x^2 + 15)$
B. $(x - 2) (x - 2) (x^2 - 15)$
C. $(x + 2) (x - 2) (x^2 - 15)$
D. $(x - 2) (x + 2) (x^2 + 15)$
E. $(x - 2) (x + 2) (x^2 - 15)$

48. If the roots of the equation $x^2- tx + t = 0$ are positive integers, what can be the value of t?
A. 4
B. -2
C. -1
D. 0
E. 2

49. I. $9x^2 - 102x + 33 = 0$
II. $2y^2 + 8y - 42 = 0$
A. if x > y B. if x ≤ y
C. if x ≥ y D. if x < y
E. if x = y or relationship between x and y can't be established

50. I. $x^2 + 14x = 207$
II. $y^2 - 26y = -153$
A. if x > y B. if x ≤ y
C. if x ≥ y D. if x < y
E. if x = y or relationship between x and y can't be established

51. I. $43x^2 + 17x + 35 = 13x^2 + 33$
II. $12y^2 + 65y + 97 = 4y^2 + 17y + 33$
A. if x > y B. if x ≤ y
C. if x ≥ y D. if x < y
E. if x = y or relationship between x and y can't be established

52. I. $6x^2 + 4x - 32 = 0$
II. $18y^2 - 41y + 21 = 0$
A. if x > y B. if x ≤ y
C. if x ≥ y D. if x < y
E. if x = y or relationship between x and y can't be established

53. I. $5x^2 - 40x = -75$
II. $11y^2 + 21y - 36 = 0$
A. if x > y B. if x ≤ y
C. if x ≥ y D. if x < y
E. if x = y or relationship between x and y can't be established

54. I. $6x^2 + 27 = -27x$
II. $2y^2 + 21y = -52$
A. if x > y B. if x ≤ y
C. if x ≥ y D. if x < y
E. if x = y or relationship between x and y can't be established

55. I. $\frac{x^2}{\sqrt{x}} - \frac{4}{\sqrt{x}} = -3\sqrt{x}$
II. $2y^2 - 23y + 38 = 0$
A. if x > y B. if x ≤ y
C. if x ≥ y D. if x < y
E. if x = y or relationship between x and y can't be established

56. I. $x^2 - 51x + 644 = 0$
II. $y^2 - 35y + 276 = 0$
A. if x > y B. if x ≤ y
C. if x ≥ y D. if x < y
E. if x = y or relationship between x and y can't be established

57. I. $x^2 + 73x + 130 = 30x - 122$
II $5y^2 + 55 = -41y + 13$
A. if x > y B. if x ≤ y
C. if x ≥ y D. if x < y
E. if x = y or relationship between x and y can't be established

58. I. $21x^2 + 14x - 54 = -51 + 4x^2$
II. $2y + \frac{16}{y} = 12$
A. if x > y B. if x ≤ y
C. if x ≥ y D. if x < y
E. if x = y or relationship between x and y can't be established

59. I. $44x^2 + 18x + 133 = 43x^2 - 8x$
II. $21y^2 + 17y + 72 = 15y^2 - 25y$
A. if x > y B. if x ≤ y
C. if x ≥ y D. if x < y
E. if x = y or relationship between x and y can't be established

60. I. $20x^2 + 76x + 102 = 19x + 62$
II. $-17y^2 + 27y + 44 = -21y^2$
A. if x > y B. if x ≤ y
C. if x ≥ y D. if x < y
E. if x = y or relationship between x and y can't be established

61. $x^2 - 8x + 13 = 0$. Find the roots.
A. + 2, + 4 B. $4 \pm \sqrt{3}$
C. – 4, + 2 D. $2 \pm \sqrt{5}$
E. none of these

62. Find the roots of the equation, $x^2 + 12x - 20 = 0$.
A. $(-6 + 2\sqrt{14})$ and $(-6 - 2\sqrt{14})$
B. $(4 + 2\sqrt{14})$ and $(4 - 2\sqrt{14})$
C. $(-6 + 2\sqrt{7})$ and $(-6 - 2\sqrt{7})$
D. $(5 + 2\sqrt{12})$ and $(5 - 2\sqrt{12})$
E. none of these

63. If one root of the equation, $2x^2 - 9x + k = 0$ is 8 times the other, find k
A. 3/2 B. 5/2
C. 6/7 D. 4
E. 6

64. What are the roots of $x^3 - 10x^2 + 27x - 18 = 0$?
A. 4, 2 and 3/2 B. 7, 1 and 6

C. 1, 3 and 6/5 D. 5 ,4 and 3/2
E. 1, 3 and 6

65. What are the roots of the equation $\frac{x^2}{2}$ + 8x + 29 = 0?
A. – 5, 8 B. – 14 $\pm\sqrt{32}$
C. 12 $\pm 2\sqrt{6}$ D. 5, -6
E. -8 $\pm \sqrt{6}$

66. Which of the following can be the value of q such that the quadratic equation x^2 – (3q + 4) x+ (8q + 9) = 0 has equal roots?
A. 10/9 B. 4
C. -10/9 D. -9/10
E. -2

67. What are the roots of the equation x^4 – $36x^2$ + 275 = 0?
A. $\sqrt{13}$, $\sqrt{11}$ B. ± 3, $\sqrt{13}$
C. ± 6, ± 7 D. $\sqrt{11}$, ±5
E. none of these

68. How many non-positive roots does the equation $415x^6$–$28x^5$–$32x^4$+$115x^3$–x^2+11x=0 have?
A. 5 B. 2
C. 3 D. 0
E. none of these

69. What are the roots of the equation x^4 – $8x^3$ + x^2 + 78x – 72 = 0
A.-2, 0, 6, -7 B. 6, -3, 4, - 8
C. 1, -3, 4, 6 D. 2, 0, -5, 6
E. none of these

70. I. -56x + $31x^2$ = $23x^2$ - 66
II. -10y + $11y^2$ +51 = 26y + 23
A. if x > y B. if x ≤ y
C. if x ≥ y D. if x < y
E. if x = y or relationship between x and y can't be established

71. I. 7x (x + 9) = 33x – 27
II. $10y^2$ + 31y = $6y^2$ - 60
A. if x > y B. if x ≤ y
C. if x ≥ y D. if x < y
E. if x = y or relationship between x and y can't be established

72. I. $143x^2$+ 43x – 66 = 0
II. $33y^2$– 37y + 10 = 0
A. if x > y B. if x ≤ y
C. if x ≥ y D. if x < y
E. if x = y or relationship between x and y can't be established

73. I. $18x^2$ - 74x + 60 = 0
II. $48y^2$– 28y + 4 = 0
A. if x > y B. if x ≤ y
C. if x ≥ y D. if x < y
E. if x = y or relationship between x and y can't be established

74. I. $56x^2$ - 30x + 4 = 0
II. $45y^2$ - 36y + 7 = 0
A. if x > y B. if x ≤ y
C. if x ≥ y D. if x < y
E. if x = y or relationship between x and y can't be established

75. I. $7x^2$ - 62x + 91 = 0
II. $5x^3$ - $57x^2$ + 36x = 0
A. if x > y B. if x ≤ y
C. if x ≥ y D. if x < y
E. if x = y or relationship between x and y can't be established

ANSWER KEY:

1) C	16) D	31) D	46) A	61) B
2) A	17) C	32) E	47) E	62) A
3) E	18) E	33) E	48) A	63) D
4) E	19) A	34) E	49) E	64) E
5) A	20) E	35) D	50) B	65) E
6) B	21) A	36) C	51) A	66) C
7) A	22) A	37) E	52) E	67) D
8) B	23) E	38) A	53) A	68) B
9) C	24) C	39) D	54) A	69) C
10) A	25) C	40) C	55) D	70) E
11) E	26) E	41) D	56) C	71) A
12) E	27) D	42) D	57) B	72) E
13) E	28) A	43) B	58) D	73) A
14) D	29) D	44) C	59) D	74) D
15) E	30) D	45) E	60) A	75) A

NUMBER SERIES

Sequence, Series & Progressions

Arithmetic Progression:

A series of number is termed to be in Arithmetic progression when the difference between two consecutive numbers remains the same.

It also means that the next number can be obtained by adding or subtracting the constant number to the previous in the sequence. Therefore, this constant number is known as the common difference(d).

For example, 4, 8, 12, 16, 20 is an AP as the difference between two consecutive terms is 4 which is fixed.

Formulae:

1. n^{th} term of an AP → t_n = a+(n-1)d
2. No. of terms = n = $\frac{l-a}{d} + 1$
3. Sum of first n terms in an AP = $S_n = \frac{n}{2}[2a + (n-1)d]$ or $\frac{n}{2}(a + l)$, where a is the first term and l is the last term.
4. While solving three unknown terms in an A.P whose sum or product is given should be assumed as a-d, a, a+d.
5. While solving four unknown terms in an A.P. whose sum or product is given should be assumed as a-2d, a-d, a+d, a+2d.

Properties:

- If a fixed number is added or subtracted from each term of an AP, then the resulting sequence is also an AP and it has the same common difference as that of the original AP.
- If each term in an AP is divided or multiply with a constant non-zero number, then the resulting sequence is also in an AP.
- If $a_1, a_2, a_3, \cdots, a_n$ and $b_1, b_2, b_3, \cdots, b_n$, are in AP. then $a_1+b_1, a_2+b_2, a_3+b_3, \cdots\cdots, a_n+b_n$ and $a_1-b_1, a_2-b_2, a_3-b_3, \cdots\cdots, a_n-b_n$ will also be in AP.
  - If n^{th} term of a series is $t_n = a_n + b_n$, then the series is in AP
- Three terms of the A.P whose sum or product is given should be assumed as a-d, a, a+d.
- Four terms of the A.P. whose sum or product is given should be assumed as a-2d, a-d, a+d, a+2d.

Some Other Formulae of Arithmetic Progression:

- Sum of first n natural numbers = S = $\frac{n(n+1)}{2}$
- Sum of squares of first n natural numbers = S = $\frac{n(n+1)(2n+1)}{6}$
- Sum of first n odd numbers = S = n^2
- Sum of first n even numbers = S = n(n+1)
- If a, b, c are in AP, then $b = \frac{a+c}{2}$ and b is called the arithmetic mean of a and c.

Solved Examples:

1. Find the first term of the AP series in which 10^{th} term is 6 and 18^{th} term is 70.

A. 76 B. –76

C. 66 D. –66

Solution

10th term = (a + 9d) = 6 ... (1)

18th term = (a + 17d) = 70 ... (2)

On solving equation 1 and 2

We get, d = 8

Put the value of d in equation 1

(a + 9d) = 6

a + 9 x 8 = 6

a + 72 = 6 = -66

Correct option: D

2. Find the n^{th} term of the series 3, 8, 13, 18, ⋯

A. 2(2n+ 1) B. 5n + 2

C. 5n – 2 D. 2(2n – 1)

Solution

The given series is in the form of AP.

first term a = 3

common difference d = 5

We know that, nth term = t_n = a + (n-1)d

Therefore, t_n = 3 + (n-1) 5

= 3 + 5n – 5

= 5n – 2

Correct option: C

3. The series 28, 25, ⋯⋯, -29 has 20 terms. Find out the sum of all 20 terms?

A. -10 B. -12

C. 10 D. 12

Solution

a =28, d= -3 (25 – 28), l = -29, n = 20

Sum of all n-terms = $S_n = \frac{n}{2}(a + l)$

$S_{20} = \frac{20}{2}\big(28 + (-29)\big) = -10$

Correct Option: A

4. Find the 10^{th} term in the series 2, 5, 8, 11, 14⋯⋯

A. 26 B. 29

C. 32 D. 27

Solution

We know that,

t_n = a + (n – 1)d

In the given series,

a = 2

d = 3 ∵ (5 – 2, 8 – 5⋯)

∴ 10^{th} term = t_{10} = a + (n-1) d

t_{10} = 2 + (10 – 1)3

t_{10} = 2 + 9 × 3

t_{10} = 29

Correct option: B

5. The sum of 3 numbers in arithmetic progression is 36 and product of their extreme is 80. Find the numbers.

A. 20, 8, 16 B. 4, 12, 20

C. 12, 20, 28 D. None of these

Solution

We know that,

t_n = a + (n – 1)d

Assume the numbers as:

a-d, a, a+d

or (a-d) + a + (a+d) = 36

or 3a = 36 → a = 12

now (a-d) (a+d) = 80

a^2-d^2 = 80

$144-d^2$ = 80

d^2 = 64 → d = 8

∴ the numbers will be: 4, 12 and 20

Correct option: B

6. If t_n = 20, d = -1 and n = 16, then the first term is ?

A. 36 B.-35

C. 35 D. 5

Solution:

We know that, t_n = a + (n – 1)d

20 = a + (16 – 1) (-1)

20 = a + (15) (-1)

20 = a – 15

-a = – 15 – 20

– a = – 35

a = 35

Correct option: C

Geometric Progression

A Geometric Progression is an arrangement where each number is obtained by either multiplying or dividing the previous number with a specific number called the common ratio "r".

Geometric Progression Questions are in the form a, ar, ar^2, ar^3 and so on.

Suppose, if 'a' is the first term and 'r' be the common ratio, then

1. Formula for n^{th} term of GP = ar^{n-1}
2. Formula for finding the common ratio: $\frac{a_2}{a_1}$
3. Geometric mean = n^{th} root of the product of 'n' terms in the GP.
4. Formula to find the geometric mean between two quantities a and b = $\sqrt{ab}$
5. Formula to find the sum of the number of terms in a GP:
6. Let 'a' be the first term, 'r' be the common ratio and 'n' be the number of terms
    - if r>1, then: $S_n = a \times \frac{r^n-1}{r-1}$
    - if r<1 , then: $S_n = a \times \frac{1-r^n}{1-r}$
7. Sum of infinite terms in a GP(-1<r<1) = $\frac{a}{1-r}$
8. While solving for three unknown Term in an G.P whose sum or product is given should be assumed as $\left(\frac{a}{r}\right), a, ar$
9. If two non-zero numbers a and b are in GP, then there GM is: GM = $ab^{\frac{1}{2}}$

   If three non-zero numbers a,b and c are in GP, then there GM is: GM = $abc^{\frac{1}{3}}$

Properties:

- If 'a' is the first term, r is the common ratio of a finite G.P. consisting of m terms, then the nth term from the end will be = ar^{m-n}
- The n^{th} term from the end of the G.P. with the last term 'l' and common ratio r is $\frac{1}{r^{n-1}}$
- Reciprocal of all the term in G.P are also considered in the form of G.P.
- If a, b, c are in GP, then b^2 = ac

When all terms is GP raised to same power, the new series of geometric progression is form.

Solved Examples:

1. Find the sum of the following infinite G.P. : $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \ldots$

A. $\frac{1}{3}$ B. $\frac{2}{3}$

C. $\frac{1}{5}$ D. $\frac{1}{2}$

Solution:

$a = \frac{1}{3}$ $r = \frac{\frac{1}{9}}{\frac{1}{3}} = \frac{1}{3}$

Required Sum = $\frac{a}{(1-r)}$

$= \frac{\frac{1}{3}}{(1-\frac{1}{3})} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$

Correct Option: D

2. Find the G.M. between $\frac{4}{25}$ and $\frac{196}{25}$

A. $\frac{28}{5}$ B. $\frac{28}{25}$

C. $\frac{8}{25}$ D. $\frac{14}{25}$

Solution:

Geometric mean = $\sqrt{ab}$

GM = $\sqrt{\frac{4}{25} \times \frac{196}{25}} = \frac{28}{25}$

Correct Option: B

3. Find the number of terms in the series 1, 3, 9 , ⋯, 19683

A. 10 B. 8

C. 6 D. 7

Solution:

In the given series,

a = 1, r = 3, t_n = 19683

$19683 = 1 \times 3^{n-1}$

$3^{n-1} = 19683$

$3^{n-1} = 3^9$

n-1 = 9

n = 10

Correct Option: A

4. Find the number of terms in the series 5, 10, 20, ..., 320?

A. 5 B. 4

C 6 D. 7

Solution:

We know that,

$t_n = ar^{n-1}$

a = 5, r = 2

t_n = 320

$320 = 5 \times 2^{n-1}$

$64 = 2^{n-1}$

$2^6 = 2^{n-1}$

n-1 = 6

n = 7

Correct Option: D

Question 5:

The sum of three numbers is 14 and their product is 64. All the three numbers are in GP. Find all the three numbers when value of r is a whole number?

A. 3, 6, 4 B. 2, 4, 8

C. 2, 4, 6 D. 4, 4, 4

Solution:

The three numbers can be written as $\frac{a}{r}, a, ar$

Sum of the three numbers = $\frac{a}{r} + a + ar = 14$ ---(1)

Product of the three numbers = $\frac{a}{r} \times a \times ar = 64$

i.e., $a^3 = 64$

$\therefore$ a = 4 ---(2)

Put (2) in (1)

$a\left(\frac{1}{r} + 1 + r\right) = 14 \rightarrow 4\left(\frac{1}{r} + 1 + r\right) = 14$

$2(1+r+r^2) = 7r$

$2r^2+2r+2=7r$

$2r^2-5r+2=0$

r = 2 or r = $\frac{1}{2}$

$\because$ the value of r is a whole number, r $\neq \frac{1}{2}$

$\therefore$ r = 2

$\therefore$ the numbers are 2, 4, 8.

Correct Option: B

Harmonic Progression:

- Harmonic progression is the series when the reciprocal of the terms are in AP.
- For example,
  $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \ldots,$ are termed as a harmonic progression as a, a + d, a + 2d are in Arithmetic progression.
- First term of a HP is $\frac{1}{a}$

Formulae:

- The n^{th} term in HP is identified by,
  $t_n = \frac{1}{a + (n-1)d}$
- To solve any problem in harmonic progression, a series of AP should be formed first, and then the problem can be solved.
- For two terms 'a' and 'b',
  - Harmonic Mean = $\frac{2ab}{a+b}$
- For 3 terms:
  - Harmonic Mean = $\frac{3abc}{ab+bc+ac}$
- For n terms:
  - Harmonic Mean = $\frac{n}{\frac{1}{a_1+a_2+\cdots+a_n}}$

Where, n = Total number of numbers or terms, a_1, a_2,$\cdots..a_n$ = Individual terms or individual values

The relation between the means of AP, GP and HP i.e., AM, GM, and HM is:

AM > GM > HM & $GM^2 = AM \times HM$

Solved Examples:

1. If the 6th term of H.P. is 10 and the 11th term is 18. Find the 16th term.

A. 90 B. 110

C. 85 D. 100

Solution:

6^{th} term = a + 5d = $\frac{1}{10}$ ---(1)

11^{th} term = a + 10d = $\frac{1}{18}$ ---(2)

On solving equation 1 and 2 we get, d = $\frac{-2}{225}$

Put value of d in equation 1

a + 5d = $\frac{1}{10}$

a + 5($\frac{-2}{225}$) = $\frac{1}{10}$

∴ a = $\frac{13}{90}$

Now, 16^{th} term = a + 15d = $\frac{13}{90} + 15 \times \frac{-2}{225}$

= $\frac{13}{90} - \frac{30}{225} = \frac{1}{90}$

∴ 16^{th} term = 90

Correct Option: A

2. Find the Harmonic mean of 6, 12, 18

A. 10.12 B. 9.62

C. 9.81 D. 8.10

Solution:

We know that,

$$HM = \frac{3abc}{ab + bc + ac}$$

$$HM = \frac{3 \times 6 \times 12 \times 18}{6 \times 12 + 12 \times 18 + 6 \times 18}$$

$$HM = \frac{3 \times 6 \times 12 \times 18}{396} = \frac{108}{11} = 9.81$$

Correct Option C

3. Find the 15^{th} term in the series $\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{12}, \ldots$

A. 45 B. $\frac{1}{70}$

C. $\frac{1}{45}$ D. 70

Solution:

We know that,

$$t_n = \frac{1}{a + (n-1)d}$$

Convert the HP series in AP

We get 3, 6, 9, 12……

In the given series,

a = 3

d = 3 → (6 – 3)

a_n = a + (n-1)d

∴ 15^{th} term = a_{15} = a + 14d

∴ t_{15} = 3 + 14(3) = 3 + 42 = 45

∴ HP (t_{15}) = $\frac{1}{45}$

Correct Option: C

4. Aarti walked first one-third of the distance at a speed of 2 km/hr. The next one-third of the distance was covered by running at the speed of 3km/hr. The last one-third of the distance was covered by cycling at the speed of 6 km/hr. Find the average speed for the whole journey covered by Aarti?

A. 5 km/hr B. 6 km/hr

C. 4 km/hr D. 3 km/hr

Solution

According to the question, the distance covered is same in all the three cases.

∴ the average speed = HM of 2, 3, and 6

Average Speed = $\frac{3}{\frac{1}{2}+\frac{1}{3}+\frac{1}{6}}$

Average Speed = 3 km/hr

Correct option – D

Exercise – 1

1. The 3rd and 9th term of an arithmetic progression are -6 and 12 respectively. What is the 16th term?
   A. 36 B. 30
   C. 27 D. 33
   E. 40
2. Arithmetic mean of 8, 4, 12, x and 10 is 12, then the value of x is
   A. 20 B. 22
   C. 24 D. 26
   E. 28
3. What is the sum of the first 9 terms of an arithmetic progression if the first term is 8 and last term is 54?
   A. 219 B. 279
   C. 231 D. 137
   E. 281
4. The 7th and 12th term of an arithmetic progression are -14 and 6 respectively. What is the 16th term?
   A. 25 B. 29
   C. 22 D. 33
   E. 35
5. For an A.P. if $t_{25} - t_{20} = 55$, then d equals to:
   A. 11 B. –11
   C. 18 D. –23
   E. 28
6. For A.P. $T_{17} – T_7$ = ...?
   A. d B. 10d
   C. 26d D. 2d
   E. None of these
7. Which term of the A.P. 27, 24, 21, ... is the first negative term?
   A. 8th B. 9th
   C. 10th D. 11th
   E. 12th
8. 15th term of A.P., x – 8, x – 3, x + 2, ⋯ is
   A. x + 62 B. x + 72
   C. x + 82 D. x + 52
   E. None of these
9. If an A.P. has a = 1, $t_n = 21$ and $S_n = 462$, then value of n is:
   A. 20 B. 32
   C. 42 D. 40
   E. 24
10. The sum of first fifteen multiples of 4 is:
    A. 420 B. 480
    C. 450 D. 490
    E. 460
11. In an A.P., if d = -4, n = 27, $t_n = 4$, then a is
    A. 120 B. 108
    C. 152 D. 96
    E. 68
12. Which term of the A.P. 138, 132, ... is 0?
    A. 23 B. 32
    C. 22 D. 24
    E. 21
13. A sequence is called _____ if $p_{n+1} = p_n * r$.
    A. arithmetic progression
    B. geometric Progression
    C. harmonic Progression
    D. Both a and c
    E. Both b and c
14. What is the nth term of a G.P.?
    A. $t_n = a + (n-1)\ d$ B. $t_n = a + (n)\ d$
    C. $t_n = a \times r^{n-1}$ D. $t_n = a \times r^n$
    E. None of these
15. If first term of a G.P. is 40 and common ratio is 4. Find the 5th term.
    A. 10240 B. 40960
    C. 5120 D. 2560
    E. None of these
16. If a sequence is in the form 2×5^n then which of the following may be the sequence?
    A. Arithmetic progression
    B. Geometric Progression
    C. Harmonic Progression
    D. Both a and c
    E. Both b and c
17. The 3rd and 8th term of a GP are 1/4 and 256, respectively. Find the 2nd term.
    A. 4 B. 1
    C. 1/64 D. 1/16
    E. None of these
18. A piece of equipment cost a certain factory 12,00,000. If it depreciates in value, 15% the first year, 13.5% the next year, 12% the third year, and so on, what will be its value at the end of 10 years, all percentages applying to the original cost?
    A. Rs. 4,00,000 B. Rs. 2,10,000
    C. Rs. 1,00,000 D. Rs. 8,10,000

E. Rs. 13,10,000

19. What is the sum of the following series?
-65, -67, -69, ..., -101
A. –1457 B. –1577
C. –1567 D. –1664
E. –1557

20. What is the sum of all positive integers up to 2000, which are divisible by 5 and are not divisible by 2?
A. 1,00,000 B. 50,000
C. 1,50,000 D. 2,00,000
E. 80,000

21. The sum of the first two terms of an infinite geometric series is 48. Also, each term of the series is equal to the sum of all the terms that follow. Find the sum of the series
A. 48 B. 64
C. 72 D. 96
E. None of these

22. An equilateral triangle is drawn by joining the midpoints of the sides of another equilateral triangle. A third equilateral triangle is drawn inside the second one joining the midpoints of the sides of the second equilateral triangle, and the process continues infinitely. Find the sum of the areas of all the equilateral triangles, if the side of the largest equilateral triangle is 12 units.
A. $32/\sqrt{3}$ units B. $48/\sqrt{3}$ units
C. 48 units D. $48\sqrt{3}$ units
E. Cannot be determined

23. In an infinite geometric progression, each term is equal to 4 times the sum of the terms that follow. If the first term of the series is 5, find the product of first three terms of the series?
A. 1 B. $\sqrt{5}$
C. 125 D. 25
E. Cannot be determined

24. If log x, log y, log z are in A.P., then x, y, z are in
A. A.P. B. G.P.
C. H.P. D. None of these
E. More than one of the above

25. The sum of an infinite G.P. whose common ratio is positive and is numerically less than 1 is 32 and the sum of the first two terms is 30. What will be the third term?
A. 1/3 B. 4/3
C. 8/3 D. 2
E. 3/2

26. What will be the value of $3^{1/3}$, $3^{1/6}$, $3^{1/12}$, ⋯, to infinity.
A. 3^2 B. $3^{2/3}$
C. $3^{3/2}$ D. 3
E. 3^3

27. Determine the fourth term of the geometric progression, the sum of whose first term and third term is 100 and the sum of the second term and fourth term is 300.
A. 240 B. 250
C. 270 D. 90
E. 180

28. If X is the sum of the n terms of the series $3 + 1/3 + 1/12 + \cdots$, and Y is the sum of 2n terms of the series $3 + 1 + 1/3 + \cdots$, then find the value of Y/X.
A. 1/3 B. 2/3
C. 4/3 D. 3/4
E. 3/2

29. How many four-digit numbers are there that exhibit the property of having their digits arranged from left to right in either an Arithmetic or Geometric Progression?
A. 15 B. 21
C. 20 D. 23
E. 25

30. A square has a side of 80 cm. Another square is formed by joining the mid-points of the sides of the given square and this process is repeated infinitely. Find the perimeter of all the squares thus formed.
A. $320 (1 + \sqrt{2})$ B. $320 (2 + \sqrt{2})$
C. $320 (2 - \sqrt{2})$ D. $320 (1 - \sqrt{2})$
E. Cannot be determined

31. Sejal started working at Crack Every Test on 8th May. After work she eats half a vada pav on her first day on her way home and doubles the amount every day. What number of vada pav does she have on 20th May?
A. 4096 B. 512
C. 1024 D. 2048
E. Cannot be determined

Exercise - 2

1. What is the next number in the series
   3, 8, 15, 24, 35, _?
   A.42 B.45
   C.48 D.51
   E. None of these
2. What is the next number in the series
   1, 3, 7, 15, 31, _?
   A.51 B.63
   C.79 D.95
   E.68
3. What is the next number in the series
   3, 10, 21, 36, 55, 78, _?
   A.105 B.115
   C.126 D.139
   E.112
4. What is the next number in the series
   1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, _?
   A.99 B.110
   C.121 D.132
   E. none of these
5. Find the missing number in the given series
   15, 26, 48, 92, 180, ?.
   A.305 B.356
   C.326 D.339
   E.312
6. Determine the missing numbers in the series
   7, 8, 11, 16, 23, ?.
   A.32 B.58
   C.48 D.46
   E.34
7. Find the missing number 36 : 50 :: 49 : ?
   A.82 B.65
   C.56 D.45
   E.64
8. 13 : 169 :: 15 : ___. Find the missing value.
   A.279 B.189
   C.256 D.361
   E.225
9. Look at this series- 4, 2, 1, (1/2), ... What number should come next?
   A.1/12 B.1/8
   C.1/4 D.1/3
   E. None of these
10. Look at this series- 8, 11, 9, 12, 10, 13, ... What number should come next?
    A.11 B.18
    C.13 D.14
    E.22
11. Next number in the given series is
    7, 7, 8, 6, 9, ?
    A. 6 B. 8
    C. 9 D. 5
    E. 7
12. What will come in place of the question mark (?) in the following number series
    656, ?, 590, 567, 548, 531
    A. 634 B. 619
    C. 629 D. 623
    E. 625
13. Choose the correct alternative that will continue the same pattern and replace 'x' in series
    5, 10, 25, 50, 125, 250, x, 1250
    A. 950 B. 700
    C. 715 D. 625
    E. 655
14. Find out the wrong term in the series
    1, 2, 6, 24, 88, 445, 2676
    A. 2 B. 2676
    C. 1 D. 24
    E. 88
15. Find the missing number in series
    1 , 4 , 27 , ? , 3125
    A. 225 B. 289
    C. 256 D. 324
    E. 512
16. Choose the correct alternative that will continue the same pattern and replace 'x' in the series 7874, 7087 , 6379 , 5742 , x
    A. 5027 B. 5168
    C. 5256 D. 5374
    E. 5582
17. Choose the correct alternative that will continue the same pattern and replace 'x' in series
    5.5, 5.55, 5.65, 5.8, x
    A. 5.95 B. 6
    C. 6.5 D. 7.2
    E. 5.9

18. The missing number in the series
9 , 81 , 729 , ? , 59049
A. 4595 B. 6561
C. 5481 D. 8325
E. 8584

19. Find the missing number in the series
54, 2, 27, 69, 3, 23, 76, 4, 19, ⋯ , 5, 15.6
A. 66 B. 68
C. 78 D. 79
E. 63

20. Choose the correct alternative that will continue the same pattern and replace 'x' in the series
51, 35, 55, 37, 59, x, 63, 41, 67, 43
A. 44 B. 62
C. 39 D. 36
E. 37

21. Find out the wrong term in the series
7, 16, 43, 124, 369
A. 43 B. 369
C. 7 D. 16
E. 124

22. Find out the wrong term in the series
18 , 21 , 23 , 32 , 48 , 73, 109
A. 18 B. 32
C. 21 D. 73
E. 109

23. Find out the wrong term in the series 99, 100, 96, 105 ,88, 114, 78
A. 96 B. 100
C. 99 D. 88
E. 114

24. Find out the wrong term in the series 75, 59, 51, 47, 45, 44.5, 43.5
A. 43.5 B. 75
C. 59 D. 45
E. 44.5

25. Find the missing number in the sequence
2, 6, 12, 20, 30, ? , 56, 72
A.37 B.42
C.45 D.39
E.47

26. Find the missing number in the following series 1, 2, 3, ? ,99, 9802
A.15 B.10
C.40 D.45
E. None

27. Find the missing number in the following series 7, 13, 24, 45, ?,
A.72 B.77
C.86 D.91
E. 83

28. Find out the wrong term in the series:
2, 3, 4, 4, 6, 8, 8, 11, 16
A.6 B.4
C.16 D.11
E.3

29. Find the missing number in the following series
1, 2, 8, 33, ?, 765
A. 263 B. 148
C. 187 D. 211
E. 184

30. What is the missing number of the following sequence?
21, 77, ?, 285, 437
A. 165 B. 155
C. 189 D. 210
E. 179

31. Find the next number in the given sequence
13, 19, 41, 87, ...
A. 155 B. 165
C. 159 D. 140
E. 149

32. Find the missing term in the following series? 10000, 11000, 9900, ? , 9801, 10781
A. 19543 B. 11573
C. 10994 D. 12056
E. 10890

33. Find the missing number in the given number series
725, 725, 700, ?, 575, 975
A. 600 B. 550
C. 700 D. 800
E. 750

34. Which fraction comes next in the sequence
$\frac{1}{3}, \frac{3}{9}, \frac{5}{27}, \frac{7}{81}, ?$
A. $\frac{1}{343}$ B. $\frac{9}{245}$
C. $\frac{1}{81}$ D. $\frac{9}{241}$
E. $\frac{1}{27}$

35. Find the missing number in the given series
13, 45, 198, 980, ?, 46452
A. 7445 B. 4724

C. 8345 D. 5910
E. 7651

36. Find the next number in the series
3, 6, 12, 15, 45, 48, ...
A. 141 B. 158
C. 152 D. 192
E. 187

37. What will come in the place of the (?) in the following number series
5, 14, 45, ?, 397, 1182
A. 214 B. 218
C. 122 D. 138
E. 130

38. 5, 9, 24, 13, 17, 19, 21, __,14
What number should fill the blank?
A. 19 B. 22
C. 25 D. 29
E. 17

39. What is the next number in series
10, 4, 60, 80, 70, 66, ?
A. 12 B. 9
C. 50 D. 86
E. 56

40. Which digits will come in the place of '?' in the following series
354, 832, 446, 285, 64?, 7?5
A. 6, 5 B. 3, 9
C. 5, 7 D. 8, 2
E. 5, 6

41. Complete the given number series
6, 11, 19.5, 33.25, ?
A. 64.573 B. 63.165
C. 52.624 D. 65.632
E. 54.875

42. Find out the wrong term in the series
1, 5, 9, 17, 25, 36, 49
A. 49 B. 25
C. 5 D. 9
E. 36

43. Find the next number in the given sequence
1, 2, 6, 21, 88, 445, ?
A. 2943 B. 2034
C. 2676 D. 1784
E. 1755

44. Which number replaces the '?' in the given series
107, 126, 2394, 2411, 40987, ?, 615030
A. 49548 B. 41002
C. 46379 D. 51248
E. 51387

45. Find out the wrong term in the series
1, 3, 4, 6, 9, 11, 16, 19, 25, 27, 36
A. 11 B. 27
C. 19 D. 36
E. 6

46. Find the next number in the given number series
2, 5, 16, 65, 326, ?
A. 1957 B. 2756
C. 1975 D. 1636
E. 2826

47. Find the next number in the given number series
20, 11, 13, 22.5, ?
A. 60 B. 52
C. 49 D. 54
E. 40

48. Complete the next number in the given number series
23, 58, 104, 162, 233, 318, ?
A. 427 B. 420
C. 492 D. 418
E. 473

49. Find the missing number in the given number series
2, 11, 48, ? , 1460, 10221
A. 342 B. 243
C. 247 D. 356
E. 276

50. Find the odd man out of the series
988, 490, 241, 116.5, 54.25, 22.125
A. 988 B. 490
C. 54.25 D. 241
E. 22.125

ANSWER KEY

Exercise – 1				
1) D	8) A	15) A	22) D	29) D
2) D	9) C	16) B	23) A	30) B
3) B	10) B	17) D	24) B	31) D
4) C	11) D	18) B	25) E	
5) A	12) D	19) B	26) B	
6) B	13) B	20) D	27) A	
7) D	14) C	21) B	28) D	
Exercise -2				
1) C	11) D	21) B	31) B	41) E
2) B	12) B	22) C	32) E	42) E
3) A	13) D	23) D	33) D	43) C
4) E	14) D	24) E	34) E	44) B
5) B	15) C	25) B	35) D	45) C
6) A	16) B	26) B	36) D	46) A
7) B	17) B	27) C	37) E	47) C
8) E	18) B	28) D	38) C	48) D
9) C	19) C	29) B	39) D	49) B
10) A	20) C	30) A	40) A	50) E

LOGARITHMS

Formulae:

- **Definition & Logarithm Formulas:**
  Logarithms are the power to which a number is raised to achieve some other number.
- **Logarithms is of 2 types:-**
  - Common logarithm
  - Natural logarithm.
- **Common Logarithm-**
  Logarithm with **base 10** is Common logarithm.
  It is expressed as $\log_{10} X$, and if any expression is not given with the base, then the **base 10** is considered.
- **Natural Logarithm-**
  Logarithm with **base e** is Natural Logarithm.
  It is expressed as $\log_e X$.
- **Very Important:** If the base is not provided, then always remember to consider base as 10.
  - $\log_a 1 = 0$
  - $\log_a a = 1$
  - $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$
  - $\log_a (x \times y) = \log_a x + \log_a y$
  - $\log_a x^n = n \times \log_a x$
  - $\log_a \sqrt[n]{x} = \frac{1}{n} \times \log_a x$
  - $\log_{b^n} a^m = \frac{m}{n} \times \log_b a$
  - $\log_a c = \frac{1}{\log_c a}$
  - $\log_a x = \frac{\log_c x}{\log_c a}$
  - $a^{\log_a x} = x$
  - $y = \log_a x \rightarrow x = a^y$

Log Values:

- log 2 = 0.301
- log 3 = 0.477= 0.48
- log 4 = 0.60
- log 5 = 0.698 = 0.7
- log 6 = 0.778 = 0.78
- log 7 = 0.845 = 0.85
- log 8 = 0.90
- log 9 = 0.954= 0.96
- log 10 = 1

Solved examples

Question 1:

If log 27= 1.431, then the value of log 9 is?

A. 0.945 B. 0.934

C. 0.958 D. 0.954

Solution:

log 27 = 1.431

$\log(3)^3 = 1.431$

3log(3) = 1.431

log(3) = 0.477

$\therefore \log(9) = \log(3)^2 = 2\log(3) = 2 \times 0.477 = 0.954$

Correct option: D

Question 2:

If $\log\frac{a}{b} + \log\frac{b}{a} = \log(a+b)$ then,

A. a – b = 1 B. a = b

C. a + b = 1 D. $a^2 - b^2 = 1$

Solution:

$$\log\frac{a}{b} + \log\frac{b}{a} = \log(a+b)$$

$$\log(a+b) = \log\left(\frac{a}{b} \times \frac{b}{a}\right) = \log(1)$$

So, a + b = 1

Correct option: C

Question 3:

Solve the equation log x= 1- log(x-3)

A. 2 B. $\frac{1}{2}$

C. 5 D. 4

Solution:

By combining both the equation we get

logx + log (x – 3) = 1

$\log_{10}(x(x-3)) = 1$

Now convert it into exponential form,

$x(x-3) = 10^1$

$x^2 - 3x - 10 = 0$

(x – 5)(x + 2) = 0

x = -2, x = 5

By solving this equation we get two values for x.

x = -2, x = 5

Put the different value of x in different equation and solve them,

x = -2

log(-2) = 1 – log (-2 – 3)

x = 5

log5 = 1 – log(5-3)

log5 = 1 – log(2)

Negative value is not considered in logarithm. So, we have a single value of x i.e., x = 5.

Correct option: C

Question 4:

$$\log_9(3\log_2(1+\log_3(1+2\log_2 x))) = \frac{1}{2}$$

A. 2 B. $\frac{1}{2}$

C. 1 D. 4

Solution:

$$\log_9(3\log_2(1+\log_3(1+2\log_2 x))) = \frac{1}{2}$$

$$3\log_2(1+\log_3(1+2\log_2 x)) = 9^{\frac{1}{2}} = 3$$

$$\log_2(1+\log_3(1+2\log_2 x)) = 1$$

$$1+\log_3(1+2\log_2 x) = 2^1$$

$$\log_3(1+2\log_2 x) = 2^1 - 1 = 1$$

$$1+2\log_2 x = 3^1 = 3$$

$$2\log_2 x = 2$$

$$\log_2 x = 1$$

$$x = 2$$

Correct option: A

Question 5:

If $\log_{10}5 + \log_{10}(5x+1) = \log_{10}(x+5) + 1$, find value of x.

A. 3 B. 1

C. 10 D. 5

Solution:

$$\log_{10}5 + \log_{10}(5x+1) = \log_{10}(x+5) + 1$$

$$\log_{10}5 + \log_{10}(5x+1) = \log_{10}(x+5) + \log_{10}10$$

$$\log_{10}(5(5x+1)) = \log_{10}(10(x+5))$$

$$5(5x+1) = (10(x+5))$$

$$(5x+1) = 2x+10$$

$$3x = 9$$

$$x = 3$$

Correct option: A

Question 6:

Which of the following statement is not correct?

A. log (1 + 2 + 3) = log 1 + log 2 + log 3

B. log(2+3) = log (2×3)

C. $\log_{10} 1 = 0$

D. $\log_{10} 10 = 1$

Solution:

A. log (1+2+3) = log6 = log (1×2×3) = log1+ log2+log3

B. log (2 + 3) = log5 and log (2 x 3) = log6 = log2 + log3

log (2 + 3) ≠ log (2 x 3)

C. Since, $\log_a 1 = 0$,

so $\log_{10} 1 = 0$

D. Since, $\log_a a = 1$

so, $\log_{10} 10 = 1$.

Correct option: B

Question 7:

If X is an integer then solve:

$(\log_2 x)^2 - \log_2 x^4 - 32 = 0$

A. 256 B. 125

C. 375 D. None of these

Solution:

$(\log_2 x)^2 - \log_2 x^4 - 32 = 0$ ...(1)

Let $\log_2 x = y$

$\therefore$ equation (1) = $y^2 - 4y - 32 = 0$

$y^2 - 8y + 4y - 32 = 0$

$y(y - 8) + 4(y - 8) = 0$

$(y + 4)(y - 8) = 0$

$\therefore y = 8$ or $y = -4$

$\log_2 x = 8$ or $\log_2 x = -4$

$x = 2^8 = 256$ or $2^{-4} = \frac{1}{16}$

Since, x is an integer so $x = 256$

Correct option: A

Exercise – 1

1. If $\log_{32} x = 0.6$, then x is equal to
   A. 10 B. 12.80
   C. 8 D. 25.6
   E. None of these
2. log 720 is equal to
   A. 2 log 2 + 3 log 3
   B. 4 log 2 + 2 log 3
   C. 4 log 2 + 2 log 3 + log 5
   D. 3 log 2 + 2 log 3 – log 5
   E. None of these
3. $(\log_6 4) \times (\log_4 1296)$ equals
   A. 2 B. 3
   C. 4 D. 5
   E. 6
4. If $\log_{10} 2 = 0.3010$, then $\log_2 10$ is equal to
   A. 0.3010 B. 0.6990
   C. $\frac{699}{301}$ D. $\frac{1000}{301}$
   E. None of these
5. If log 2 = 0.30103, the number of digits in 2^{64} is
   A. 17 B. 18
   C. 19 D. 20
   E. 25
6. Which of the following statements is not correct?
   A. $\log_{10} 10 = 1$
   B. log (2 + 3) = log (2 x 3)
   C. $\log_{10} 1 = 0$
   D. log (1 + 2 + 3) = log 1 + log 2 + log 3
   E. None of these
7. Provided $\log_{27} x + \log_3 x = 4$, then x is equal to:
   A.14 B. 15
   C. 27 D. 35
   E. None of these
8. What is the value of $\log_2 (\log_8 512)$ is:
   A. $\log_2 5$ B. 3
   C. $\log_2 3$ D. 5
   E. None of these
9. What is the value of $\log_6 \left(\frac{1}{216}\right)$ is:
   A. 2 B. 3
   C. –2 D. –3
   E. None of these
10. The value of $\log_3 243$ is?
    A. 2 B. 3
    C. 5 D. 7
    E. None of these
11. The value of log 2 is 0.3010, then the value of log 2^7 is:
    A. 1 B. 1.8
    C. 2 D. 2.4
    E. 2.1
12. Provided $\log_6 36 + \log_6 6 = x$, then what is the value of x?
    A. 3 B. 5
    C. 7 D. 8
    E. 10
13. If $\log_{2401} x = \left(-\frac{1}{4}\right)$, then x is equal to:
    A. 7^4 B. -7^4
    C. $1/7^4$ D. -1/7
    E. 1/7
14. Provided $\log_{10} 2 = 0.3010$, then the value of $\log_{10} 25$ is:
    A. 1.398 B. 0.301
    C. 0.699 D. 2.301
    E. None of these
15. If $\log_4 x = -2$. then, x is equal to:
    A. $\frac{1}{4}$ B. $\frac{1}{16}$
    C. 4 D. 16
    E. None of these
16. What is the value of $25^{\log_5 4}$?
    A. 12 B. 16
    C. 18 D. 24
    E. 21
17. What is the value of $\log_4 5.\log_{25} 64$?
    A. 1/2 B. 2
    C. 3/2 D. 3
    E. 5/2
18. What is the value of $\log_{25} 125 - \log_{125} 25$?
    A. 1 B. $\frac{1}{2}$
    C. $\frac{3}{2}$ D. $\frac{2}{3}$
    E. $\frac{5}{6}$
19. What is the value of $[\log_{10} (3.33 \log_{10} 1000)]^2$?
    A. 5 B. 4
    C. 3 D. 1
    E. 2
20. What is the value of $1/2 \log_{10} 36 - 2\log_{10} 3 + \log_{10} 15$?
    A. 2 B. 3
    C. 1 D. 0
    E. 4

21. What is the logarithm of 0.00001 with respect to base 10?
    A. 5 B. –5
    C. 3 D. –3
    E. –4
22. What is the value of $\{\log_{14}(10)\} / \{\log_{196}(10)\}$?
    A. $\frac{1}{2}$ B. 2
    C. 1 D. $\log_{10} 14$
    E. 3
23. What is the value of $\log_{100} 0.001$?
    A. $-\frac{3}{2}$ B. $-\frac{1}{2}$
    C. 2 D. –2
    E. 3
24. What is the value of $2log\left(\frac{6}{16}\right) + log\left(\frac{512}{216}\right) + log\left(\frac{6}{2}\right)$
    A. 0 B. 1
    C. 2 D. 5
    E. 3
25. If $9^{x+1} - 9^{x-1} = 80$, find x
    A. 0 B. 1
    C. 2 D. 5
    E. 3
26. Calculate: $\log_2\left(\frac{2}{3}\right) + \log_4\left(\frac{9}{4}\right)$
    A. 0 B. 1
    C. 2 D. 5
    E. 3
27. Find the value of the expression $1/\log_3 2 + 2/\log_9 4 - 3/\log_{27} 8$
    A. –1 B. 1
    C. 2 D. –2
    E. None of these
28. log 56700 =?
    A. log 5.67 + 4 B. log 5.67 + 2
    C. 2 log 567 D. 100 x log 567
    E. None of these
29. log 0.0487 =?
    A. log 4.87 + 2 B. log 4.87 – 2
    C. $\log\left(\frac{487}{1000}\right)$ D. –2 log 4.87
    E. None of these
30. If $\log_{10} 2 = 0.301$, find $\log_{10} 250$
    A. 2.097 B. 2.398
    C. 2.490 D. 2.087
    E. None of these

ANSWER KEY:

1) C	7) C	13) E	19) D	25) B
2) C	8) C	14) A	20) C	26) A
3) C	9) D	15) B	21) B	27) E
4) D	10) C	16) B	22) B	28) A
5) D	11) E	17) C	23) A	29) B
6) B	12) A	18) E	24) A	30) B

GEOMETRY

Point:

The point should be visualised as a singular dot.

A point can be defined as a single dot that can be created on plain paper by a very sharp pencil.

It could also be visualised as a singular prick on a piece of paper by a very sharp nail or pin.

Line:

Mathematically, lines are defined as a group of points which are straight one after another.

All lines are supposed to extend infinitely in two directions.

Segment of a Line:

If a part of a line is cut out, we get a segment of a line.

Physically, the closest representations of a segment of a line would be a tight thread or the straight

crease of a piece of paper.

Plane:

The surface of a smooth wall or a tabletop is the closest representation of a portion of a plane.

Lines Between Points:

infinite number of lines can be drawn in the plane passing through any one of these two points. However, only one line can be drawn which passes through both the points.

Collinearity of Points:

Collinear or non-collinear points are only defined in the context of three or more points.

There can only be two cases with respect to three points:

1. All the points lie on the same line. (Here, the points are said to be collinear).

2. All the three points do not lie on the same line (In this case the points are said to be noncollinear).

Points in Common between Distinct Lines:

In the case of distinct lines, there can only be two cases:

A. There is one point in common: In such a case, the common point is called the point of intersection, and the two lines are called as intersecting lines.

B. There is no point in common: In such a case, the two lines are non-intersecting and are also called as parallel lines. Two distinct lines in a plane cannot have more than one point in common.

Lines:

Two intersecting lines cannot be parallel to the same line.

Multiple lines:

1. No two lines intersect each other
2. Some lines intersect
3. Every line intersects all other lines, but their respective points of intersection are different from each other.
4. Every pair of lines is intersecting, and all the points of intersection coincide.

Basic Unit Conversions

Prefix	Symbol	Magnitude
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}
deka	da	10^{1}
unit	-	1
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}

Bigger Unit to Smaller Unit → Multiply

Smaller Unit to Bigger Unit → Divide

Basic Conversions

A. 1 m = 100 cm = 1000 mm
1 km = 1000 m
= 5/8 miles
1 inch = 2.54 cm

B. 1 m = 39.37 inches
1 mile = 1760 yd
= 5280 ft
1 nautical mile (knot)
= 6080 ft

C. 100 kg = 1 quintal
10 quintal = 1 tonne
= 1000 kg
1 kg = 2.2 pounds (approx.)

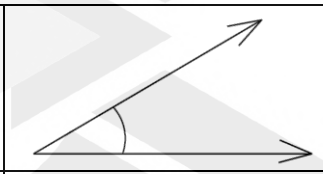
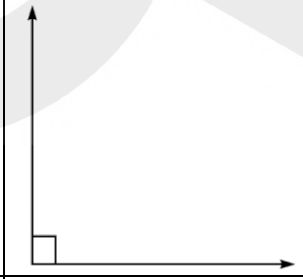
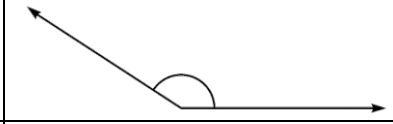
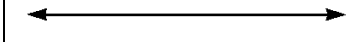
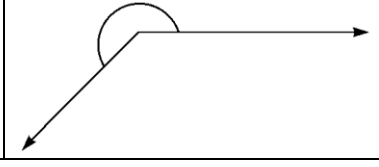
D. 1 litre = 1000 cc
1 acre = 100 sq m

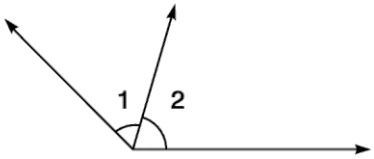
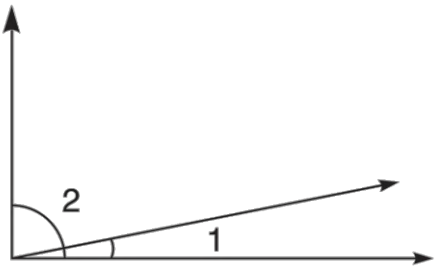
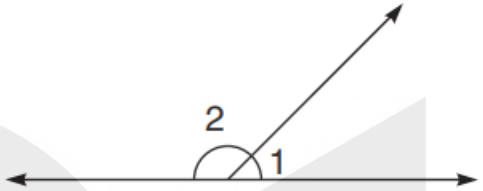
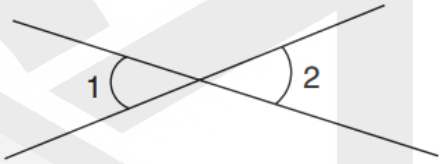
1 hectare = 10000 sq m

METRIC TABLE OF LENGTH		
10 millimetres	=	1 centimetre
10 centimetres	=	1 decimetre
10 decimetres	=	1 metre
10 metres	=	1 decametre
10 decametres	=	1 hectometre
10 hectometres	=	1 kilometre

METRIC TABLE OF WEIGHT		
10 milligrams	=	1 centigram
10 centigrams	=	1 decigram
10 decigrams	=	1 gram
10 grams	=	1 decagram
10 decagrams	=	1 hectogram
10 hectograms	=	1 kilogram

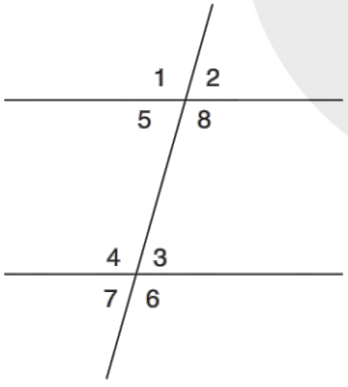
Types of Angles:

Acute Angles	An angle whose measures less than 90°.	
Right angle	An angle whose measure is 90°.	
Obtuse Angle	An angle whose measure is bigger than 90° but less than 180°. Thus, it is between 90° and 180°.	
Straight Angle	Is an angle whose measure is 180°.	
Reflex Angle	An angle whose measure is more than 180° but less than 360°.	

Adjacent Angles	Angles with a common vertex and one common side. In the figure, ∠1 and ∠2 are adjacent angles.	
Complementary Angles:	Two angles whose measures add to 90°	
Supplementary Angles:	Two angles whose measures add up to 180°.	
Vertical Angles	Angles that have a common vertex and whose sides are formed by the same lines. Vertically Opposite angles are equal.	

Angles formed when two parallel lines, are crossed by a transversal:

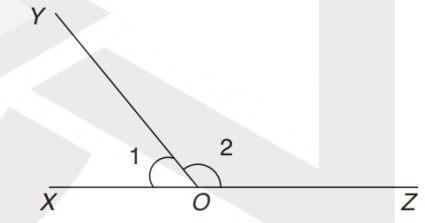
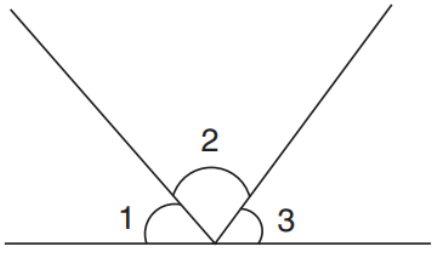
When two parallel lines are crossed by a third line, (transversal), 8 angles are formed.

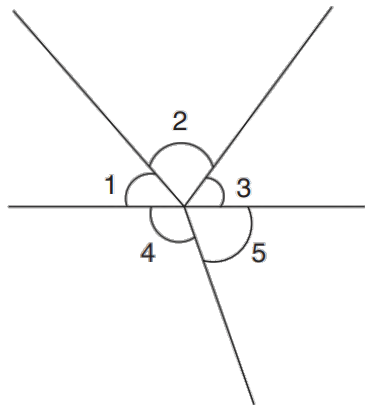
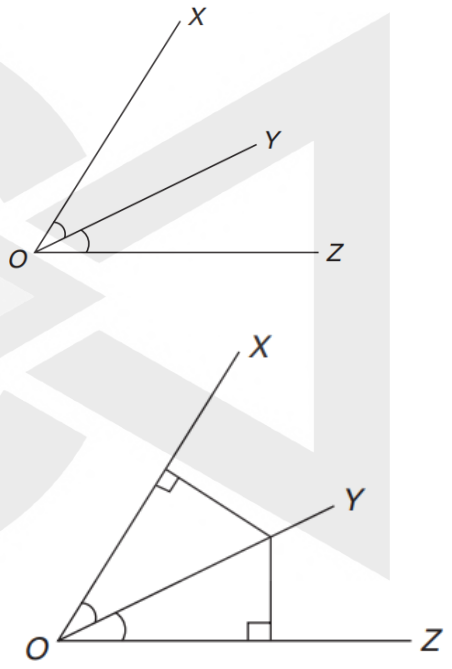


Angles 3,4,5,8 are interior angles.

Angles 1,2,6,7 are exterior angles.

Alternate Interior Angles	Pairs of interior angles on opposite sides of the transversal. Angle 3 and Angle 5 are alternate interior angles. Angle 4 and angle 8 are also alternate interior angles. Both the angles in a pair of alternate interior angles are equal. ∠3 = ∠5 ∠4 = ∠8

Alternate Exterior Angles	Pairs of exterior angles on opposite sides of the transversal. Angle 2 and angle 7 are alternate exterior angles. Angles 1 and 6 are also alternate exterior angles. Both the angles in a pair of alternate exterior angles are equal. $\angle 2 = \angle 7$ $\angle 1 = \angle 6$	
Co-interior Angles	When two lines are cut by a third line (transversal) co-interior angles are between the pair of lines on the same side of the transversal. If the lines that are being cut by the transversal are parallel to each other, the co-interior angles are supplementary (add up to 180°). Angles 3 and 8 are co-interior angles. → $\angle 3 + \angle 8 = 180°$ Angles 4 and 5 are also co-interior angles → $\angle 4 + \angle 5 = 180°$	
Corresponding angles	Pairs of angles that are in similar positions when two parallel lines are intersected by a transversal. Corresponding angles are equal. Angle 3 and angle 2 are corresponding angles. → $\angle 3 = \angle 2$ Similarly, Angles 1 and 4; → $\angle 1 = \angle 4$ Angles 5 and 7; → $\angle 5 = \angle 7$ Angles 6 and 8 → $\angle 6 = \angle 8$ are corresponding angles.	
Linear Pair	$\angle XOY$ and $\angle YOZ$ are linear pair angles. One side must be common (e.g., OY) and these two angles must be supplementary.	
Angles on the side of a line	$\angle 1 + \angle 2 + \angle 3 = 180°$	

Angles around the point	∠1 + ∠2 + ∠3 + ∠4 + ∠5 = 360°	
Angle Bisector:	OY is the angle bisector for the ∠XOZ. i.e., ∠XOY = ∠ZOY = ½ ∠XOZ When a line segment divides an angle equally into two parts, then it is said to be the angle bisector (OY). (Angle bisector is equidistant from the two sides of the angle.) The distance between the lines OX & OY and the lines OY & OZ are equal to each other.	

Polygons:

Polygons are plane figures formed by a closed series of rectilinear (straight) segments. The following are examples of polygons:

Number of Sides	Name of the polygon	Number of Sides	Name of the polygon
3	Triangle	17	Heptakaidecagon or Heptadecagon
4	Rectangle	18	Octakaidecagon or Octadecagon
5	Pentagon	19	Enneakaidecagon or Enneadecagon
6	Hexagon	20	Icosagon
7	Heptagon	30	Triacontagon
8	Octagon	40	Tetracontagon
9	Nonagon	50	Pentacontagon
10	Decagon	60	Hexacontagon
11	Undecagon	70	Heptacontagon
12	Dodecagon	80	Ontacontagon
13	Triskaidecagon or Tridecagon	90	Enneacontagon
14	Terakaidecagon or Tetradecagon	100	Hecatontagon or Hectogon
15	Pentakaidecagon or Pentadecagon	1000	Chiliagon
16	Hexakaidecagon, Hexadecagon	10000	Myriagon

Polygons can broadly be divided into two types:

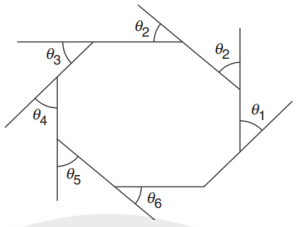
A. Regular polygons: Polygons with all the sides and angles equal.

B. Irregular polygons: Polygons in which all the sides or angles are not of the same measure.

No. of sides	Name of the polygon	Sum of all the angles	No. of sides	Name of the polygon	Sum of all the angles
3	Triangle	180°	7	Heptagon	500°
4	Rectangle	360°	8	Octagon	1080°
5	Pentagon	540°	9	Nonagon	1260°
6	Hexagon	720°	10	Decagon	1440°

Properties:

1. Sum of all the angles of a polygon with n sides
= $(2n - 4)\ \pi/2$ or $(n - 2)\pi^C$
= $(n - 2)\ 180°$
2. Sum of all exterior angles = 360°
i.e., $\theta_1 + \theta_2 + \ldots + \theta_6 = 360°$
In general, $\theta_1 + \theta_2 + \ldots + \theta_n = 360°$
3. No. of sides = 360°/exterior angle.
(Note: This property is true only for regular polygons)
4. Perimeter = n × s.
Where s = length of side, n = no. of sides
(Note: This property is true only for regular polygons)



Triangles:

A triangle is a polygon having three sides. Sum of all the angles of a triangle = 180°.

Types

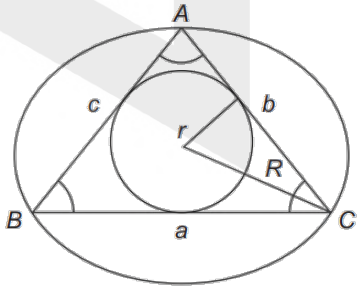
1. Acute angle triangle: Triangles with all three angles acute (less than 90°).
2. Obtuse angle triangle: Triangles with one of the angles obtuse (more than 90°).

Note: We cannot have more than one obtuse angle in a triangle.

3. Right angle triangle: Triangle with one of the angles equal to 90°.
4. Equilateral triangle: Triangle with all sides equal. All the angles in such a triangle measure 60°.
5. Isosceles triangle: Triangle with two of its sides equal and consequently the angles opposite the equal sides are also equal.
6. Scalene Triangle: Triangle with none of the sides equal to any other side.

Properties:

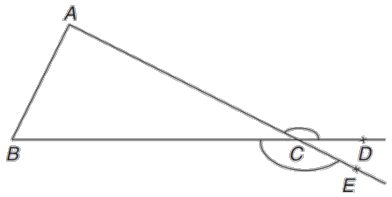
- Sum of the length of any two sides of a triangle has to be always greater than the third side.
- Difference between the lengths of any two sides of a triangle has to be always lesser than the third side.
- Side opposite to the greatest angle will be the greatest and the side opposite to the smallest angle the smallest.
- The sine rule: a/sin A = b/sin B = c/sin C = 2R (where R = circumradius.)
- The cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$ This is true for all sides and respective angles.



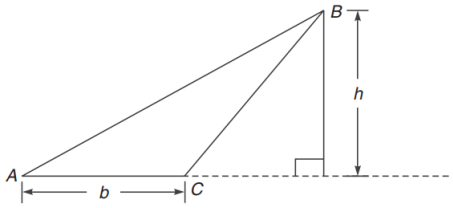
In case of a right triangle, the formula reduces to $a^2 = b^2 + c^2$

Since cos 90° = 0

- The exterior angle is equal to the sum of two interior angles not adjacent to it.
∠ACD = ∠BCE = ∠A + ∠B



Area



Area = 1/2 base × height or 1/2 bh.

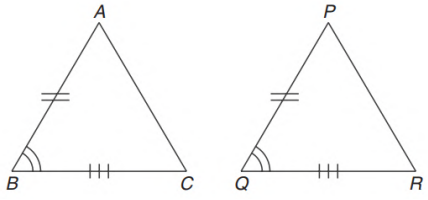
Height = Perpendicular distance between the base

Congruency of triangles

Two triangles are congruent if all the sides of one are equal to the corresponding sides of another. It follows that all the angles of one are equal to the corresponding angles of another. The notation for congruency is (≅).

Conditions of Congruency

1. **SAS Congruency:**
   If two sides and an included angle of one triangle are equal to two sides and an included angle of another, the two triangles are congruent.
   AB = PQ
   BC = QR
   ∠B = ∠Q
   ∴ △ ABC ≅ △ PQR



2. **ASA Congruency:**
   If two angles and the included side of one triangle is equal to two angles and the included side of another, the triangles are congruent.

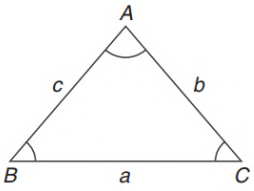
and vertex opposite to it

Area = $\sqrt{s(s-a)(s-b)(s-c)}$ (Heron's formula)

where s = $\frac{a+b+c}{2}$

(a, b and c being the length of the sides)

Area = rs (where r is in radius)



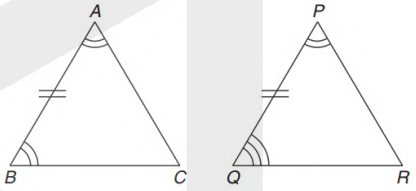
Area = 1/2 × product of two sides × sine of the included angle

= 1/2 ac sin B = 1/2 ab sin C

= 1/2 bc sin A

Area = abc/4R

where R = circumradius



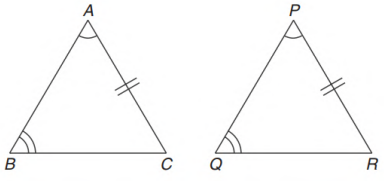
∠A = ∠P

∠B = ∠Q

AB = PQ

∴ △ ABC ≅ △ PQR

3. **AAS Congruency:**
   If two angles and side opposite to one of the angles is equal to the corresponding angles and the side of another triangle, the triangles are congruent.



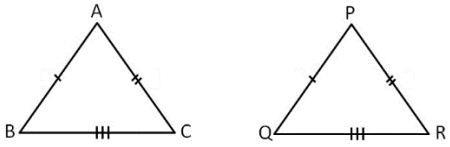
∠A = ∠P

∠B = ∠Q

AC = PR

∴ △ ABC ≅ △ PQR

4. **SSS Congruency:**
   If three sides of one triangle are equal to three sides of another triangle, the two triangles are congruent.



AB = PQ

AC = PR

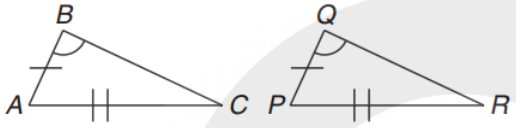
BC = QR

∴ △ ABC ≅ △ PQR

5. **SSA Congruency:**

If two sides and the angle opposite the greater side of one triangle are equal to the two sides and the angle opposite to the greater side of another triangle, then the triangles are congruent.

The congruency doesn't hold if the equal angles lie opposite the shorter side.



∠B = ∠Q

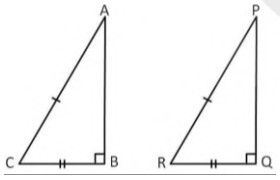
AB = PQ

AC = PR

∴ △ ABC ≅ △ PQR

6. **RHS Congruency**

Two right triangles are congruent if the hypotenuse and one side of one triangle are equal to the corresponding hypotenuse and one side of the other triangle.



∠B = ∠Q = 90°

BC = RQ

AC = PR

∴ △ ABC ≅ △ PQR

Similarity Of Triangles

Similarity of triangles is a special case where if either of the conditions of similarity of polygons holds, the other will hold automatically. The notation for similarity is (~).

Types Of Similarity

1. **AAA Similarity:**

If in two triangles, corresponding angles are equal, that is, the two triangles are equiangular then the triangles are similar.

2. **AA Similarity:**

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar. The reason being the third angle becomes equal automatically.

3. **SSS Similarity:**

If the corresponding sides of two triangles are proportional, then they are similar.

For △ ABC to be similar to △PQR,

AB/ PQ = BC/ QR = AC/ PR, must hold true.

4. **SAS Similarity:**

If in two triangles, one pair of corresponding sides are proportional, and the included angles are equal then the two triangles are similar.

△ ABC ~ △ PQR

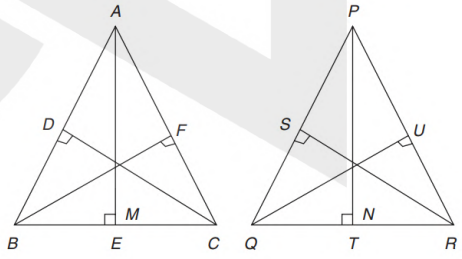
If AB/BC = PQ/QR and ∠B = ∠Q

In similar triangles, the following identity holds:

Ratio of medians = Ratio of heights = Ratio of circumradii = Ratio of inradii = Ratio of angle bisectors

Properties of similar triangles

If the two triangles are similar, then for the proportional/ corresponding sides we have the following results:



1. Ratio of sides
   = Ratio of heights (altitudes)
   = Ratio of medians
   = Ratio of angle bisectors
   = Ratio of inradii
   = Ratio of circumradii
2. Ratio of areas = Ratio of square of corresponding sides.
   i.e., if △ ABC ~ △ PQR, then

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{(AB)^2}{(PQ)^2} = \frac{(BC)^2}{(QR)^2} = \frac{(AC)^2}{(PR)^2}$$

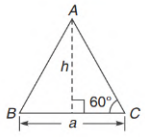
Equilateral Triangles:

1. sin 60 = 3 /2 = h/side

$\therefore h = \frac{a\sqrt{3}}{2}$

2. Area = (1/2) × base × height

$= (1/2) \times a \times \frac{a\sqrt{3}}{2} = \frac{\sqrt{3}}{4} \times a^2$



3. R (circumradius) = $\frac{2h}{3} = \frac{a}{\sqrt{3}}$
4. r (inradius) = $\frac{h}{3} = \frac{a}{2\sqrt{3}}$

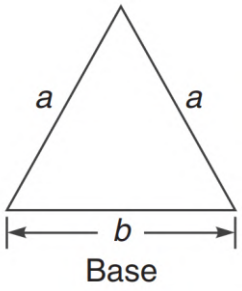
Properties:

1. The incentre and circumcentre lies at a point that divides the height in the ratio 2 : 1.
2. The circumradius is always twice the in radius. [R = 2r.]
3. Among all the triangles that can be formed with a given perimeter, the equilateral triangle will have the maximum area.
4. An equilateral triangle in a circle will have the maximum area compared to other triangles inside the same circle.

Isosceles Triangle

Area = $\frac{b}{4}\sqrt{4a^2 - b^2}$

In an isosceles triangle, the angles opposite to the equal sides are equal.



Right Angle Triangle

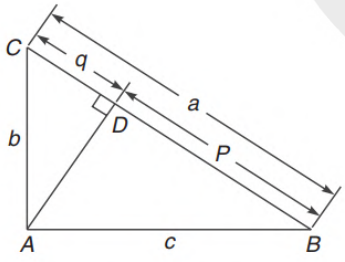
Pythagoras Theorem

In the case of a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

$\therefore$ In ΔABC,

$BC^2 = AB^2 + AC^2$

$\therefore a^2 = b^2 + c^2$



$AD^2 = CD^2 + DB^2$

R(circumradius) = $\frac{\text{hypotenuse}}{2}$

Area = rs

Where, r = inradius, s = $\frac{a+b+c}{2}$, (where a, b and c are sides of the triangle)

1/2 bc = r(a + b + c)/2

r = (bc)/(a + b + c)

Basic Pythagorean triplets:

3 – 4 – 5	11 – 60 – 61
5 – 12 – 13	12 – 35 – 37
7 – 24 – 25	16 – 63 – 65
8 – 15 – 17	20 – 21 – 29
9 – 40 – 41	28 – 45 – 53

Any triplet formed by either multiplying or dividing one of the basic triplets by any positive real number will be another Pythagorean triplet.

Thus, since 3, 4, 5 form a triplet so also will 6, 8 and 10 as also 3.3, 4.4 and 5.5.

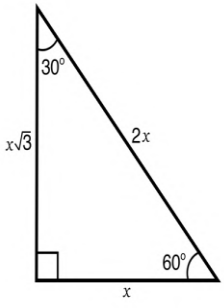
30°-60°-90° Triangle:

$= 1 : \sqrt{3} : 2$

Side Opposite 30° = x

Side Opposite 60° = $x\sqrt{3}$

Side Opposite 90° = $2x$



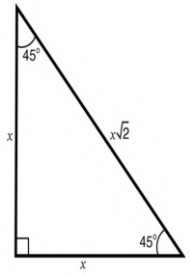
45°-45°-90° Triangle:

$= 1 : 1 : \sqrt{2}$

Side Opposite 45° = x

Side Opposite 45° = x

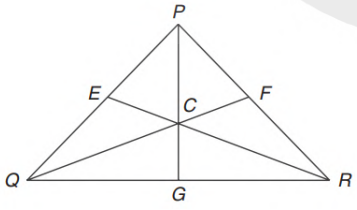
Side Opposite 90° = $\sqrt{2}$x



Terms Related to Triangle:

Median:

- A line joining the mid-point of a side of a triangle to the opposite vertex is called a median.
- In the figure the three medians are PG, QF and RE where G, E and F are mid-points of their respective sides.
- A median divides a triangle into two parts of equal area.
- The point where the three medians of a triangle meet is called the centroid of the triangle.
- The centroid of a triangle divides each median in the ratio 2 : 1.
  i.e. PC : CG = 2 : 1 = QC : CF = RC : CE



Formulae:

$2 \times (\text{median})^2 + 2 \times (1/2 \text{ the third side})^2$

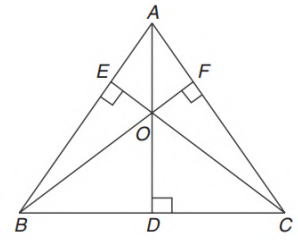
= Sum of the squares of other two sides

$= 2(PG)^2 + 2 \times (\frac{QR}{2})^2$

$= (PQ)^2 + (PR)^2$

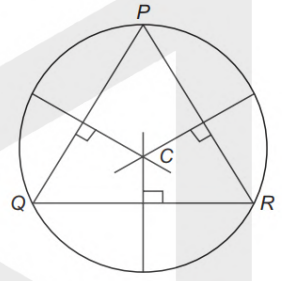
Altitude/ Height:

- A perpendicular drawn from any vertex to the opposite side is called the altitude. (In the figure, AD, BF and CE are the altitudes of the triangles).
- All the altitudes of a triangle meet at a point called the orthocentre of the triangle.
- The angle made by any side at the orthocentre and the vertical angle make a supplementary pair (i.e. they both add up to 180°).
- In the figure below:
  $\angle A + \angle BOC = 180° = \angle C + \angle AOB$



Perpendicular Bisectors:

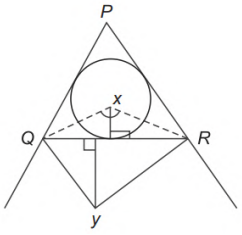
- A line that is a perpendicular to a side and bisects it is the perpendicular bisector of the side.



- The point at which the perpendicular bisectors of the sides meet is called the circumcentre of the triangle.
- The circumcentre is the centre of the circle that circumscribes the triangle.
- There can be only one such circle.
- Angle formed by any side at the circumcentre is two times the vertical angle opposite to the side.
- This is the property of the circle whereby angles formed by an arc at the centre are twice that of the angle formed by the same arc in the opposite arc.
  $\angle QCR = 2\angle QPR$
  (when we consider arc QR and it's opposite arc QPR)

Incentre

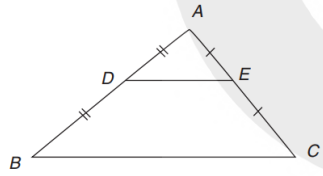
- The lines bisecting the interior angles of a triangle are the angle bisectors of that triangle.
- The angle bisectors meet at a point called the incentre of the triangle.
- The incentre is equidistant from all the sides of the triangle.



- From the incentre with a perpendicular drawn to any of the sides as the radius, a circle can be drawn touching all the three sides.
- This is called the incircle of the triangle.
- The radius of the incircle is known as inradius.
- The angle formed by any side at the incentre is always a right angle more than half the angle opposite to the side.
  $\angle QXR = 90 + \frac{1}{2}\angle P$
- If QY and RY are the angle bisectors of the exterior angles at Q and R, then:
  $\angle QYR = 90 - \frac{1}{2}\angle P$

Mid-Point theorem

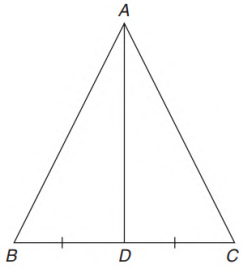
- The line segment joining the mid-points of two sides of a triangle is parallel to the third side and equal to half the third side.



- AD = BD and AE = CE
  DE || BC

Apollonius' Theorem

- "The sum of the squares of any two sides of any triangle equals twice the square on half the third side plus twice the square of the median bisecting the third side".
- In ΔABC, AD is the median:

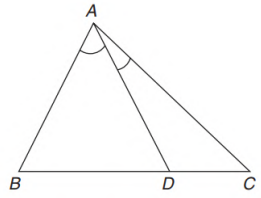


BD = CD

$AB^2 + AC^2 = 2(AD^2 + BD^2)$.

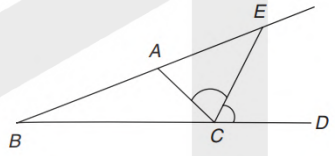
Angle Bisector Theorem

- In a triangle the angle bisector of an angle divides the opposite side to the angle in the ratio of the remaining two sides. i.e.,
  $\frac{BD}{CD} = \frac{AB}{AC}$ &
  $BD \times AC - CD \times AB = AD^2$



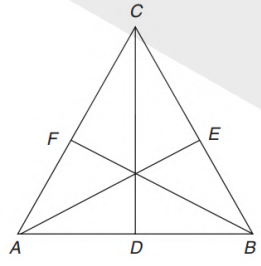
Exterior angle bisector theorem

- In a triangle the angle bisector (represented by CE in the figure) of any exterior angle of a triangle divides the side opposite to the external angle in the ratio of the remaining two sides i.e., $\frac{BE}{AE} = \frac{BC}{AC}$.

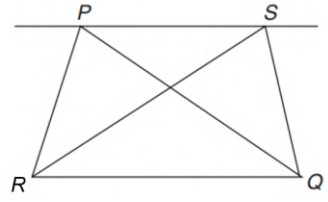


Important Results:

1. In a triangle AE, CD and BF are the medians then,
   $3(AB^2 + BC^2 + AC^2) = 4(CD^2 + BF^2 + AE^2)$



2. If the two triangles have the same base and lie between the same parallel lines, then the area of two triangles will be equal.

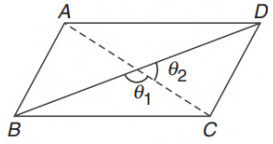


i.e., Area (ΔPQR) = Area(ΔPQS)

Quadrilaterals

Area:

1. Area = 1/2 (product of diagonals) × (sine of the angle between them)

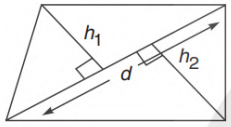


If θ_1 and θ_2 are the two angles made between themselves by the two diagonals, we have by the property of intersecting lines → $\theta_1 + \theta_2 = 180°$

Then, the area of the quadrilateral

$= ½ \times d_1 \times d_2 \times \sin\theta_1 = ½ \times d_1 \times d_2 \times \sin\theta_2$

2. Area = 1/2 × diagonal × sum of the perpendiculars to it from opposite vertices = $\frac{d(h_1 + h_2)}{2}$



3. Area of a circumscribed quadrilateral

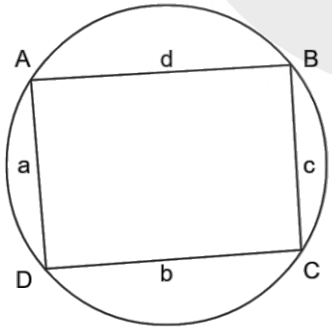
$A = \sqrt{(S-a)(S-b)(S-c)(S-d)}$

where, $S = \frac{a+b+c+d}{2}$

(where, a, b, c and d are the lengths of the sides.)

Properties:

1. In a quadrilateral inscribed in a circle, the product of the diagonals is equal to the sum of the products of the opposite sides.



$(a \times c) + (b \times d) = AC \times BD$

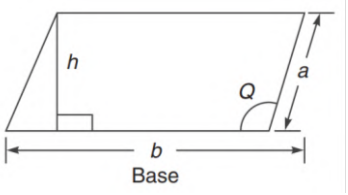
2. Sum of all the angles of a quadrilateral = 360°.

Types of Quadrilaterals:

1. **Parallelograms ($||^{gm}$)**

A parallelogram is a quadrilateral with opposite sides parallel.

- Area = Base (b) × Height (h) = bh



- Area = product of any two adjacent sides × sine of the included angle. = ab sin Q
- Perimeter = 2(a+b)

  Where a and b are any two adjacent sides.

Properties:

a. Diagonals of a parallelogram bisect each other.

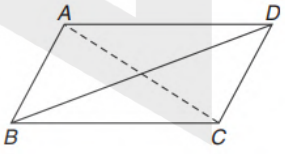
b. Bisectors of the angles of a parallelogram form a rectangle.

c. A parallelogram inscribed in a circle is a rectangle.

d. A parallelogram circumscribed about a circle is a rhombus.

e. The opposite angles in a parallelogram are equal.

f. The sum of the squares of the diagonals is equal to the sum of the squares of the four sides in the figure:

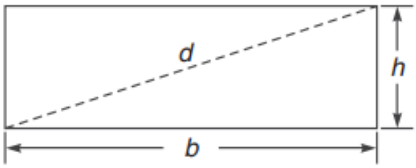


$AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + AD^2 = 2(AB^2 + BC^2)$

2. Rectangles

A rectangle is a parallelogram with all angles 90°

A. Area = Base × Height = b × h



Base and height are also referred to as the length and the breadth in a rectangle.

B. Diagonal D. = $\sqrt{b^2 + h^2}$

(By Pythagoras Theorem)

Properties of a rectangle

A. Diagonals are equal and bisect each other.

B. Bisectors of the angles of a rectangle (a parallelogram) form another rectangle.

C. All rectangles are parallelograms but the reverse is not true.

3. Rhombus

A parallelogram having all the sides equal is a rhombus.

(a) Area = 1/2 × product of diagonals × sine of the angle between them.
= 1/2 × d_1 × d_2 × sin 90°
(Diagonals in a rhombus intersect at right angles)
= 1/2 × d_1 × d_2 (since sin 90° = 1)

(b) Area = product of adjacent sides × sine of the angle between them.

Properties:

- Diagonals bisect each other at right angles.
- All rhombuses are parallelograms but the reverse is not true.
- A rhombus may or may not be a square but all squares are rhombuses.

4. Square

A square is a rectangle with adjacent sides equal or a rhombus with each angle 90°.

A. Area = base × height = a^2

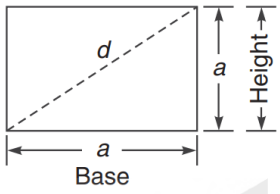
B. Area = 1/2 (diagonal)^2 = 1/2 d^2 (square is a rhombus too).

C. Perimeter = 4a

(a = side of the square)

D. Diagonal = $a\sqrt{2}$

(e) Inradius = $\frac{a}{2}$



Properties:

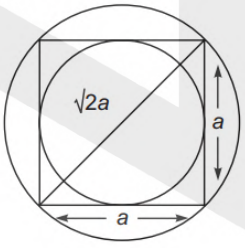
A. Diagonals are equal and bisect each other at right angles.

B. Side is the diameter of the inscribed circle.

C. Diagonal is the diameter of the circumscribing circle.

Diameter = $a\sqrt{2}$

Circumradius = $\frac{a}{\sqrt{2}}$

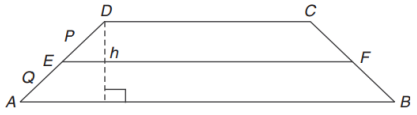


5. Trapezium:

A trapezium is a quadrilateral with only two sides parallel to each other.

(a) Area = 1/2 × sum of parallel sides × height = 1/2 (AB + DC) × h

(b) Median = 1/2 × sum of the parallel sides (median is the line equidistant from the parallel sides) For any line EF parallel to AB



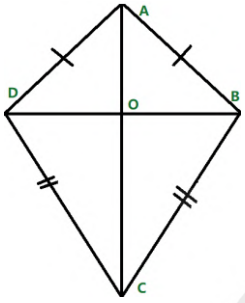
$$EF = \frac{\{[P \times AB] + [Q \times (DC)]\}}{AD}$$

Properties

If the non-parallel sides are equal then diagonals will be equal too.

6. **Kite**

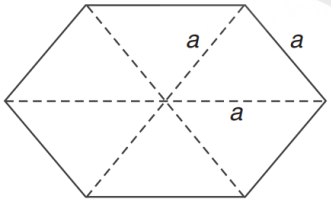
A Kite is a quadrilateral in which four sides can be grouped into two pairs of equal-length sides that are adjacent to each other and the diagonals intersect each other at right angles. The quadrilateral is a 4-sided polygon. The figure shown below represents a kite:



- Kite has 2 diagonals that intersect each other at right angles.
- A kite is symmetrical about its main diagonal.
- Angles opposite to the main diagonal are equal.
- The kite can be viewed as a pair of congruent triangles with a common base.
- The shorter diagonal divides the kite into 2 isosceles triangles.
- The area of the kite is 1/2 × d1 × d2

Regular Hexagon

a. Area = $\frac{3\sqrt{3}}{2} \times \text{side}^2 = \frac{3\sqrt{3}}{2} \times a^2$



b. A regular hexagon is actually a combination of 6 equilateral triangles all of side 'a'.
Hence, the area is also given by: 6 × Area of an equilateral triangle having the same side as the side of the hexagon = $6 \times \frac{\sqrt{3}}{4} \times a^2$

c. Circumradius (R) = a;
i.e., the side of the hexagon is equal to the circumradius.

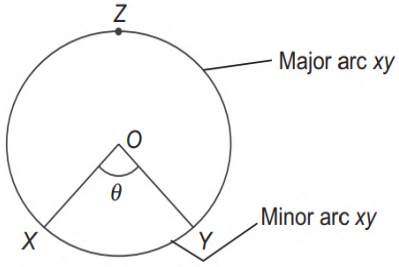
Circles

a. Circumference = $2\pi r$

b. Area = ½ × Circumference × r = πr^2

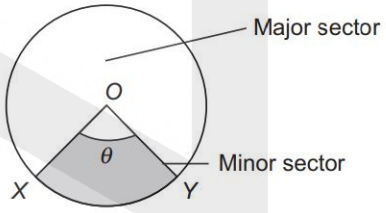
c. Arc: It is a part of the circumference of the circle. The bigger one is called the major arc and the smaller one the minor arc.

d. Length (Arc XY) = $\frac{\theta}{360} \times 2\pi r$



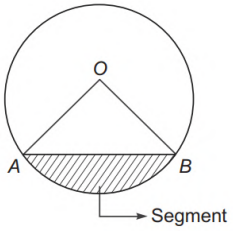
e. Sector of a circle is a part of the area of a circle between two radii.

f. Area of a sector = $\frac{\theta}{360} \times \pi r^2$,
(where θ is the angle between two radii)
= (½)r × length (arc xy)
(∵ πrθ/180 = length arc xy)
= $\frac{1}{2} \times r \times \frac{\pi r \theta}{360}$



g. Segment: A sector minus the triangle formed by the two radii is called the segment of the circle.

h. Area = Area of the sector – Area △OAB = $\frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times r^2 \sin\theta$



i. Perimeter of segment = length of the arc + length of segment AB
= $\frac{\theta}{360} \times 2\pi r + 2r \sin\left(\frac{\theta}{2}\right)$

j. Congruency: Two circles can be congruent if and only if they have equal radii.

Properties:

a. The perpendicular from the centre of a circle to a chord bisects the chord. The converse is also true.

b. The perpendicular bisectors of two chords of a circle intersect at its centre.

c. There can be one and only one circle passing through three or more non-collinear points.

d. If two circles intersect in two points then the line through the centres is the perpendicular bisector of the common chord.

e. If two chords of a circle are equal, then the centre of the circle lies on the perpendicular bisector of the two chords.

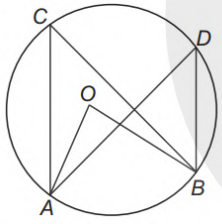
f. Equal chords of a circle or congruent circles are equidistant from the centre.

g. Equidistant chords from the centre of a circle are equal to each other in terms of their length.

h. The degree measure of an arc of a circle is twice the angle subtended by it at any point on the alternate segment of the circle.

With respect to the arc AB,

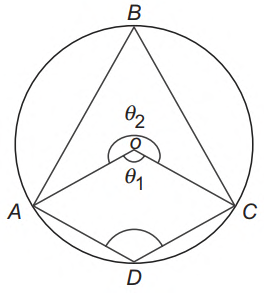
$\angle AOB = 2\angle ACB$



i. Any two angles in the same segment are equal. Thus, $\angle ACB = \angle ADB$.

j. The angle subtended by a semi-circle is a right angle. Conversely, the arc of a circle subtending a right angle at any point of the circle in its alternate segment is a semi-circle.

k. Any angle subtended by a minor arc in the alternate segment is acute, and any angle subtended by a major arc in the alternate segment is obtuse.



$\angle ABC$ is acute and $\angle ADC$ is obtuse.

Also:

$\theta_1 = 2\angle B$

$\theta_2 = 2\angle D$

$\theta_1 + \theta_2 = 2(\angle B + \angle D)$

or

$\angle B + \angle D = 180°$

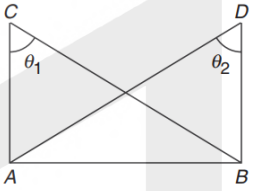
or

sum of opposite angles of a cyclic quadrilateral is 180°

l. If a line segment joining two points subtends equal angles at two other points lying on the same side of the line, the four points are concyclic.

If, $\theta_1 = \theta_2$

Then ABCD are concyclic, that is, they lie on the same circle.



m. Equal chords of a circle (or of congruent circles) subtend equal angles at the centre (at the corresponding centres.) The converse is also true.

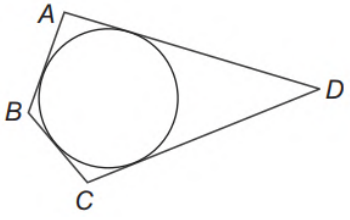
n. If the sum of the opposite angles of a quadrilateral is 180°, then the quadrilateral is cyclic.

Secant: A line that intersects a circle at two points.

Tangent: A line that touches a circle at exactly one point.

o. If a circle touches all the four sides of a quadrilateral, then the sum of the two opposite sides is equal to the sum of other two.

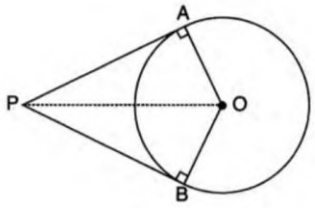
AB + DC = AD + BC



p. In two concentric circles, the chord of the larger circle that is tangent to the smaller circle is bisected at the point of contact.

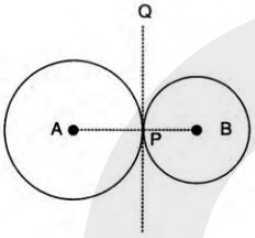
Tangents

- The tangent at any point of a circle and the radius through this point are perpendicular to each other.

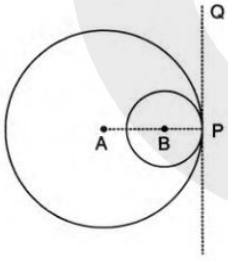


PA = PB, ∠AOP = ∠BOP,
∠APO = ∠BPO

- If two circles touch each other, the point of contact lies on the straight line through the centres.
  - When the given two circles touch each other externally.



  - When the given two circles touch each other internally.

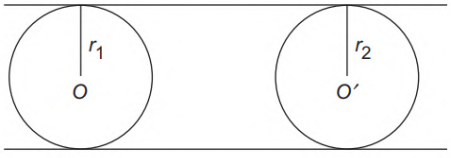


- Length of direct common tangents is:

$= \sqrt{(\text{Distance between their centres})^2 - (r_1 - r_2)^2}$

where r_1 and r_2 are the radii of the circles.

$= \sqrt{(OO')^2 - (r_1 - r_2)^2}$

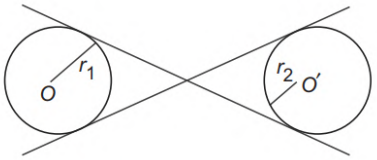


- Length of transverse common tangents is:

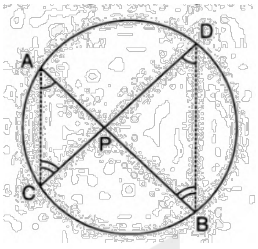
$= \sqrt{(\text{Distance between their centres})^2 - (r_1 + r_2)^2}$

where r_1 and r_2 are the radii of the circles.

$= \sqrt{(OO')^2 - (r_1 + r_2)^2}$

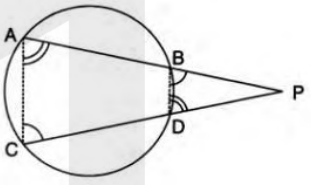


- If two chords of a circle intersect internally or externally then the product of the lengths of their segments is equal.
  - When chords intersect internally:



PA × PB = PC × PD

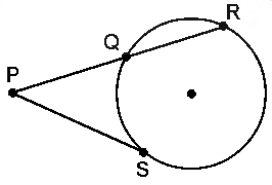
  - When chords intersect externally:



PA × PB = PC × PD

Tangent – Secant Theorem

When a tangent and a secant are drawn from one single external point to a circle, square of the length of tangent segment must be equal to the product of lengths of whole secant segment and the exterior portion of secant segment.

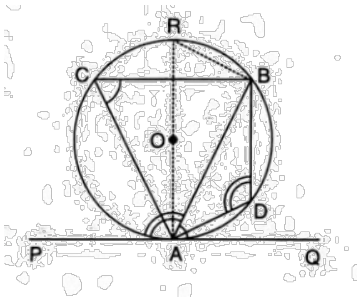


$PS^2 = PR \times PQ$

or

$(\text{Tangent})^2 = \text{Whole Secant} \times \text{external secant}$

- The angle between a tangent and a chord through the point of contact is equal to an angle in the alternate segment.



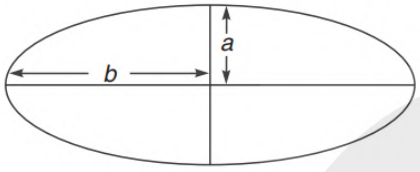
∠BAQ = ∠ACB

∠BAP = ∠ADB

Ellipse

Perimeter = π(a+b)

Area = πab

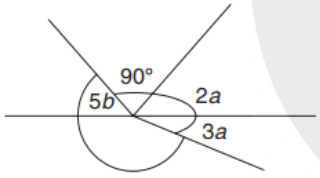


Star

Sum of angles of a star = (2n – 8) × π/2 = (n – 4) π

Question 1:

In the given figure, find the value of (a + b)



A. 50° B. 54°

C. 60° D. None of these

Solution

90° + 2a + 3a + 5b = 360°

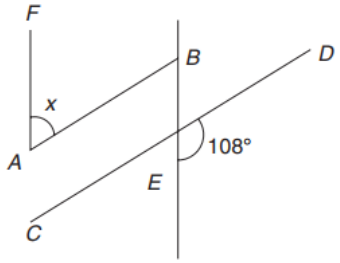
5a + 5b = 270°

a + b = 54°

∴ Option B.

Question 2:

If AB||CD and AF||BE then the value of x is:



A. 108° B. 72°

Solved Examples:

C. 88° D. 82°

Solution

If AB||CD then ∠CEB + ∠ABE = 180°

∠CEB = 108°

108° + ∠ABE = 180°

∠ABE = 72°

If AB||BD then ∠ABE = x [Alternate angles]

x = 72°

∴ Option B.

Question 3:

Each interior angle of a regular octagon is:

A. 90° B. 115°

C. 125° D. 135°

Solution

Total number of sides in octagon = 8

Each interior angle =

$$\frac{(n-2)\times 180^\circ}{n} = \frac{(8-2)\times 180^\circ}{8}$$

$$= \frac{6\times 180^\circ}{8} = 135^\circ$$

∴ Option B.

Question 4:

Difference between interior and exterior angle of a polygon is 100°. Then the number of sides in the polygon is:

A. 8 B. 9

C. 10 D. 11

Solution

Let the internal angle be x and external angle be y,

According to the question

x + y = 180° (i)

x – y = 100° (ii)

x = 140°, y = 40°

Number of sides = 360° / 40° = 9

∴ Option B.

Question 5:

The ratio of the measure of an angle of a regular octagon to the measure of its exterior angle is:

A. 2:1 B. 1:3

C. 3:1 D. 1:1

Solution

Interior angle of a regular octagon = 135°

Exterior angle of a regular octagon = 45°

Required ratio = 135° / 45° = 3:1

∴ Option C.

Question 6:

Number of diagonals of a 6-sided polygon is

A. 6 B. 9

C. 12 D. 15

Solution

Number of diagonals = ${}^{n}C_2 - n = {}^{6}C_2 - 6$

$= \frac{6!}{2!4!} - 6 = 15 - 6 = 9$

∴ Option B.

Question 7:

If height of an equilateral triangle is 10 cm, its area will be equal to:

A. $100\sqrt{3}$ cm^2 B. $\frac{100\sqrt{3}}{3}$ cm^2

C. $\frac{100}{3}$ cm^2 D. $\frac{200\sqrt{3}}{3}$ cm^2

Solution

h = 10 cm

$h = \frac{a\sqrt{3}}{2}$

$a = \frac{10\times2}{\sqrt{3}} = \frac{20}{\sqrt{3}}$ cm

Area = $\frac{1}{2} \times \frac{20}{\sqrt{3}} \times 10 = \frac{100}{\sqrt{3}}$ cm^2 = $\frac{100\sqrt{3}}{3}$ cm^2

∴ Option B.

Question 8:

AD is the median of the triangle ABC and O is the centroid such that AO = 12 cm. The length of OD in cm is

A. 4 B. 5

C. 6 D. 8

Solution

D, is the mid-point of side BC.

Centroid 'O' divides AD in the ratio 2:1

∴ OD = 12/2 = 6 cm

∴ Option C.

Question 9:

The sides of a triangle are in the ratio 4:5:6. The triangle is:

A. acute-angled

B. right-angled

C. obtuse-angled

D. either acute-angled or right angled.

Solution

Let the sides of the triangle be 3x, 4x and 6x units.

Clearly, $(4x)^2+(5x)^2 > (6x)^2$

∴ The triangle will be acute angled.

∴ Option A.

Question 10:

The sum of three altitudes of a triangle is

A. equal to the sum of three sides

B. less than the sum of sides

C. 1/ 2 times of the sum of sides

D. half the sum of sides

Solution

For a triangle PQR, let the altitudes be AP, BR and CQ respectively. Then:

AP < PR

BR < RQ

CQ < PQ

∴ AP + BR + CQ < PQ + QR + PR

∴ Option B.

Question 11:

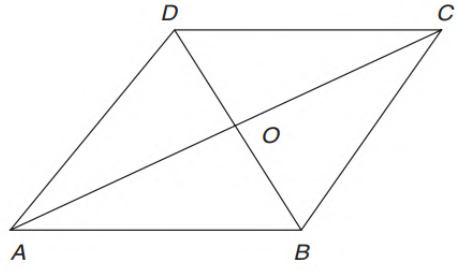
If one diagonal of a rhombus is equal to it's side, then the diagonals of the rhombus are in the ratio.

A. $\sqrt{3}$: 1 B. 3 : 1

C. 2 : 1 D. None of these

Solution

Let AB = BD = DC = a, AC = b



In △ COD:

$(CD)^2 = (OC)^2 + (OD)^2$

$a^2 = \left(\frac{b}{2}\right)^2 + \left(\frac{a}{2}\right)^2$

$\frac{3a^2}{4} = \frac{b^2}{4}$

$b = a\sqrt{3}$

$\frac{b}{a} = \frac{\sqrt{3}}{1}$

∴ Option A.

Question 12:

A triangle and a parallelogram are constructed on the same base such that their areas are equal. If the altitude of the parallelogram is 100 m, then the altitude of the triangle is

A. 50 m B. 100 m

C. 200 m D. None of these

Solution

If 'b' is the base and h_1, h_2 are altitudes of the triangle and parallelogram respectively. Then according to the question:

$\frac{1}{2} \times b \times h_1 = b \times h_2$

$h_1 = 2h_2 = 2 \times 100 = 200$ m

∴ Option C.

Question 13:

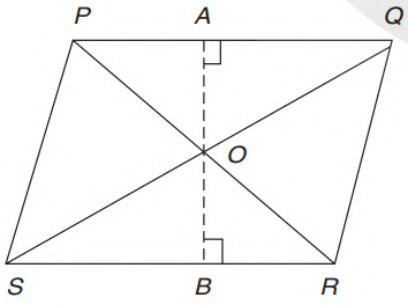
□ PQRS is a parallelogram. 'O' is a point within it, and area of parallelogram PQRS is 50 cm^2. Find the sum of areas of DOPQ and DOSR (in cm^2):

A. 15 B. 20

C. 25 D. 30

Solution

Draw OA ⊥ PQ and OB ⊥ SR



If OA = x, OB = y and PQ = SR = a, QR = PS = b

Then, Area of ΔOPQ = $\frac{1}{2} \times x \times a = \frac{ax}{2}$

Area of ΔOSR = $\frac{1}{2} \times y \times a = \frac{ay}{2}$

Area of ΔOPQ + Area of ΔOSR = $\frac{ax}{2} + \frac{ay}{2}$

$= \frac{1}{2}a(x + y)$

x + y = Altitude of parallelogram PQRS

Area of PQRS = a(x + y)

Area of (ΔOPQ + ΔOSR) = $\frac{1}{2}$ Area of □ PQRS

$= \frac{1}{2} \times 50 = 25$ cm^2

∴ Option C.

Question 14:

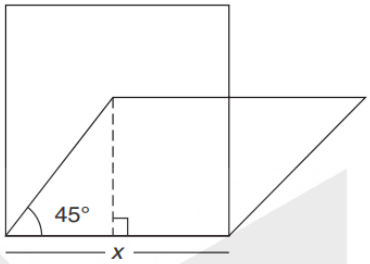
A square and a rhombus have the same base and the rhombus is inclined at 45o then what will be the ratio of area of the square to the area of the rhombus?

A. 2:1 B. $\sqrt{2} : 1$

C. $1 : \sqrt{2}$ D. $\sqrt{3} : 1$

Solution

Let the length of base be 'x' units. Area of square = x^2



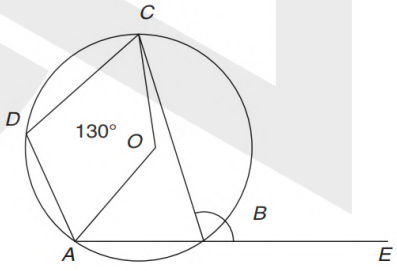
Area of rhombus = x × sin45° = x × $\frac{x}{\sqrt{2}} = \frac{x^2}{\sqrt{2}}$

Required ratio = $x^2 : \frac{x^2}{\sqrt{2}} = 1 : \frac{1}{\sqrt{2}} = \sqrt{2} : 1$

∴ Option B.

Question 15:

In the given figure, AOC = 130°, where O is the center. Find ∠CBE.



A. 100° B. 70°

C. 115° D. 130°

Solution

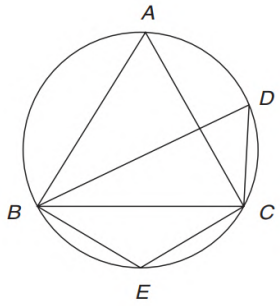
∠ABC = $\frac{130°}{2} = 65°$

∠CBE = 180° - 65° = 115°

∴ Option C.

Question 16:

In the given figure, ΔABC is an equilateral triangle. Find ∠BEC.



A. 120° B. 60°

C. 80° D. None of the above

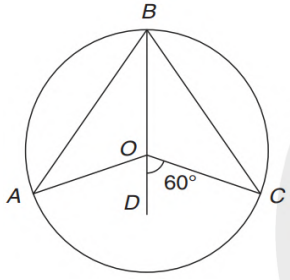
Solution

∠BAC = 60°

∠BEC = 180° – ∠BAC = 180° – 60° = 120°

Question 17:

O is the center of the circle, line segment BOD is the angle bisector of AOC, COD = 60°. Find ABC.



A. 30° B. 40°

C. 50° D. 60°

Solution

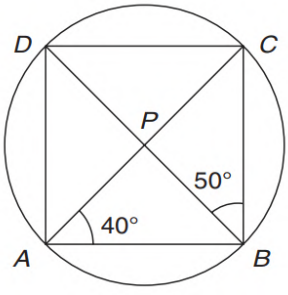
∠COD = 60°

∠AOC = 2 × 60° = 120°

∠ABC = 120° / 2 = 60°

Question 18:

In the given figure, ABCD is a cyclic quadrilateral and the diagonals bisect each other at P. If CBD = 50° and CAB = 40°, then BCD is:



A. 60° B. 75°

C. 90° D. 105°

Solution

∠CDB = ∠CAB = 40°

In △BDC = ∠BCD + 50° + 40° = 180°

∠BCD = 90°

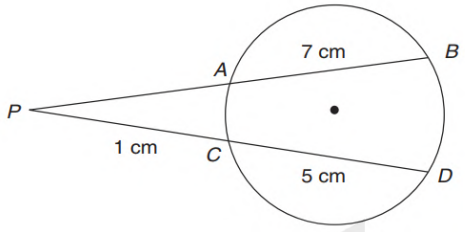
Question 19:

Chords BA and DC of a circle intersect externally at P. If AB = 7 cm, CD = 5 cm and PC = 1 cm, then the length of PB is

A. 11 cm B. 10 cm

C. 9 cm D. 8 cm

Solution



AB = 7 cm, CD = 5 cm PC = 1 cm, PA = x cm

PA× PB = PC × PD

∴ x(x+7) = 6 × 5 = 30

By solving we get x = 3 cm PB = 3 + 7 = 10 cm

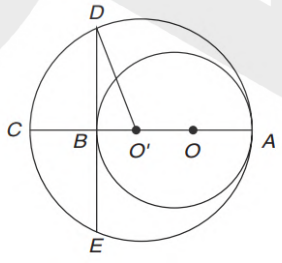
Question 20:

Two circles touch each other internally. Their radii are 3 cm and 4 cm. The biggest chord of the greater circle which is outside the inner circle is of length:

A. $2\sqrt{3}$ cm B. $3\sqrt{2}$ cm

C. $4\sqrt{3}$ cm D. $4\sqrt{2}$ cm

Solution



O'A = 4 cm

AB = 6 cm

O'B = AB – O'A = 6 – 4 = 2 cm

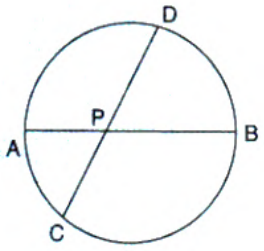
BD = $\sqrt{4^2 - 2^2} = 2\sqrt{3}$ cm

∴ DE = $4\sqrt{3}$ cm

Question 21:

From each of the following figures, find the value of x.

i.



PA = 4 cm, PB = 6 cm,
PC = 5 cm, PD = x cm

Solution

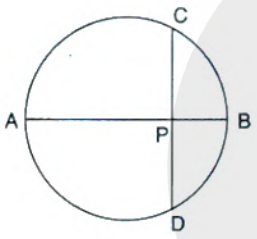
Since, chords AB and CD intersect each other at point P, inside the circle:

PA × PB = PC × PD

4 × 6 = 5 × x

$x = \frac{24}{5}\ \text{cm} = 4.8\ \text{cm}$

ii.



PA = 2PB = 12 cm,
PC = PD = x cm

Solution

Given, PA = 12 cm, PB = 12/2 cm = 6 cm; chords AB and CD intersect each other at point P, inside the circle:

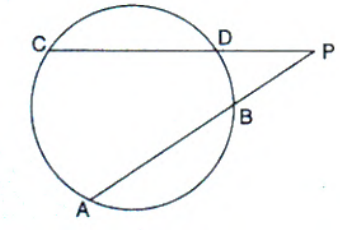
PA × PB = PC × PD

12 × 6 = x × x

$x^2 = 72$

$x = \sqrt{72} = 6\sqrt{2}\ \text{cm}$

iii.



AB = 10 cm, PB = 6 cm,
CD = x cm, PD = 4 cm

Since, chords AB and CD intersect each other, externally at point P

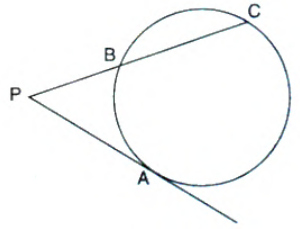
PA × PB = PC × PD

PA = AB + PB = 10 + 6 =16

16 × 6 = (4+x) × 4

x = 20 cm

iv.



PA = 20 cm, PB = 16 cm,

BC = x cm

Solution

Since, chord BC and the tangent at point A intersect each other at point P, outside the circle:

$PA^2 = PB \times PC$

$(20)^2 = 16 \times (16 + x)$

[PC = PB +BC = (16+x) cm]

$\therefore x = 9$ cm.

Exercise -1

1. In a circle of radius 25 cm, two parallel chords of length 48 cm and 14 cm are drawn. If both the chords are on the same side of the centre, then the distance between the chords is
   A. 8 B. 34
   C. 17 D. 15
   E. 12
2. Two circles touch each other externally. The distance between their centre is 15 cm. If the diameter of one circle is 16 cm, then the radius of the other circle is
   A. 3 B. 4
   C. 5 D. 7
   E. 6
3. A, B and C are the three points on a circle such that the angles subtended by the chords AB and AC at the centre O are 90° and 120° respectively. ∠BAC is equal to
   A. 80° B. 75°
   C. 85° D. 95°
   E. None of these
4. A, B, C and D are four points on a circle. AC and BD intersect at a point E such that ∠BEC = 140° and ∠ECD = 30°, ∠BAC is
   A. 110° B. 120°
   C. 130° D. 140°
   E. None of these
5. If two concentric circles are of radii 6 cm and 4 cm, then the length of the chord of the larger circle which touches the smaller circle is
   A. 4√5 B. 2√5
   C. 3√ D. 5√5
   E. None of these
6. A chord 24 cm long is drawn in a circle of diameter 30 cm. The distance of the chord from the centre is
   A. 8 B. 6
   C. 9 D. 12
   E. None of these
7. 'O' is the centre of the circle, AB is a chord of the circle, OM ⊥ AB. If AB = 30 cm, OM = √64cm, then radius of the circle is
   A. 8 B. 6
   C. 17 D. 12
   E. None of these
8. In ΔABC, ∠ABC = 80°, ∠BCA = 40°. O is the point of intersection of the perpendicular bisectors of the sides, then the angle ∠BOC is
   A. 130^o B. 120^o
   C. 100^o D. 110^o
   E. None of these
9. The sum of all interior angles of a regular polygon is thrice the sum of all its exterior angles. The number of sides of the polygon is
   A. 6 B. 8
   C. 10 D. 12
   E. None of these
10. If each interior angle of a regular polygon is 140°, the number of sides of the polygon is
    A. 8 B. 12
    C. 10 D. 9
    E. 6
11. The sum of interior angles of a regular polygon is 1260°. The number of sides of the polygon is
    A. 9 B. 10
    C. 8 D. 11
    E. 12
12. Among the angles 40°, 100°, 90°, 72°,45^o one angle cannot be an exterior angle of a regular polygon. The angle is
    A. 45° B. 90°
    C. 100° D. 40°
    E. 72^o
13. An interior angle of a regular polygon is 4 times its exterior angle. Then the number of sides of the polygon is
    A. 10 B. 12
    C. 8 D. 6
    E. 9
14. In a regular polygon if one of its internal angle is greater than the external angle by 144°, then the number of sides of the polygon is
    A. 22 B. 12
    C. 16 D. 20
    E. 18

15. If ABCD be a cyclic quadrilateral in which ∠A = 6x°, ∠B = 5x°, ∠C = 3y°, ∠D = 4y°, then x : y is
A. 1:2 B. 2:1
C. 1:1 D. 3:2
E. 2:3

16. ABCD is a cyclic quadrilateral and AB is a diameter. If ∠BAC = 65° then value of ∠ADC is
A. 155° B. 165°
C. 145° D. 55°
E. None of these

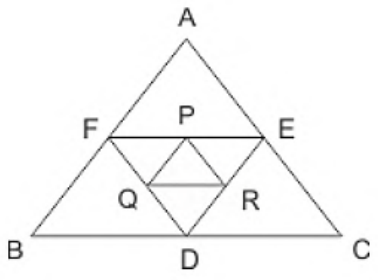
17. The difference between the exterior and interior angles at a vertex of a regular polygon is 160°. The number of sides of the polygon is
A. 30 B. 40
C. 36 D. 32
E. None of these

18. If the sum of the interior angles of a regular polygon be 1440°, the number of sides of the polygon is
A. 12 B. 10
C. 15 D. 14
E. 16

19. If an exterior angle of a cyclic quadrilateral be 70°, then the interior opposite angles is:
A. 70° B. 75°
C. 130° D. 110°
E. None of these

20. ABCD is a cyclic quadrilateral and O is the centre of the circle. If ∠COD = 120° and ∠BAC = 50°, then the value of ∠BCD is equal to
A. 70° B. 75°
C. 130° D. 110°
E. None of these

21. In the given figure, if the area of triangle ABC is 128 sq. units, then find the area of triangle PQR, where D, E and F are midpoints of sides of ΔABC and P, Q and R are midpoints of sides of ΔDEF.



A. 32 B. 4
C. 8 D. 16
E. 64

22. The ratio between the number of sides of two regular polygons is 1 : 2 and the ratio between their interior angles is 4 : 5. The number of sides of these polygons is respectively
A. (3,6) B. (5,10)
C. (6,12) D. (4,8)
E. None of these

23. ABCD is a cyclic trapezium such that AD || BC, if ∠ABC = 80°, then the value of ∠BCD is:
A. 90° B. 100°
C. 80° D. 70°
E. None of these

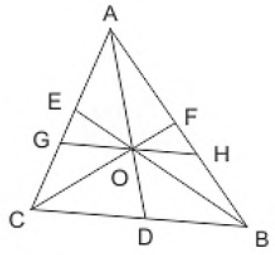
24. The ratio of the angles of a quadrilateral is 5 : 4 : 3 : 6. The smallest angle of a triangle is one-fourth the largest angle of the quadrilateral and the largest angle of the triangle 40° more than the second largest angle of the triangle. What is the second largest angle of the triangle?
A. 60° B. 55°
C. 50° D. 65°
E. None of these

25. ABC is an equilateral triangle inscribed in a circle with AB = 12 cm. Suppose bisector of angle B meets AC at X and circle at Y, then what is the value of 2 × BX × BY?
A. 230 cm^2 B. 238 cm^2
C. 228 cm^2 D. 220 cm^2
E. None of these

26. Longest side of a triangle is 20 cm and another side is 10 cm. If area of the triangle is 120 cm^2, then what is the length (in cm) of its third side?
A. 4√10 cm B. 5√10 cm
C. 8√10 cm D. 6√10 cm
E. None of these

27. Find the sum of the medians of isosceles triangle, whose sides are 8, 8 and 10.
A. (√41 + 2√66) B. (√39 + 2√66)
C. (√39 + 2√65) D. (√41 + 2√65)
E. None of these

28. In the figure shown below AD, BE and CF are all medians of triangle ABC, and GH is parallel to BC. If GC = 20 cm, what is the length (in cm) of AC?



A. 40 cm B. 60 cm
C. 50 cm D. 80 cm
E. None of these

29. If the geometric mean of the lengths of three sides of a triangle is 4 and the area of the circumcircle is 36π, what is the area of the triangle (in sq. units)?
A. 8/5 B. 4/3
C. 8/3 D. 4/5
E. None of these

30. If the hypotenuse of a right triangle is 61 cm and the sum of the other two sides is 71 cm, find the difference between the other sides.
A. 49cm B. 50cm
C. 48cm D. 52cm
E. None of these

31. The sides of the triangle are 5 cm, 12 cm and 13 cm. The area (in cm^2) of the triangle formed by joining the mid-points of the sides of the triangle is
A. 15 cm^2 B. 3 cm^2
C. 7 cm^2 D. 7.5 cm^2
E. None of these

32. In a triangle ABC, the side BC is extended up to D. Such that CD = AC, if ∠BAD = 112° and ∠ACB = 68° then the value of ∠ABC is
A. 36° B. 44°
C. 40° D. 34°
E. None of these

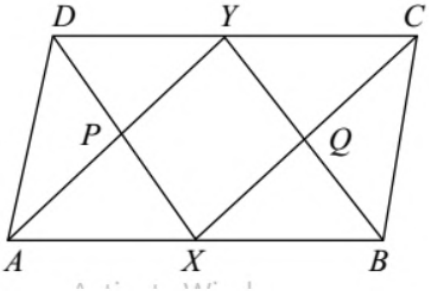
33. In a triangle ABC, if ∠A + ∠C = 150° and ∠A + 3∠B = 180°, then ∠A is equal to
A. 80° B. 90°
C. 60° D. 45°
E. None of these

34. A tower standing on a horizontal plane subtends a certain angle at a point 120 m apart from the foot of the tower. On advancing 80 m towards it, the tower is found to subtend an angle twice as before. The height of the tower is
A. 100 / √3 m B. 120 / √3 m
C. 150 / 2√3 m D. 150 / √3 m
E. None of these

35. The length of the shadow of a vertical tower on level ground increases by 20 metres when the altitude of the sun changes from 45 to 30°. Then the height of the tower is
A. 15 (2√3 + 1) m B. 10 (2√3 + 1) m
C. 10 (√3 + 1) m D. 15 (√3 + 1) m
E. None of these

36. From two points on the ground lying on a straight line through the foot of a chimney, the two angles of elevation of the top of the chimney are complementary to each other. If the distance of the two points from the foot of the chimney are 160 metres and 360 metres and the two points lie on the same side of the chimney, then the height of the chimney is
A. 280 m B. 240 m
C. 30 m D. 200 m
E. None of these

37. X, Y are the mid-points of opposite sides AB and DC of a parallelogram ABCD. AY and DX are joined intersecting in P; CX and BY are joined intersecting in Q. Then PXQY is a :

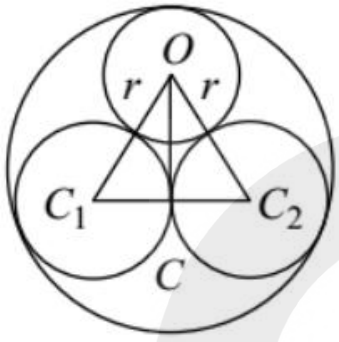


A. Rectangle B. Parallelogram
C. Square D. Rhombus
E. None of these

38. If the ratio of number of sides of two regular polygons be 2 : 3 and the ratio of their interior angles be 6 : 7, find the number of sides of the two polygons.

A. 7 and 8 B. 6 and 7
C. 6 and 9 D. 6 and 8
E. 6 and 4

39. Two circles of unit radius touch each other and each of them touches internally a circle of radius two, as shown in the following figure. The radius of the circle which touches all the three circles:



A. 375 B. 380
C. 390 D. 365
E. 385

40. If P and Q are the mid points of the sides CA and CB respectively of a triangle ABC, right-angled at C. Then the value of 4 ($AQ^2 + BP^2$) is equal to:

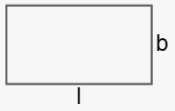
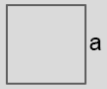
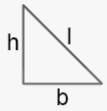
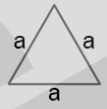
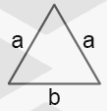
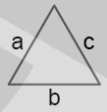
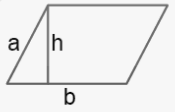
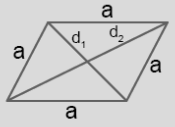
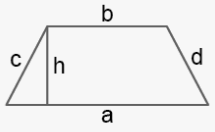
A. 5 AB^2 B. 4 BC^2
C. 2 BC^2 D. 5 BC^2
E. None of these

ANSWER KEY:

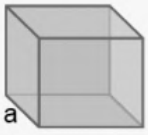
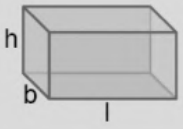
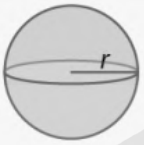
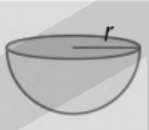
1) C	11) A	21) C	31) D
2) D	12) C	22) C	32) D
3) B	13) A	23) C	33) B
4) A	14) D	24) B	34) B
5) A	15) C	25) C	35) C
6) C	16) A	26) C	36) B
7) C	17) C	27) B	37) B
8) B	18) B	28) B	38) C
9) B	19) A	29) C	39) C
10) D	20) A	30) A	40) A

MENSURATION

Two – Dimensional Figures

Shape	Area (in square units)	Perimeter (in units)	Figure
Rectangle l = length b = breadth	l x b	2(l + b)	
Square a = side	a^2	4a	
Right Angled Triangle b = base h = height l = hypotenuse	½ x b x h	b + h + l	
Equilateral Triangle a = side	$(\sqrt{3}/4) \times a^2$	3a	
Isosceles Triangle a = length of one of the two equal sides b = length of the unequal side	$½[\sqrt{(a^2 - b^2/4)} \times b]$	2a + b	
Scalene Triangle a, b and c = sides of the triangle s = (a +b +c)/2	$\sqrt{[s(s-a)(s-b)(s-c)]}$, where, s = (a +b +c)/2	a + b + c	
Parallelogram b = base h = height	b x h	2(a + b)	
Rhombus a = side d_1, d_2 = diagonals	½ x d_1x d_2	4a	
Trapezium a, b = parallel sides c, d = non parallel sides h = height	½ x (a + b) x h	a + b + c + d	
Circle r = radius	πr^2	$2\pi r$	

Three – Dimensional Figures

Solid	Volume	CSA	TSA	Figure
Cube a=side	a^3	$4\ a^2$	$6\ a^2$	
Cuboid l = length b = breadth h = height	$l \times b \times h$	$2h(l + b)$	$2\ (lb + bh + hl)$	
Sphere r = radius	$(4/3)\ \pi\ r^3$	$4\ \pi\ r^2$	$4\ \pi\ r^2$	
Hemisphere r = radius	$(⅔)\ \pi\ r^3$	$2\ \pi\ r^2$	$3\ \pi\ r^2$	
Cylinder r = radius h = height	$\pi\ r^2\ h$	$2\pi\ r\ h$	$2\pi rh + 2\pi r^2$	
Cone r = radius l = slant height h = height	$(⅓)\ \pi\ r^2\ h$	$\pi\ r\ l$	$\pi r\ (r + l)$	

Solved Examples:

1. Calculate the area of a square with side 7 cm.

Solution

The area of a square of side 5cm is $A = L^2 = 7^2 = 49\ cm^2$

2. Calculate the perimeter of a rectangle having a length of 12 cm and breadth of 10 cm.

Solution

The perimeter of a rectangle with length 10 cm and breadth 5 cm is $2(l + b) = 2(12 + 10) = 11$ cm.

3. What is the area of a right-angled triangle with two of its perpendicular sides as 3 and 4 cm?

Solution

The area of a right angled triangle with two of its perpendicular sides as 8 cm and 6 cm is:

$A = ½ \times b \times h = ½ \times 8 \times 6 = 24\ cm^2$

4. Calculate the volume of a cone having a radius of 21 cm and height of 10 cm?

Solution

The volume of a cone with a radius of 21 cm and height of 10 cm is:

$(1/3)\pi r^2h = 4620\ cm^3$

5. What is the total surface area and curved surface area of a cylinder with a radius of 7 cm and height of 20 cm?

Solution

The total surface area of a cylinder with radius 7 cm and height 20 cm is:

$2\pi rh + 2\pi r^2 = 378\pi\ cm^3 = 1187.522\ cm^3$

The curved surface area of a cylinder with radius 14cm and height 10cm is:

$2\pi rh = 280\pi = 879.6459\ cm^2$

Exercise – 1

1. Three cubes whose edges measure 3 cm, 4cm and 5cm respectively are melted to form a new cube. The surface area of the new cube is:
   A. 50 m^2 B. 216 cm^2
   C. 250 cm^2 D. 300 cm^2
   E. None of these
2. If each edge of a cube is increased by 100%, find the percentage increase in Its surface area.
   A. 125% B. 400%
   C. 300% D. 100%
   E. 200%
3. 40 men took a dip in a water tank 50 m long and 20 m broad on a religious day. If the average displacement of water by a man is $6m^3$, then the rise in the water level in the tank will be:
   A. 20 cm B. 24 cm
   C. 32 cm D. 50 cm
   E. 48 cm
4. A right triangle with sides 6 cm, 8 cm and 10 cm is rotated the side of 6 cm to form a cone. The volume of the cone so formed is:
   A. $96\pi\ cm^3$ B. $36\pi\ cm^3$
   C. $48\pi\ cm^3$ D. $64\pi\ cm^3$
   E. $24\pi\ cm^3$
5. A rectangular block 6 cm by 9 cm by 12 cm is cut up into an exact number of equal cubes. Find the least possible number of cubes.
   A. 36 B. 40
   C. 16 D. 20
   E. 24
6. A cistern 8 m long and 4 m wide contains water up to a depth of 1 m 25 cm. The total area of the wet surface is:
   A. 52 B. 56
   C. 65.5 D. 62
   E. 52
7. A hall is 10 m long and 6 m broad. If the sum of the areas of the floor and the ceiling is equal to the sum of the areas of four walls, the volume of the hall is:
   A. 150 B. 250
   C. 125 D. 225
   E. 400
8. How many bricks each measuring 25 × 10.25cm × 8cm, will be needed to build a wall 8m × 4m × 2.05m
   A. 56000 B. 6000
   C. 64000 D. 72000
   E. 32000
9. How many cubes of 4 cm edge can be cut out of a cube of 20 cm edge?
   A. 343 B. 232

C. 125 D. 216
E. 81

10. The surface area of a cube is 1536 sq. cm. Find its volume.
A. 2334 cm^3 B. 4096 cm^3
C. 4816 cm^3 D. 3478 cm^3
E. 4126 cm^3

11. The diagonal of a rectangle is $\sqrt{97}$ cm. and its area is 12 sq. cm. The perimeter of the rectangle must be:
A. 22 cm B. 18 cm
C. 20 cm D. 41 cm
E. 25 cm

12. The volume of a hollow cube is $343x^3$. What surface area of the largest sphere is enclosed in it?
A. $18\pi x^2$ B. $27\pi x^2$
C. $36\pi x^2$ D. $72\pi x^2$
E. $49\pi x^2$

13. If 64 identical small spheres are made of a big sphere of diameter 16 cm, then what is the surface area of each small sphere?
A. π cm^2 B. 2π cm^2
C. 4π cm^2 D. 8π cm^2
E. 16π cm^2

14. A cube has each edge 3 cm, and a cuboid is 1 cm long, 2 cm wide and 4 cm high. The paint in a certain container is sufficient to paint an area equal to 82 cm^2. Which one of the following is correct?
A. Both cube and cuboid can be painted.
B. Only one cube can be painted.
C. Only cuboid can be painted.
D. Neither cube nor cuboid can be painted.
E. Either B or C

15. Find the volume of a sphere whose radius is $\sqrt{2}$ times the radius of another sphere which exactly fits in a cube of side 12 cm.
A. 529 $\pi\sqrt{2}$ B. 676 $\pi\sqrt{2}$
C. 576 $\pi\sqrt{2}$ D. 482 $\pi\sqrt{2}$
E. 324 $\pi\sqrt{2}$

16. The volume of a wall, 10 times as high as it is broad and 2 times as long as it is high, is 25 cu. meters. Find the breadth of the wall.
A. 50cm B. 30cm
C. 20cm D. 400cm
E. None of these

17. Find the number of bricks, each measuring 24 cm x 16 cm x 10 cm, required to construct a wall 36 m long, 8m high and 60 cm thick, if 10% of the wall is filled with mortar?
A. 36500 B. 45500
C. 55000 D. 40500
E. 65000

18. The dimensions of an open box are 50 cm, 40 cm and 23 cm. Its thickness is 3 cm. If 1 cubic cm of metal used in the box weighs 1.5 gms, find the weight of the box.
A. 24.12kg B. 22.14kg
C. 24.24kg D. 24.04kg
E. 22.12kg

19. A large cube is formed from the material obtained by melting three smaller cubes of 1, 6 and 8 cm side. What is the ratio of the total surface areas of the smaller cubes and the large cube?
A. 29 : 24 B. 3 : 2
C. 25 : 18 D. 97 : 43
E. 101 : 81

20. A hollow iron pipe is 18 cm long and its external diameter is 10 cm. If the thickness of the pipe is 1 cm and iron weighs 6 g/cu.cm, then the weight of the pipe is:
A. 475.2 kg B. 4752 kg
C. 4.752 kg D. 47.52 kg
E. None of these

21. What is the total surface area of a right conical tent of base radius 7 cm and height equal to the side of a cube of volume 2744 cm^3?
A. 344.39 cm^2 B. 462 cm^2
C. 440.50 cm^2 D. 498.35 cm^2
E. None of these

22. How many iron rods, each of length 14 mts and diameter 2 cm can be made of 1.87 cubic meters of iron?
A. 425 B. 675
C. 525 D. 550
E. 350

23. In a shower, 4 cm of rain falls. The volume of water that falls on 1.8 hectares of ground is:
A. 72 m^3 B. 720 m^3
C. 7200 m^3 D. 72000 m^3
E. Cannot be determined.

24. A sphere is melted and moulded into a solid cylinder. If the radius of both solids is equal, then find the ratio of total surface area of the cylinder to the total surface area of the sphere.
A. 2 : 3 B. 4 : 3
C. 3 : 7 D. 6 : 7
E. 7 : 6

25. The side of a square exceeds the side of the other square by 5 cm and the sum of the areas of the two squares is 325 cm^2. The dimensions of the square are?
A. 8 cm and 13 cm B. 10 cm and 15 cm
C. 5 cm and 10 cm D. 15 cm and 20 cm
E. None of these

26. Area and perimeter of a rectangle is 480 cm^2 and 92 cm respectively. If the rectangle is inscribed in a circle of maximum possible area, then find the circumference of the circle.
A. 106 6/7 cm B. 168 3/7 cm
C. 56 5/7 cm D. 94 6/7 cm
E. 84 3/7 cm

27. A solid sphere of radius 6 cm is melted to form a sphere of radius 3 cm and a right circular cylinder of same radius. The height of the cylinder so formed is:
A. 56 cm B. 81 cm
C. 28 cm D. 105 cm
E. 54 cm

28. A rectangular prism has a surface area of 432 cm^2 and a height of 6 cm. If the length and width of the base are in the ratio 2:3, find the ratio of dimensions of the prism. (In order, length : width : height)
A. 6:8:9 B. 1:3:2
C. 2:4:3 D. 5:6:2
E. 4:6:3

29. Water flows into a tank 200 m x 150 m through a rectangular pipe of 1.5m x 1.25 m at 20 kmph . In what time (in minutes) will the water rise by 2 meters?
A. 92 min B. 93 min
C. 95 min D. 96 min
E. none of these

30. Ratio of volume of conical tent to total surface area of hemispherical bowl is 7 : 1. Radius of the bowl and tent is equal, and the slant height of the tent is 65cm. If the radius of the conical tent is equal to the side of a cube, then find the curved surface area of the cube.
A. 1536 cm^2 B. 1764 cm^2
C. 1852 cm^2 D. 1024 cm^2
E. 1512 cm^2

31. Diameter of a cylinder is 2/3rd of the sum of its radius & height, while the ratio of curved surface area to volume of the cylinder is 1 : 8. If the radius and height of a cone is the same as that of a cylinder, then find the slant height of the cone?
A. $21\sqrt{5}$ cm B. $10\sqrt{5}$ cm
C. $7\sqrt{5}$ cm D. $14\sqrt{5}$ cm
E. $16\sqrt{5}$ cm

32. The surface area of the sphere and total surface area of the hemisphere is in ratio 3:1. What is the volume of a sphere if the sum of squares of radius of sphere and hemisphere is 42 cm^2. (In cm^3)
A. 272 π B. 288 π
C. 232 π D. 236 π
E. 218 π

33. Length of a rectangular land is twice the radius of a circle of circumference 176 cm. The land area increases by 169 sq.cm, when a square land is attached along the breadth of the rectangle. Find the area of the rectangle in sq.cm.
A. 1008 B. 728
C. 504 D. 756
E. 1512

34. Difference between the circumference of a circular vessel (candy maker) and its diameter is 90 cm. If 161 cm^2 is waste material and 25 square boxes are required to pack the remaining quantity of candy, then find the side of each square box?
A. 4 cm B. 8 cm
C. 10 cm D. 5 cm
E. 7 cm

35. A toy is formed such that a cone is mounted on a hemispherical base. Radius of the cone and hemispherical base is equal and the ratio of radius to height of the cone is 9 : 10. If the total height of the toy is 38 cm, then find the volume of the toy.
A. 9856 cm^3 B. 19008 cm^3
C. 9936 cm^3 D. 9882 cm^3
E. 9996 cm^3

ANSWER KEY:

1) B	8) E	15) C	22) A	29) D
2) C	9) C	16) A	23) B	30) D
3) B	10) B	17) D	24) E	31) E
4) A	11) A	18) A	25) B	32) B
5) E	12) E	19) E	26) A	33) B
6) D	13) E	20) E	27) C	34) E
7) D	14) A	21) D	28) E	35) B

TRIGONOMETRY

Angles: The measure of space between two intersecting lines are known as angles.

Right-angle Triangle: A triangle with one of its interior angles being the right angle i.e., 90°, is called right angles triangle.

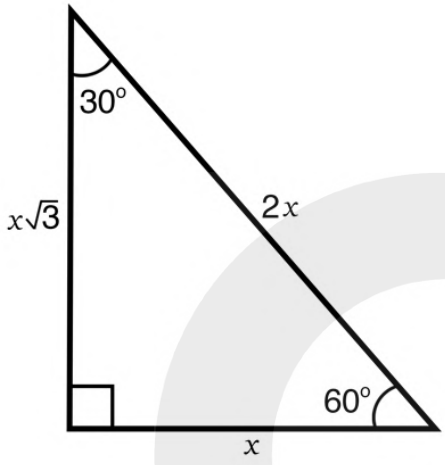
Pythagoras Theorem: In right angles triangle, according to the Pythagoras theorem, the square of the hypotenuse is equal to the sum of squares of the other two sides.

30°-60°-90° Triangle: $= 1 : \sqrt{3} : 2$

Side Opposite 30° = x

Side Opposite 60° = $x\sqrt{3}$

Side Opposite 90° = $2x$

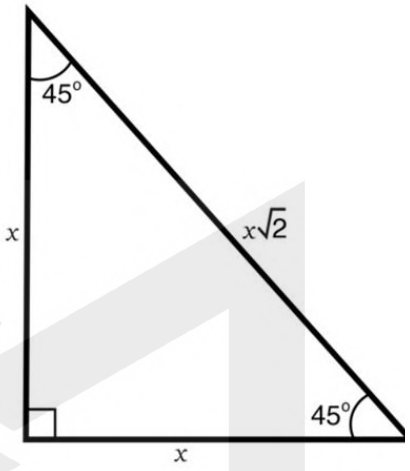


45°-45°-90° Triangle: $= 1 : 1 : \sqrt{2}$

Side Opposite 45° = x

Side Opposite 45° = x

Side Opposite 90° = $\sqrt{2}x$

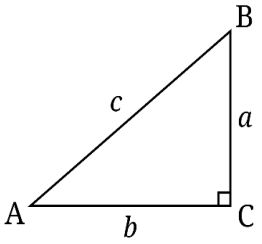


Trigonometric Ratios: Trigonometric Ratios are defined as the ratio of the sides of the right angle triangles. As there are 3 ways to choose two sides out of three and two ways for each chosen pair to arrange in ratio, thus there are 3×2 =6 trigonometric ratios which are defined for each possible pair of sides of the right angle triangle.

The important trigonometric functions include sin and cos, as all the other trigonometric ratios can be defined in terms of sin and cos.

- tangent or tan can be defined as the ratio of sin and cos i.e., $\tan x = \frac{\sin x}{\cos x}$,
- cotangent or cot can be defined as the ratio of cos and sin i.e., $\cot x = \frac{\cos x}{\sin x}$, and
- cosecant or cosec is the inverse of the sin .i.e, $\operatorname{cosec} x = \frac{1}{\sin x}$,
- secant or sec is the inverse of cos i.e., $\sec x = \frac{1}{\cos x}$.

These trigonometric ratios can also be defined as the ratios between two sides of a right-angle triangle.



AB = c, BC = a, AC = b

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ → eg: $\sin A = \frac{a}{c}$, $\sin B = \frac{b}{c}$	$\text{cosec}\,\theta = \frac{\text{hypotenuse}}{\text{opposite}}$ → eg: $\text{cosec}\,A = \frac{a}{c}$, $\text{cosec}\,B = \frac{b}{c}$
$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ → eg: $\cos A = \frac{b}{c}$, $\cos B = \frac{a}{c}$	$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$ → eg: $\sec A = \frac{c}{b}$, $\sec B = \frac{c}{a}$
$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ → eg: $\tan A = \frac{a}{b}$, $\tan B = \frac{b}{a}$	$\cot\theta = \frac{\text{adjacent}}{\text{opposite}}$ → eg: $\cot A = \frac{b}{a}$, $\cot B = \frac{a}{b}$

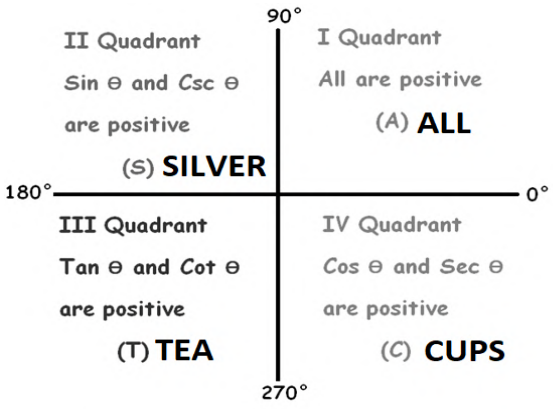
The Acronym to remember this is:

O S H A C H O T A

O → Opposite	S → sin = $\frac{O}{H}$, cosec = $\frac{H}{O}$
H → Hypotenuse	C → cos = $\frac{A}{H}$, sec = $\frac{H}{A}$
A → Adjacent	T → tan = $\frac{O}{A}$, cot = $\frac{A}{O}$

Trigonometry Table:

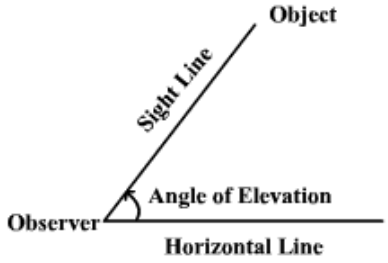
Angle (in Degrees)	**0°**	**30°**	**45°**	**60°**	**90°**	**120°**	**180°**	**270°**	**360°**
Angles (in Radians)	$\mathbf{0^C}$	$\frac{\pi^C}{6}$	$\frac{\pi^C}{4}$	$\frac{\pi^C}{3}$	$\frac{\pi^C}{2}$	$\frac{2\pi^C}{3}$	π^C	$\frac{3\pi^C}{2}$	$2\pi^C$
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	0	-1	0
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$\frac{-1}{2}$	-1	0	1
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	0	∞	0
Cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$\frac{-1}{\sqrt{3}}$	∞	0	∞
Sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	-2	-1	∞	1
Cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	$\frac{2}{\sqrt{3}}$	∞	-1	∞



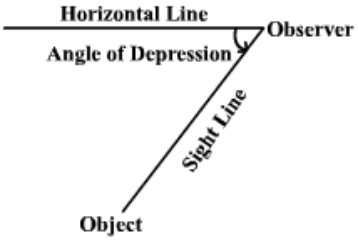
Trigonometry Formulae List:

- Pythagorean Trigonometric Identities
  - $\sin^2\theta + \cos^2\theta = 1$
  - $1+\tan^2\theta = \sec^2\theta$
  - $1 + \cot^2\theta = \operatorname{cosec}^2\theta$
- Trigonometric Identities
  - $\sin(-\theta) = -\sin\theta$
  - $\cos(-\theta) = \cos\theta$
- Sum and Difference Identities
  - $\sin(A+B) = \sin A\cos B + \cos A\sin B$
  - $\sin(A-B) = \sin A\cos B - \cos A\sin B$
  - $\cos(A+B) = \cos A\cos B - \sin A\sin B$
  - $\cos(A-B) = \cos A\cos B + \sin A\sin B$
  - $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$
  - $\tan(A-B) = \frac{\tan A - \tan B}{(1 + \tan A\tan B)}$
- Double angle Identities
  - $\sin 2\theta = 2\sin\theta\cos\theta$
  - $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - \sin^2\theta$
  - $\tan 2\theta = \frac{2\tan\theta}{1 - \tan 2\theta}$
- Half Angle Identities
  - $\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$
  - $\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$
  - $\tan\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{\sin\theta}{1+\cos\theta} = \frac{\theta}{\sin\theta}$
- Product Sum Identities
  - $\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
  - $\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
  - $\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
  - $\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
- Product Identities
  - $\sin A\cos B = \frac{\sin(A+B) + \sin(A-B)}{2}$
  - $\cos A\cos B = \frac{\cos(A+B) + \cos(A-B)}{2}$
  - $\sin A\sin B = \frac{\cos(A-B) - \cos(A+B)}{2}$
- Triple Angle Identities
  - $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$
  - $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$
  - $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

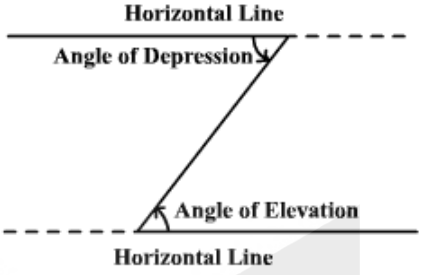
The term angle of elevation denotes the angle from the horizontal upward to an object. An observer's line of sight would be above the horizontal.



The term angle of depression denotes the angle from the horizontal downward to an object. An observer's line of sight would be below the horizontal.



Note that the angle of elevation and the angle of depression are congruent



Solved Examples:

1. A ladder is leaning against a wall. The angle between the ladder and the ground is 45 degrees, and the length of the ladder is 20 meters. How far is the ladder from the wall?

   Solution

   Let the distance between the ladder and the wall be x meters.

   Here, ladder, wall and ground together makes a right angle triangle, where for given angle,

   Length of ladder = hypotenuse = 20 meter,

   Distance between wall and ladder

   = base = x meter

   Using trigonometric ratio cos, we get

   $\Rightarrow \cos 45° = \frac{x}{10} = 1$

   $\Rightarrow \frac{1}{\sqrt{2}} = \frac{x}{20}$

   $\Rightarrow x = \frac{20}{\sqrt{2}} = 10\sqrt{2}$ meters.

   Therefore, the ladder is $10\sqrt{2}$ meters away from the wall.

2. A right-angled triangle has a hypotenuse of length 20 cm and one of its acute angle measures 30°. What are the lengths of the other two sides?

   Solution

   Let's call the side opposite to the 30° angle as 'a' and the side adjacent to it as 'b'.

   Now, sin (30°) = perpendicular/hypotenuse = a/20

   $\Rightarrow a = 20 \times \sin(30°)$

   = 10 cm [sin(30°) = 1/2]

   and cos(30°) = b/20

   $\Rightarrow b = 20 \times \cos(30°) = 20 \times \sqrt{3}/2 \approx 17.32$ cm

   Therefore, the lengths of the other two sides are 10 cm and 17.32 cm (approx.).

   Or

   30°-60°-90° : $= 1 : \sqrt{3} : 2$

   $= a : \sqrt{3}a : 2a$

   $2a = 20$

   $a = 10$

   $\sqrt{3}a = 1.732 \times 10 = 17.32$

3. Prove that (cos x/sin x) + (sin x/cos x) = sec x × cosec x.

   Solution

   LHS = (cos x/sin x) + (sin x/cos x)

   $\Rightarrow$ LHS = [cos2x + sin2x]/(cos x sin x)

   $\Rightarrow$ LHS = 1/(cosx sinx)

   [Using cos2x + sin2x = 1]

   $\Rightarrow$ LHS = (1/cosx) × (1/sinx)

   $\Rightarrow$ LHS = secx × cosecx

   = RHS [1/cosx = sec x and 1/sinx = cosec x]

4. A person is standing at a distance of 20 meters from the base of a building. The person measures the angle of elevation to the top of the building as 60°. What is the height of the building?

   Solution

   Let h be the height of the building.

   All the distances here in the question make a right-angle triangle, with a base of 10 meters and height h meter.

   As tan θ = Perpendicular/Base

   $\Rightarrow \tan(60°) = h / 20$

   $\Rightarrow h = 20 \tan(60°)$

   Using the values of $\tan(60°) = \sqrt{3}$, we get:

   $h = 20\sqrt{3} \approx 34.64$ m

   Therefore, the height of the building is approximately 34.64 meters.

5. Find the value of x in the equation $\cos^{-1}(x) + \sin^{-1}(x) = \pi/4$.

   Solution

   For, $\cos^{-1}(x) + \sin^{-1}(x) = \pi/4$

   As we know, $\cos^{-1}(x) + \sin^{-1}(x) = \pi/2$, above equation becomes

   $\pi/2 = \pi/4$, which is not true.

   Thus, the given equation has no such value of x, which can satisfy the equation.

Exercise 1:

1. If $0° < q < 90°$, the value of $\sin q + \cos q$ is
   A. equal to 1 B. greater than 1
   C. less than 1 D. equal to 2
   E. None of these
2. The value of $\frac{\cot 30° - \cot 75°}{\tan 15° - \tan 60°}$ is :
   A. 0 B. 1
   C. $\sqrt{3} - 1$ D. -1
   E. None of these
3. If cos q + sec q = 2, the value of $\cos^6 q + \sec^6 q$ is
   A. 4 B. 8
   C. 1 D. 2
   E. None of these
4. If tan 7q tan 2q = 1, then the value of tan 3q is
   A. $\sqrt{3}$ B. $-\left(\frac{1}{\sqrt{3}}\right)$
   C. $\frac{1}{\sqrt{3}}$ D. $-\sqrt{3}$
   E. None of these
5. If $\frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} = \frac{5}{3}$ then $\sin\theta$ is equal to?
   A. $\frac{1}{4}$ B. $\frac{1}{3}$
   C. $\frac{2}{3}$ D. $\frac{3}{4}$
   E. None of these
6. If $\tan q + \cot q = 2$, then the value of $\tan^2 q + \cot^2 q$ is
   A. 2 B. 1
   C. $\sqrt{2}$ D. 0
   E. None of these
7. The value of $\frac{1}{1+\tan^2\theta} + \frac{1}{1+\cot^2\theta}$ is
   A. $\frac{1}{4}$ B. 1
   C. $\frac{5}{4}$ D. $\frac{4}{3}$
   E. None of these
8. If $r \sin q = 1, r \cos q = 3$, then the value of $(3 \tan q + 1)$ is
   A. $\sqrt{3}$ B. $\frac{1}{\sqrt{3}}$
   C. 1 D. 2
   E. None of these
9. If $\tan q + \cot q = 2$ then the value of q is
   A. $45°$ B. $60°$
   C. $90°$ D. $30°$
   E. None of these
10. In ABC, angle $B = 90°$ and $AB : BC = 2 : 1$. The value of $\sin A + \cot C$ is
    A. $3 + \sqrt{5}$ B. $\frac{2+\sqrt{5}}{2\sqrt{5}}$
    C. $2 + \sqrt{5}$ D. $3\sqrt{5}$
    E. None of these
11. Which one of the following is true for $0° < q < 90°$?
    A. $\cos q \leq \cos^2 q$ B. $\cos q \geq \cos^2 q$
    C. $\cos q < \cos^2 q$ D. $\cos q > \cos^2 q$
    E. None of these
12. If $\tan x = \sin 45° . \cos 45° + \sin 30°$ then the value of x is
    A. 30° B. 45°
    C. 60° D. 90°
    E. None of these
13. The top of two poles of height 24 m and 36 m are connected by a wire. If the wire makes an angle of 60° with the horizontal, then the length of the wire is
    A. 6 B. $8\sqrt{3}$
    C. 8 D. $6\sqrt{3}$
    E. None of these
14. what is the value of $\tan 45°$
    A. 1 B. 0
    C. $\frac{1}{\sqrt{3}}$ D. $\sqrt{3}$
    E. None of the above
15. Find the value of $\cos^2 60° + \cos^2 45°$
    A. 1 B. 2
    C. 0 D. 3
    E. None of the above
16. Find the value of tan(60°) + cot(30°).
    A. 2 B. $\sqrt{3}$
    C. $3\sqrt{3}$ D. $2\sqrt{3}$
    E. None of the above
17. If cos(x) = 1/2 and x is in the first quadrant, find the value of sin(x).
    A. $\frac{1}{2}$ B. 1
    C. 0 D. $\frac{1}{\sqrt{2}}$
    E. $\frac{\sqrt{3}}{2}$
18. Find the value of sec(45°) + sec(60°).
    A. 2 B. 0
    C. $\sqrt{2} + 1$ D. $\sqrt{2} + 2$
    E. None of the above
19. If sin(x) = 3/5 and x is in the second quadrant, find the value of cos(x).
    A. $\frac{4}{5}$ B. $\frac{1}{4}$
    C. $-\frac{4}{5}$ D. $-\frac{1}{4}$
    E. None of the above

20. Find the value of tan(π/3) + sin(π/6).
    A. 1.732 B. 4.323
    C. 1.3485 D. 1.8999
    E. None of the above
21. If cos(x) = 3/5 and x is in the third quadrant, find the value of sin(x).
    A. $-\frac{1}{5}$ B. $\frac{4}{5}$
    C. 0 D. $\frac{1}{5}$
    E. $-\frac{4}{5}$
22. The minimum value of $\sin^2 q + \cos^2 q + \sec^2 q + \operatorname{cosec}^2 q + \tan^2 q + \cot^2 q$ is
    A. 1 B. 3
    C. 5 D. 7
    E. None of these
23. The value of $\frac{4}{1+\tan^2 \alpha} + \frac{3}{1+\cot^2 \alpha} + 3\sin^2 \alpha$ is
    A. 4 B. -1
    C. 2 D. 3
    E. None of these
24. If $7\sin^2 q + 3\cos^2 q = 4, (0° < q < 90°)$, then the value of $\tan q$ is
    A. $\frac{1}{\sqrt{3}}$ B. $\frac{1}{2}$
    C. 1 D. $\sqrt{3}$
    E. None of these
25. If $\sin A + \sin^2 A = 1$, then the value of $\cos^2 A + \cos^4 A$ is
    A. 1 B. 6
    C. 2 D. 4
    E. None of the above
26. If tan(x) = 5/12 and x is in the second quadrant, find the value of sec(x).
    A. $\frac{12}{5}$ B. $-\frac{12}{5}$
    C. 0 D. 1
    E. None of the above
27. The angles of depression of two ships from the top of a light house are 45° and 30° toward east. If the ships are 200m apart, the height of the light house is
    A. 273 m B. 270 m
    C. 253 m D. 263 m
    E. None of the above
28. If cos q + sin q = m and sec q + cosec q = n then the value of n ($m^2 - 1$) is equal to :
    A. 2m B. mn
    C. 4mn D. 2n
    E. None of the above
29. From the top of a building 60 metre high, the angles of depression of the top and bottom of a tower are observed to be 30° and 60° respectively. The height of the tower in metre is:
    A. 40 B. 45
    C. 50 D. 55
    E. none of the above
30. A boat is moving away from an observation tower. It makes an angle of depression of 60° with an observer 's eye when at a distance of 50 metre from the tower. After 8 seconds, the angle of depression becomes 30°. By assuming that it is running in still water, the approximate speed of the boat is:
    A. 33 km/hr B. 42 km/hr
    C. 45 km/hr D. 50 km/hr
    E. None of the above

ANSWER KEY:

1) B	11) D	21) E
2) D	12) B	22) D
3) D	13) B	23) A
4) C	14) A	24) A
5) A	15) E	25) A
6) A	16) D	26) E
7) B	17) E	27) A
8) D	18) D	28) A
9) A	19) C	29) A
10) B	20) E	30) C

CO-ORDINATE GEOMETRY

In coordinate geometry, points are placed on the coordinate plane. There are two scales, one is running across the plane called the x-axis and another scale is a right-angles to it called the y – axis.

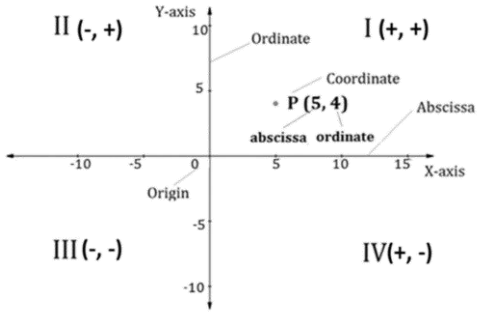
- The point of intersection of the x and the y-axis is known as the origin. At this point, both x and y are 0.
- The values on the right-hand side of the x-axis are positive and the values on the left-hand side of the x-axis are negative.
- Similarly, on the y-axis, the values located above the origin are positive and the values located below the origin are negative.
- By a set of two numbers a point on the plane can be located. First value will be of x axis and second value will be of y-axis which will determine the unified position on the plane.
- Coordinate is represented as (x, y).
- Origin: Intersection of the axes usually at (0,0). Origin is used as a reference for measuring the distances for coordinates.

Types of Coordinate Systems:

- **Cartesian coordinate system:** A cartesian coordinate system specifies a point by two numbers, which signifies the distance from two perpendicular axes, i.e., X-axis and Y-axis.
    - Ordinate: Vertical Axis i.e., Y-axis
    - Abscissa: Horizontal Axis i.e., X-axis

The ordinate and abscissa divide the plane into four parts, known as Quadrants.

- In Quadrant I both X and Y coordinates are positive
- In Quadrant II X coordinate is negative while Y coordinate is positive
- In Quadrant III X coordinate is positive while Y coordinate is negative
- In Quadrant IV both X and Y coordinates are negative



The distance between two points whose coordinates are A (x_1, y_1)and B (x_2, y_2) is:

$$\text{Distance} = \sqrt{(x_{2-} x_1)^2 + (y_{2-} y_1)^2}$$

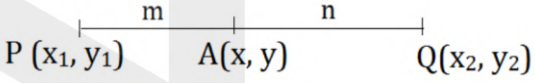
Note: The distance of any point P(x, y) from the origin can be given as

$$\text{Distance} = \sqrt{(x)^2 + (y)^2}$$

Section Formula: Section formula tells us about the coordinates of the Point, which divides a line segment into the ratio m : n.

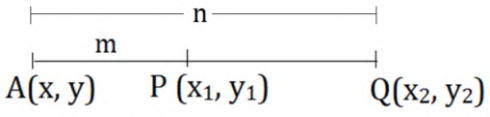
Internal division: The coordinates of the Point A(x, y) which divides the line segment PQ internally in the ratio m : n, as shown in the figure, will be given as:

$$(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$



External division: The coordinates of the Point A(x, y) which divides the line segment PQ externally such that AP : AQ = m : n, as shown in the figure, will be given as:

$$(x, y) = \frac{(mx_2 - nx_1)}{m-n}, \frac{my_2 - ny_1}{m-n}$$



NOTE:

1. The coordinates of a point A(x, y) which divides the line segment PQ into two equal halves (i.e., m : n = 1 : 1) is given as:

$$(x, y) = \frac{(x_2 + x_1)}{2}, \frac{y_2 + y_1}{2}$$

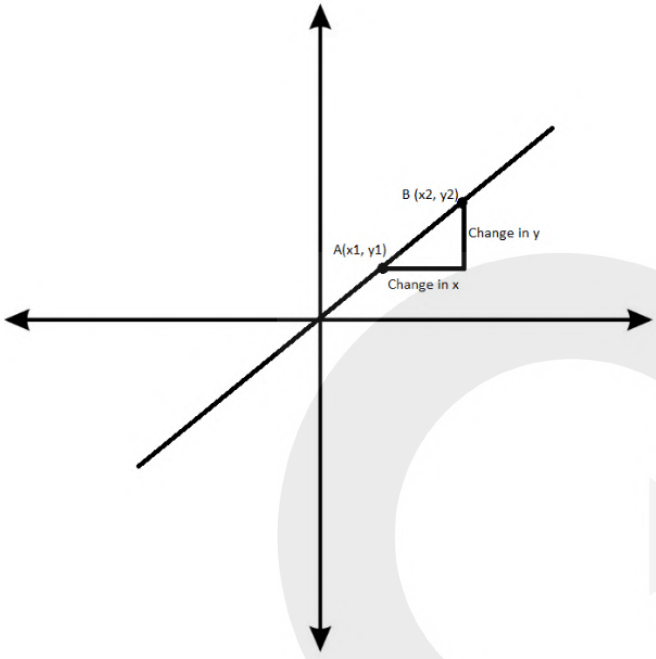
2. If a point A divides line PQ internally in the ratio m : n and the point B divides the line segment PQ externally in the ratio m : n, then

$$\frac{1}{AP}+\frac{1}{BP}=\frac{2}{PQ}$$

Slope of a Straight Line (m)

The slope of a line denotes the steepness of a line. Slope is the tangent of the angle made by straight line with the positive direction of X-axis.

Slope can also be defined as the change in Y-coordinate per unit change in X-coordinate.



The slope of a line AB having coordinates A(x_1, y_1) and B(x_2, y_2) is given as

$$m = tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$$

Note: Slope of X-axis is 0 and the slope of Y-axis is not defined.

Equation of a Straight Line

The equation of line parallel to X-axis is given as y = d

The equation of line parallel to Y-axis is given as x = c

There are many forms a straight line can be represented:

1. **Slope form:** The equation of straight-line having slope m is given as $y = mx + c$, where c is the intercept of a straight line on Y-axis.
2. **Intercept form:** The intercept form of a straight line having a and b as the intercepts on X-axis and Y-axis respectively is given as
$$\frac{x}{a}+\frac{y}{b}=1$$
3. **Point slope form:** The equation of line passing through Point (x_1, y_1) and having slope m is given as:
$$y - y_1 = m(x - x_1)$$
4. **Two-point form:** The equation of line passing through points (x_1, y_1) and (x_2, y_2) is given as
$$y - y_1 = \frac{(y_2 - y_1)}{x_2 - x_1}(x - x_1)$$
5. **The General form** of a straight line is given as ax + by + c = 0
6. **Normal Form:** Normal form of any straight line is defined by the length of the perpendicular (p) from the origin to the line and angle (θ) which perpendicular makes with the positive direction of X-axis. Any straight line can be represented in normal form as $xcos\,\theta + ysin\,\theta = p$

Note: The slope of a line can also be defined as the coefficient of x when the coefficient of y is 1.

Position of point A(x1, y1) and B(x2, y2) with respect to line ax + by + c = 0

If the value of $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ are of the same sign than the points lie on the same side of straight-line otherwise on the different side of the line.

Note: For a point (x_1, y_1), if $ax_1 + by_1 + c$ is equal to zero, then the point lies on the line ax + by + c = 0

Angles between two lines

The acute angle between two lines having slopes m_1 and m_2 is given as:

$$tan\theta = \left|\frac{(m_2 - m_1)}{1 + m_1 m_2}\right|$$

And, the obtuse angle can be found out directly by subtracting acute angle from 180^o

The acute angle between two lines having equation $ax_1 + by_1 + c$ and equation $ax_2 + by_2 + d = 0$ will be

$$tan\theta = \frac{(b_1 a_2 - a_1 b_2)}{a_1 a_2 + b_1 b_2}$$

Equation of a line parallel to a line

The family of equation of lines perpendicular to line ax + by + c = 0 is given as ax + by + d = 0

1. When two lines are parallel to each other then, the angle between them is zero and slopes of them are equal, i.e., $m_1 = m_2$

Equation of a Line perpendicular to a line

The family of equation of lines perpendicular to line ax + by + c = 0 is given as bx – ay + d = 0

1. When two lines are perpendicular, then the angle between them is 90^0

2. When two lines $a_1x + b_1y + c = 0$ and $a_2x + b_2y + d$ are perpendicular, then $a_1a_2 + b_1b_2 = 0$
3. The product of the slopes of two perpendicular lines is always 1, i.e., $m_1m_2 = -1$

Distance of a point from a line

The shortest distance from a point P (x_1, y_1) to the straight line having equation Ax + By + c = 0 will be the length of the perpendicular drawn from point P to the line Ax + By + C = 0

$$Distance = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Shortest distance between two parallel lines

The shortest distance between two parallel lines having equations $y = mx + c_1$ and $y = mx + c_2$ is given as

$$Distance = \frac{|c_1 - c_2|}{\sqrt{1 + m^2}}$$

The shortest distance between two parallel lines having equation $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is given as

$$Distance = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$$

Condition for collinear points

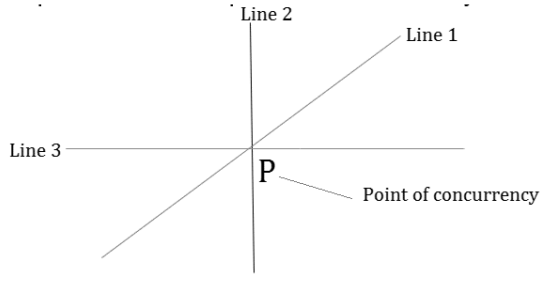
The points are said to be collinear when all the points lie in a straight line.

The points A (x_1, y_1), B (x_2, y_2) and C(x_3, y_3) will be collinear to each other if slope of line segment AB is equal to the slope of line segment BC. Equating slope of both the lines we get a direct result

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_3 - y_2}{x_3 - x_2}$$

Condition for concurrent lines

The pair of lines is said to be concurrent to each other if all the lines intersect each other at the common point P, which is known as Point of concurrency.



The lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$, and $a_3x + b_3y + c_3 = 0$ will be concurrent to each other if and only if

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

Projection of a point on any straight line

To find the projection of a point P (x_1, y_1) on a line Ax + By + C = 0 can be found using the following steps:

Step 1: First find the equation of line perpendicular to Ax + By + C = 0

The line perpendicular to Ax + By + C = 0 will be Bx – Ay + D = 0

Step 2: Now, put the coordinates of point P (x_1, y_1) in the line Bx – Ay + D = 0 as the points satisfy the equation to find the value of D.

$\Rightarrow Bx_1 - Ay_1 + D = 0$

Step 3: Now, find the intersection point of line Ax + By + C = 0 and Bx – Ay + D = 0 to obtain the projection of a point on a line Ax + By + C = 0

Reflection of a point

- The reflection of a point (x_1, y_1) about X-axis is $(x_1, -y_1)$
- The reflection of a point (x_1, y_1) about Y-axis is $(-x_1, y_1)$
- The reflection of a point (x_1, y_1) about line y = x is (y_1, x_1)
- The reflection of a point (x_1, y_1) about line y = – x is $(-y_1, -x_1)$
- The reflection of a point (x_1, y_1) about origin (0, 0) is $(-x_1, -y_1)$

How to find reflection of a point about any straight line

Step 1: First we have to find the projection of the Point on the line (SEE ABOVE)

Step 2: Now, distance between Point and its projection is similar to the distance between the reflection of Point and projection of Point so using either section formula or formula for distance between two points we can get the coordinates of reflection of a point.

Coordinates of Centroid of a triangle

The coordinates of a centroid of a triangle ABC having vertices as A (x_1, y_1), B (x_2, y_2) and C(x_3, y_3) is given as

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Coordinates of Incentre of a triangle

The coordinates of an Incentre of a triangle ABC having vertices as A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) and sides a = BC, b = CA and c = AB is given as

$$\left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cx_3}{a + b + c}\right)$$

NOTE: Centroid divides the line formed by joining Orthocentre and Circumcentre internally in the ratio 2:1.

Area of a triangle

The area of a triangle ABC formed from the vertices A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) will be

$$Area = {}^1/_2 \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$
$$= \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Note: The vertices should be taken in either a clockwise direction or an anticlockwise direction.

NOTE: If the area of the triangle is zero, then all the vertices A, B, and C are collinear to each other, i.e., all the points lie on a straight line.

NOTE: The Area of triangle for line $y = m_1x + c_1$, $y = m_2x + c_2$ and $x = 0$ is given as

$$Area = \frac{|{c_1}^2 + {c_2}^2|}{2(m_1 - m_2)}$$

Area of a Polygon

The area of a polygon having n vertices in the form (x_i, y_i) where i = 1, 2, 3, ……, n can be calculated as

$$Area = \frac{1}{2} |x_i y_i + 1 - y_i x_i + 1|$$

Solved Examples:

1. Find the slope of the line passing through the coordinates A(1, 9) and B(3, –5)?

Solution

We know,

$$Slope\ (m) = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\therefore Slope = \frac{-5-9}{3-1} = -\frac{14}{2} = -7$$

Thus, slope of the given line will be –7.

2. Find the slope of the equation 3y = 4x + 9

Solution

Step 1: Firstly, make the coefficient of y as 1

3y = 4x + 9

$$y = \frac{4x}{3} + \frac{9}{3}$$

Step 2: Now, the slope of equation is the coefficient of x i.e. slope (m) = 4/3

3. If the line formed by the points (1, 4) and (3, k) is perpendicular to the line 3x – 4y = 5, then what will be the value of k?

Solution

Step 1: Find the slope of line 3x – 4y = 5

Slope is the coefficient of x when coefficient of y is 1. $y = \frac{3x}{4} - \frac{5}{4}$

So, slope for line will be $m_1 = 3/4$

Step 2: Find the slope for line formed by points (1, 4) and (3, k)

$$m_2 = \frac{k-4}{3-1} = \frac{k-4}{2}$$

Step 3: for two lines perpendicular to each other $m_1 * m_2 = -1$

$$\frac{3}{4} \times \frac{k-4}{2} = -1$$

$$3k - 12 = -8$$

$$k = \frac{4}{3}$$

4. What will be the projection of (3, 1) on the line 5x – 3y = 4?

Solution

Step 1: Equation of Line perpendicular to 5x – 3y = 4 will be

⇒ 3x – 5y = D

Step 2: Putting coordinates of (3, 1) in the above equation to find the value of D

⇒ D = 3 * 3 – 5 * 1 = 9 – 5 = 4

Step 3: Find the intersection point of 5x – 3y = 4 and 3x – 5y = 4

Solving above equation for x and y we get

x = 1/2 and y = –1/2

Thus, projection of (3, 1) on the line 5x – 3y = 4 is $(\frac{1}{2}, \frac{-1}{2})$

5. Find the area of the quadrilateral having vertices (1, 4), (2, 6), (5, 4) and (4, 3)

Solution

Step 1: write x coordinates of all the vertices in a column and write y coordinates of all the vertices in another column. Repeat the coordinates of first point in the last row to complete the calculation of area.

x_i	y_i
1	4
2	6
5	4
4	3
1	4

Step 2: Now multiply the first row coordinate of x with second-row coordinate of y and then from the obtained result subtract product of first row coordinate of y and second row coordinate of x. Do the step for all the coordinates and then add all the numbers thus obtained

⇒ (1*6 – 2*4) + (2*4 – 5*6) + (5*3 – 4*4) + (4*4 – 1*3)

⇒ –2 – 22 – 1 + 13

⇒ – 12

Thus, area of quadrilateral will be

$$\frac{1}{2}\ Obtained\ result = \frac{1}{2} * |-12| = 6 \text{ sq. unit}$$

Exercise – 1

1. Slope of the line AB is -4/3. Co-ordinates of points A and B are (x, -5) and (-5, 3) respectively. What is the value of x?
   A. -1 B. 2
   C. -2 D. 1
   E. 0
2. Find k, if the line 2x – 3y = 11 is perpendicular to the line 3x + ky = 4?
   A. -2 B. -1
   C. 1 D. 2
   E. 0
3. What is the slope of the line perpendicular to the line passing through the points (-5, 1) and (-2, 0)?
   A. -3 B. 3
   C. -1/3 D. 1/3
   E. None of these
4. At what point does the line 3x + y = -6 intercept the x-axis?
   A. (2, 0) B. (-2, 0)
   C. (0, -6) D. (0, 6)
   E. None of these
5. The point Q (a, b) is first reflected in y-axis to Q1 and Q1 is reflected in x-axis to (-6, 2). The co-ordinates of point Q are
   A. (–6, –2) B. (2, –6)
   C. (6, –2) D. (–2, 6)
   E. None of these
6. Point A divides segment BC in the ratio 4:1 co-ordinates of B are (6, 1) and C are (7/2, 6). What are the co-ordinates of point A?
   A. (4, 3) B. (4, 5)
   C. (2, 5) D. (3, 5)
   E. None of these
7. What is the equation of the line if its slope is -2/5 and y-intercept is 6?
   A. 2x + 5y = 6 B. 2x + 5y = 30
   C. 2x – 5y = 6 D. 2x + 5y = -30
   E. None of these
8. What is the reflection of the point (–3, 1) in the line x = –2?
   A. (–1, 1) B. (–3, –5)
   C. (1, 1) D. (–3, 5)
   E. (–1, –1)
9. What is the reflection of the point (5, –1) in the line y = 2?
   A. (5, -5) B. (-5, -5)
   C. (5, 5) D. (-5, 5)
   E. None of these
10. What is the slope of the line parallel to the line passing through the points (5, -1) and (4, -4)?
    A. -3 B. -1/3
    C. 3 D. 1/3
    E. 0
11. Point P (-2, 5) is the midpoint of segment AB. Co-ordinates of A are (-5, y) and B are (x, 3). What is the value of x?
    A. 1 B. -1
    C. 2 D. -2
    E. 3
12. A line cuts the x-axis at the point (-3, 0) and the y-axis at the point (0, 6). What is the equation of the line?
    A. x = 2y + 6 B. y = 2x – 6
    C. x = 2y – 6 D. y = 2x + 6
    E. Cannot be determined
13. What is the slope of the line 2x – 5y = 12?
    A. 2/5 B. 5/2
    C. -2/5 D. -5/2
    E. 1/5
14. The point P (a, b) is first reflected in origin to P1 and P1 is reflected in Yaxis to (4, 3). The coordinates of point P are
    A. (4, –3) B. (4, 6)
    C. (3, 4) D. (3, 6)
    E. (–4, 3)
15. What is the reflection of the point (5, 2) in the line x = -3?
    A. (–11, 2) B. (–11, –2)
    C. (11, –2) D. (11, 2)
    E. Cannot be determined
16. What are the co-ordinates of the centroid of a triangle, whose vertices are A (1, -5), B (4, 0) and C (-2, 2)?
    A. (1, -1) B. (-1, 1)
    C. (2, -2) D. (-2, 2)
    E. (1, 2)

17. If ax – 4y = –6 has a slope of –3/2. What is the value of a?
A. 6 B. 3
C. –6 D. –3
E. 2

18. What is the slope of the line parallel to the line passing through the points (6, 3) and (2, 1)?
A. ½ B. 1
C. 2 D. 1
E. 3/2

19. What is the reflection of the point (5, –2) in the line x = –1?
A. (–7, –2) B. (5, 0)
C. (7, –2) D. (5, 2)
E. (–5, 2)

20. What is the equation of a line having a slope –1/3 and y-intercept equal to 6?
A. x + 3y = 18 B. x – 3y = 6
C. x + 3y = –18 D. x – 3y = –6
E. None of these

21. What is the equation of the line perpendicular to the line 5x + 3y = 6 and having Y-intercept –3?
A. 3x – 5y = 15 B. 3x + 5y = 15
C. 3x – 5y = –15 D. 3x + 5y = –15
E. Cannot be determined

22. What is the reflection of the point (4, –3) in the line y = –2?
A. (4, 1) B. (–4, 1)
C. (–4, –1) D. (4, –1)
E. None of these

23. What is the slope of the line parallel to the line passing through the points (4, –2) and (–3, 5)?
A. 3/7 B. 1
C. –3/7 D. –1
E. 2

24. What is the reflection of the point (–2, 5) in the line x = –1?
A. (–2, –7) B. (0, 5)
C. (2, 5) D. (–2, 7)
E. None of these

25. The distance of the point P (2, 3) from the x-axis is
A. 2 B. 3
C. 1 D. 4
E. 5

26. In what ratio is the segment joining (–1, –12) and (3, 4) divided by the x-axis?
A. 1:3 B. 3:2
C.3:1 D. 2:3
E. None of these

27. What is the equation of the line whose yintercept is 3/4 and making an angle of 45° with the positive x axis?
A. 4x – 4y = 2 B. 4x – 4y = –3
C. 3x – 3y = 4 D. 3x – 4y = 4
E. Cannot be determined

28. A line passing through the origin perpendicularly cuts the line 3x – 2y = 6 at point M. Find M?
A. (18/13, 12/13) B. (18/13, –12/13)
C. (–18/13, –12/13) D. (–18/13, 12/13)
E. None of these

29. Point P is the midpoint of segment AB. Co-ordinates of P are (3, 1) and B are (5, –4) What are the coordinates of point A?
A. (–1, 7) B. (1, –7)
C. (1, 6) D. (–1, –7)
E. (1, –6)

30. At what point does the line 4x – 3y = –6 intercept the y-axis?
A. (0, 2) B. (0, 3/2)
C. (2, 0) D. (3/2, 0)
E. (0, 3)

ANSWER KEY:

1) D	11) A	21) A
2) D	12) D	22) D
3) B	13) A	23) D
4) B	14) A	24) B
5) C	15) A	25) B
6) B	16) A	26) C
7) B	17) C	27) B
8) A	18) A	28) B
9) C	19) A	29) C
10) C	20) A	30) A

PERMUTATION & COMBINATION

Factorial:

- n! = n(n-1)(n-2)1

E.g. – 5! = 5(5-1)(5-2)(5-3)(5-4) = 5(4)(3)(2)(1)

- 0! = 1

n! Only exists of n≥0 and doesn't exist for n<0.

n	n!	n	n!
0	1	6	720
1	1	7	5040
2	2	8	40,320
3	6	9	362880
4	24	10	3628800
5	120		

Combination:

- Each of the different groups or selections which can be formed by taking some or all of a number of objects is called a combination.
- This is denoted by ${}^{n}C_{r}$.
- Combination is the selection of items in which order does not matters.
- Number of ways of selection of items in which order does not matter.
  ${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$, where n ≥ r (n is greater than or equal to r)
- We use combinations if an issue arises for the number of tricks of choosing things along with the series of choice is not to be considered.
- The values of the given combinations of n factors, reserved r at a time is:
  ${}^{n}C_{r} = \frac{n!}{r!(n-r)!} = n(n-1)(n-2)\ldots\frac{\text{To r factors}}{r!}$
- While solving combinations, when n = r, the value of combinations is constantly equal to 1.

Formulae:

- Number of all combinations of n things, taken r at a time, is given by:
  ${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$
- N-combinations from a set with n elements (without repetition):
  ${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$
- n-combinations from a set with n elements (with repetition)
  ${}^{r+n-1}C_{r} = {}^{n}Cr$
- ${}^{n}C_{n} = 1$
- ${}^{n}C_{0} = 1$
- ${}^{n}C_{r} = {}^{n}C_{n-r}$
- ${}^{n}C_{0} + {}^{n}C_{1} + {}^{n}C_{2} + {}^{n}C_{3} + \cdots + {}^{n}C_{n} = 2^{n}$
- Combination is an arrangement of objects where order does not matter.
- There are two easy methods for solving combination questions:
  - **Repetition is allowed:**
    - Formula: ${}^{r+n-1}C_{r}$
  - **Repetition is not allowed:**
    - Formula: ${}^{n}C_{r}$

Shortcut:

Regular method:

${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$

${}^{5}C_{2} = \frac{5!}{2!(5-2)!} = \frac{5!}{2!(3)!} = \frac{5\times4\times3\times2\times1}{(2\times1)\times(3\times2\times1)} = 10$

Shortcut:

${}^{n}C_{r} = \frac{n!}{r(n-r)!}$

${}^{5}C_{2} = \frac{5!}{2!(5-2)!} = \frac{5\times4}{2\times1}$

i.e., write factorial of 5 only upto 2 & in numerator write only 2!

${}^{5}C_{2} = 10$

E.g.

- Selections for people from total numbers who want to go out on a picnic.
- Filling posts with people.
- Selection for sports team out of available players
- Selection of balls from a bag

Solved examples:

Type 1: how to solve combination question (with or without repetition)

1.There was a flock of sheep in which 6 were male sheep 5 were female sheep. Now we need to select 4 sheep to take out wool from them. In how many different ways can they be selected such that at least one male sheep should be there?

A. 325 B. 302

C. 295 D. 154

Solution

The selection can be made in following manner

(1 male sheep and 3 female sheep) or (2 male sheep and 2 female sheep) or (3 male sheep and 1 female sheep) or (4 male sheep)

required number of ways =

$({}^6C_1 \times {}^5C_3) + ({}^6C_2 \times {}^5C_2) + ({}^6C_3 \times {}^5C_1) + ({}^6C_4)$

Required number of ways =

$(6 \times 10) + (15 \times 10) + (20 \times 5) + 15$

Required number of ways = 60 + 150 + 100 + 15

Required number of ways = 325

Correct Option: A

2.Among a set of 5 white balls and 3 blue balls, how many selections of 5 balls can be made such that at least 3 of them are white balls.

A. 45 B. 46

C. 44 D. 40

Solution

The selection can be made in following manner

(3 white ball and 2 blue ball) or (4 white ball and 1 blue ball) or (5 white ball)

$= {}^5C_3 \times {}^3C_2 + {}^5C_4 \times {}^3C_1 + {}^5C_5 = (10 \times 3) + (5 \times 3) + 1$

$= 30 + 15 + 1 = 46$

Correct Option: B

3.There are 7 consonants and 4 vowels. Find out how many words of 3 consonants and 2 vowels can be formed?

A. 120 B. 102

C. 20 D. 210

Solution

Number of ways of selecting 3 consonants from 7 = ${}^7C_3$

Number of ways of selecting 2 vowels from 4 = ${}^4C_2$

Number of ways of selecting 3 consonants from 7 and 2 vowels from 4 = ${}^7C_3 \times {}^4C_2 = 35 \times 6 = 210$

Correct Option: D

4.In how many ways can a team of 5 cricketers can be formed out of a total of 10 cricketers such that two particular cricketers should be included in each team?

A. 66 B. 65

C. 56 D. 22

Solution

Two particular cricketers should be included in each team.

Therefore, select remaining 5-2=3 cricketers from 10-2=8 cricketers = ${}^8C_3 = \frac{8!}{3!5!} = 56$

Correct Option: C

5.There are 3 types of t-shirts available in a clothing store. In how many ways can 5 t-shirts be selected?

A. 21 B. 42

C. 5 D. 12

Solution

${}^{r+n-1}C_r = {}^{5+3-1}C_{15} = {}^7C_5 = \frac{7!}{2!5!} = 21$

Correct Option: A

Permutations

- The different arrangements of a given number of things by taking some or all at a time, are called permutations.
- Permutation is the arrangement of items in which order matters.
- Number of ways of selection and arrangement of items in which order matters.
- This is denoted by ${}^{n}P_{r}$.
- Number of all permutations of n things, taken r at a time, is given by:
  ${}^{n}P_{r} = {}^{n}C_{r} \times r! = \frac{n!}{(n-r)!}$, where $n \geq r$
- The permutation of n events, taken r at a time, is represented as:
  ${}^{n}P_{r} = {}^{n}C_{r} \times r! = \frac{n!}{(n-r)!} = n(n-1)(n-2)\ldots(n-r+1)$

E.g.

- Arrangement of letters/alphabets to form words with meaning or without meaning.
- Arrangements of balls on a table.

Shortcut:

Regular method:

${}^{n}P_{r} = \frac{n!}{(n-r)!}$

${}^{5}P_{2} = \frac{5!}{(5-2)!} = \frac{5!}{(3)!} = \frac{5\times4\times3\times2\times1}{3\times2\times1} = 20$

Shortcut:

${}^{n}P_{r} = \frac{n!}{(n-r)!}$

${}^{5}P_{2} = = 5 \times 4 = 20$

Solved examples:

Type 1: different ways to arrange (with repetition)

1.How many 3 letter words with or without meaning can be formed out of the letters of the word Monday when repetition of words is allowed?

A. 125 B. 216

C. 120 D 320

Solution

6 × 6 × 6 = 216 or

We can solve directly by formula $n^{r} = 6^{3} = 216$

Correct Option: B

2.In how many ways the letters in the word tooth can be arranged?

A. 120 B. 40

C. 20 D. 30

Solution

$\frac{5!}{2!\times2!} = \frac{5\times4\times3\times2\times1}{2\times1\times2\times1} = \frac{120}{4} = 30$

Correct Option: D

3.how many three-digit numbers can be formed using digits 2, 3, 4, 7, 9 so that the digits can be repeated?

A. 125 B. 360

C. 24 D. 6

Solution

Each place can be filled by any one of 5 digits

Total numbers = 5 × 5 ×5 =125

Or

We can solve directly by formula $n^{r} = 5^{3} = 125$

Correct Option: A

Type 2: Different ways to arrange (without repetition)

1. How many five letter words with or without meaning, can be formed from the word 'complexify', if repetition of letters is not allowed?

A. 43200 B. 30240

C. 12032 D. 36000

Solution

${}^{10}P_{5} = \frac{10!}{(10-5)!} = 10 \times 9 \times 8 \times 7 \times 6 = 30240$

Correct Option: B

2. In how many different ways can the letters of the word 'logarithms' be arranged so that the vowels always come together?

A. 6720 B. 241920

C. 40320 D. 360344

Solution

In such questions we treat vowels as one letter.

So, the word becomes lgrthms (oai)

It means there are total 8 letters. Therefore, number of ways of arranging these letters = 8! = 40320

Now, there are three vowels (oai), number of ways of these letters can be arranged = 3! = 6
Required number of words = 40320 × 6 = 241920
Correct Option: B

3. How many three-digit numbers can be formed from the digits 3, 4, 5, 7, 8, and 9. Also, the number formed should be divisible by 5 and no repetition is allowed?

A. 20 B. 24
C. 25 D. 10

Solution

The number which is divisible by 5 has 5 or 0 at one's place. In this case we must have 5 at the unit place as 0 is not in the list. There are total 6 digit out of which last digit is fixed by 5. Therefore, we are left with 5 digits (3, 4, 7, 8, 9) at the tens place.
Similarly, the hundred place can be filled by 4 digits.
So, required number = 4 × 5 × 1 = 20
Correct Option: A

4. In how many ways can the letters of the word 'leader' be arranged?

A. 720 B. 360
C. 200 D. 120

Solution

Letter 'e' appears twice and all other letters 1l, 1a, 1d and 1r appears once in the word.
Required number of ways:
$\frac{6!}{2!} = \frac{6\times5\times4\times3\times2\times1}{2\times1} = \frac{720}{2} = 360$
Correct Option: B

Type 3: different ways to arrange (without repetition)

1. How many different ways are there to arrange your first three classes if they are math, English and Hindi?

A. 4 B. 6
C. 120 D. 36

Solution

n! = 3! = 6
Correct Option: B

Type 3: Different ways to select (with repetition)

1. In a shop there are 4 types of sweets. In how many ways can Shekhar buy 19 sweets?

A. 480 B. 540
C. 720 D. 1540

Solution

$^{r+n-1}C_r = {}^{19+4-1}C_{19} = {}^{22}C_{19} = \frac{22!}{(22-19)!19!} = 1540$
Correct Option: D

Type 4: Different ways to select (without repetition)

1. How many different 4-digit numbers can be formed using the digits 2,3,4,5,6,7,8 no digit being repeated in any number

A. 720 B. 120
C. 24 D. 840

Solution

The thousand places can be filled in 7 ways, the hundredth place can be filled in 6 ways, the tens place can be filled in 5 ways, and the ones place can be filled in 5 ways.
Total ways = 7×6×5×4 = 840
Correct Option: D

Circular permutation:

Circular permutation are arrangements in the closed loops.

- If clockwise and anti-clock-wise orders are different, then total number of circular-permutations is given by $(n-1)!$
- If clock-wise and anti-clock-wise orders are taken as not different, then total number of circular-permutations is given by: $\frac{(n-1)!}{2}$
- Number of circular-permutations of 'n' different things taken 'r' at a time:-
  - Case 1: if clock-wise and anti-clockwise orders are taken as different, then total number of circular-permutations = ${}^{n}P_{r}$ / r
  - Case 2: if clock-wise and anti-clockwise orders are taken as not different, then total number of circular – permutation = ${}^{n}P_{r}$ / 2r
- There are two cases of circular-permutations:
  - If clockwise and anti-clock-wise orders are different, then total number of circular-permutations is given by (n-1)!
  - If clock-wise and anti-clock-wise orders are taken as not different, then total number of circular-permutations is given by: $\frac{(n-1)!}{2!}$
- The number of ways to arrange distinct objects along a fixed (i.e., cannot be picked up out of the plane and turned over) circle is (n-1)!
- There are two easy methods for solving permutation questions:
  - Repetition is allowed:
    - Formula: n^r
  - Repetition is not allowed
    - Formula: ${}^{n}P_{r}$

Important properties:

- Property 1:
  Number of permutations (or arrangements) of n different things taken all at a time = n!
- Property 2:
  For objects in which p1 are alike and are of one type, p2 are alike or other different type and p3 are alike or another different type and the rest must be all different, number of permutations = $\frac{n!}{p1!p2!p3!}$
- Property 3:
  When repetition is allowed number of permutations of n different things taken r at a time = n × n× n ×⋯ (r times) = n^r
- Property 4:
  Here, we are counting the number of ways in which k balls can be distributed into n boxes under various conditions.

The conditions which are generally asked are

1. The balls are either distinct or identical.
2. No box can contain more than one ball or any box may contain more than one ball.
3. No box can be empty or any box can be empty.

Distribution of		How many balls can the box contain			
K balls	Into n boxes	No restrictions	≤1 (At most 1)	≥1 (at least 1)	= 1 (Exactly one)
Distinct	Distinct	n^k	${}^{n}P_{k}$	S(k,n)×n!	${}^{n}P_{n}$ = n! If k = n 0 if k ≠ n
Identical	Distinct	${}^{(k+n-1)}C_{(n-1)}$	${}^{n}C_{k}$	${}^{(k-1)}C_{(n-1)}$	1 if k =n 0 if k ≠ n

Other properties:

- ${}^{n}P_r = r! \times {}^{n}C_r$
- ${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$
- ${}^{n}C_x = {}^{n}C_y$ when x=y or x+y=n
- ${}^{n}C_r = {}^{n}P_{n-r}$
- $r \times {}^{n}C_r = n \times {}^{n-1}C_{r-1}$
- ${}^{n}C_r / r+1 = {}^{n-1}C_{r+1} / n+1$
- For ${}^{n}C_r$ to be greatest,
    - If n is even, $r = \frac{n}{2}$
    - If n is odd, $r = \frac{n+1}{2}$ or $\frac{n-1}{2}$

Solved examples

Type 1: When clockwise and anticlockwise arrangements are different.

Trick: number of circular permutations (arrangements) of n distinct things when arrangements are different = (n–1)!

1. In how many ways can 5 girls be seated in a circular order?

A. 45 B. 24

C. 12 D. 120

Solution

Number of arrangements possible = (5 – 1)! = 4! = 4 x 3 x 2 x 1 = 24

Correct Option: B

2. Determine the number of ways in which 5 married couples are seated on a circular round table if the spouses sit opposite to one another.

A. 120 B. 320

C. 384 D. 387

Solution

5 Married couples mean we have to arrange 10 peoples

First women can be placed anywhere in a circular round table and her husband in 1 way.

Second women can be placed in 8 ways and her husband in 1 way.

Third woman can be placed in 6 ways and her husband in 1 way.

Fourth women can be placed in 4 ways and her husband in 1 way.

Fifth women can be placed in 2 ways and her husband in 1 way.

Total number of ways = $1 \times 8 \times 1 \times 6 \times 1 \times 4 \times 1 \times 2 \times 1 = 384$ ways

Correct Option: C

Type 2: When clockwise and anticlockwise arrangements are not different

Tips & trick: number of circular permutations (arrangements) of n distinct things when arrangements are not different = $\frac{1}{2} \times (\mathbf{n-1})!$

1. In how many ways can 8 beads can be arranged to form a necklace?

A. 2520 B. 5040

C. 360 D. 1200

Solution

Since in formation of necklace the clockwise and anti-clockwise arrangements are same so we divide by 2

Number of arrangements possible = $\frac{1}{2} \times (N-1)! = = \frac{1}{2} \times (8-1)! = \frac{1}{2} \times 7! = \frac{1}{2} \times 5040 = 2520$

Correct Option: A

Exercise – 1

1. In how many different ways can the letters of the word 'dealing' be arranged in such a way that the vowels always come together
   A. 720 B. 480
   C. 360 D. 5040
   E. None of these
2. From a group of 7 boys and 6 girls, five persons are to be selected to form a group so that at least 3 boys are there on the group. In how many ways can it be done?
   A. 456 B. 786
   C. 756 D. 735
   E. None of these
3. How many 3-digit numbers can be formed from the digits 1, 2, 3, 5, 6, and 7, which are divisible by 5 and none of the digits is repeated?
   A. 5 B. 10
   C. 15 D. 20
   E. 25
4. A box contains 2 white marbles, 3 black marbles and 4 red marbles. In how many ways can 3 marbles be drawn from the box, if at least one black marble is to be included in the draw?
   A. 36 B. 48
   C. 69 D. 64
   E. None of these
5. In how many different ways can the letters of the word 'retail' be arranged in such a way that the vowels occupy only the odd positions?
   A. 32 B. 48
   C. 120 D. 60
   E. 36
6. Calculate in how many ways can a group of 5 men and 2 women be made out of a total of 7 men and 3 women?
   A. 78 B. 63
   C. 90 D. 64
   E. 145
7. In how many different ways can the letters of the word 'tropical' be arranged so that the vowels always come together?
   A. 120 B. 720
   C. 4320 D. 2160
   E. None of these
8. How many 5-letter words with or without meaning, can be formed out of the letters of the word, 'logarithms', if repetition of letters is not allowed?
   A. 40 B. 400
   C. 5040 D. 2520
   E. 30240
9. How many numbers are there between 98 and 1001, having at least one of their digits 6?
   A. 562 B. 344
   C. 272 D. 252
   E. None of these
10. A box contains 3 pink balls, 4 white balls and 3 red balls. The number of ways in which three balls can be drawn from the box so that at least one of the balls is white is
    A. 50 B. 100
    C. 150 D. 200
    E. 300
11. In how many ways can 6 prizes be distributed to 8 students, if each student can get any number of prizes?
    A. 40 B. 6^8
    C. 120 D. 8^6
    E. 360
12. In how many ways can 5 Indians, 7 Pakistanis and 6 Dutch be seated in a row so that all persons of the same nationality sit together?
    A. 3! B. 7!×6!×5!
    C. 3!×7!×6!×5! D. 182
    E. None of these
13. How many numbers of four digits can be formed with the digits 0, 3, 4, 5 (repetition of digits being allowed)?
    A. 12 B. 256
    C. 108 D. 384
    E. 192
14. For the arrangements of the letters of the word Patna, how many words would start with the letter P?
    A. 24 B. 12
    C. 60 D.120
    E. 72
15. How many numbers between 2000 and 3000 can be formed with the digits 0, 1, 2, 3, 4, 5, 6, 7 (repetition of digits not allowed)?
    A. 210 B. 420
    C. 252 D. 130
    E. 256

16. How many 4-digit numbers are possible, criteria being all the digits are even?
A. 625 B. 125
C. 45 D. 500
E. 225

17. There are 6 types of envelopes and 5 types of stamps in the post office. How many ways are there to buy an envelope and a stamp?
A. 20 B. 40
C. 30 D. 10
E. 50

18. In how many ways are there to choose 4 cards of different suite but same values from a deck of 52 cards?
A. 39 B. 42
C. 65 D. 13
E. None of these

19. There are 6 plates and 4 cups. In how many can they be arranged in a row so that no cups are place together?
A. $6! \times {}^7P_4$ B. 10!/(4!)(6!)
c. 6^4 D. 120
E. None of these

20. Several players took part in a tournament, each of which must play one game with all other players. How many players took part in the tournament if they played 55 games in all?
A. 5 B. 10
C. 11 D. 15
E. 20

21. In how many ways, five watches can be chosen from an unlimited number of titan, reebok and citizen watches?
A. 27 B. 81
C. 243 D. 9
E. 125

22. If we have to make 8 guys sit alternately with 8 girls around a round table which is numbered, then the number of ways in which this can be done is
A. $8! \times 7!$ B. $2 \times (8!)^2$
C. $8! \times 8!$ D. $7! \times 6!$
E. None of these

23. Nine points are marked on a straight line and ten points are marked on another straight line. How many triangles can be constructed with vertices from among the above points?
A. 495 B. 765
C. 1045 D. 550
E. 2475

24. If we have to make 6 boys sit with 6 girls around a round table, then the number of different relative arrangements of boys and girls that we can make so that there are no two boys or any two girls sitting next to each other is
A. $6! \times 6!$ B. $6! \times 7!$
C. $(6!)^2$ D. $6! \times 5!$
E. None of these

25. A college has 9 basketball players. A 5-member team and a captain will be selected out of these 9 players. How many different selections can be made?
A. 504 B. 635
C. 717 D. 736
E. 516

26. There are 18 stations between Mumbai and Delhi. How many second-class tickets have to be printed, so that a passenger can travel from any station to any other station?
A. 280 B. 380
C. 480 D. 680
E. Cannot be determined

27. How many numbers between 200 and 1200 can be formed with the digits 0, 1, 2, 3 (repetition of digits not allowed)?
A. 6 B. 4
C. 3 D. 20
E. 14

28. For the above question, how many numbers can be formed with the same digits if repetition of digits is allowed?
A. 14 B. 63
C. 48 D. 33
E. 49

29. How many distinct words can be formed out of the word 'crawling' which start and end with 'a' or 'w'?
A. 6! B. 24
C. 120 D. 6!/2
E. 6!×2

30. How many numbers can be formed with the digits 3, 4, 5, 6, 5, 4, 3 so that the odd digits always occupy the odd places?
A. 20 B. 12
C. 15 D. 18
E. 21

31. A set of 15 different numbers are given. In how many ways is it possible to choose a subset of not more than five numbers?
A. 4^{15} B. 4944
C. 15^4 D. 4954
E. None of these

Directions for questions 32 and 33: Answer these questions based on the information given below. Each of the eleven letters A, H, I, M, O, T, U, V, W, X and Z appear same when looked at in a mirror. They are symmetric while other being asymmetric.

32. How many four lettered passwords can be formed using only the symmetric letters (no repetition allowed?
A. 419430 B. 330
C. 7920 D. 14640
E. None of these

33. How many 3 letter passwords can be formed (no repetition allowed) with at least one symmetric letter?
A. 990 B. 2730
C. 15600 D. 12780
E. 12870

34. If the letters of the word 'chase' are rearranged to form 5 letter words such that none of the word repeat and the results arranged in ascending order as in a dictionary what is the rank of the word 'chase'?
A. 40 B. 28
C. 32 D. 36
E. 38

35. In how many ways can 5 different caps be placed on 3 identical hooks such that no hook is empty, if any of the hooks may hold all of the caps?
A. 10 B. 15
C. 25 D. 20
E. 35

36. A polygon has 35 diagonals, then the number of its sides are?
A. 10 B. 11
C. 9 D. 12
E. Cannot be determined

37. Consider the word Nitin. Whichever way you read it, from left to right or from right to left, you get the same word. Such a word is known as palindrome. Find the maximum possible number of 5-letter palindromes.
A. 17756 B. 17675
C. 17576 D. 12657
E. 12666

38. Find the total numbers greater than 3000 that can be formed with digits 1, 2, 3, 4, 5 no digit being repeated in any number?
A. 120 B. 256
C. 244 D. 182
E. 192

39. Compute the sum of 4-digit numbers which can be formed with the four digits 2,4,6,8 if each digit is used only once in each arrangement.
A. 1355553 B. 1333320
C. 1366654 D. 1366568
E. 1383337

40. From a total of 7 men and 4 ladies a committee of three is to be formed. If Mrs. A is not willing to join the committee in which Mr. B is a member, whereas Mr. B is willing to join the committee only if Mrs. C is included, how many such committees are possible?
A. 91 B. 104
C. 109 D. 128
E. 98

41. On if a, b and c can only take the values 1, 2, 3, 4, 5, 6, 7 then find the number of Solutions of the equation a + b + c = 12.
A. 37 B. 31
C. 38 D. 36
E. 39

42. How many rectangles can be formed out of a chessboard?
A. 204 B. 174
C. 1230 D. 1246
E. None of these

43. How many symbols need to be there if one were to make 10 lakhs distinct 3-digit initials using the symbols?
A. 1000 B. 100
C. 500 D. 999
E. 998

44. Roshan wants to buy a total of 100 t-shirts using exactly a sum of Rs 1000. He can buy red t-shirts at Rs 20 per t-shirt or orange or yellow t-shirts at Rs 5 and Re 1 per t-shirt respectively. If he has to buy at least one of each t-shirt and cannot buy any other colour of t-shirt, then in how many distinct ways can Roshan make his purchase?
A. 3 B. 6
C. 5 D. 4
E. 2

45. A pizza delivery company has three orders to deliver to three different flats. If the orders are delivered at random to the three flats, how many ways are there for at least one flat to get the wrong order?
    A. 4! B. 25
    C. 5 D. 3!
    E. 5!
46. Suresh and Mukesh are in a horse race with 4 other contestants. How many different arrangements could the race end with if Mukesh always finishes before Suresh and if all of the horses finish the race?
    A. 720 B. 240
    C. 360 D. 120
    E. 640
47. If the letters of the word 'Kochin' are arranged in all possible ways and these words are written out as in dictionary, then the word 'Kochin' appears at serial number?
    A. 456 B. 602
    C. 357 D. 455
    E. 457
48. If no repetition is not allowed then how many numbers between 1000 and 2000 can be formed using the digits from 0 to 7?
    A. 42 B. 336
    C. 210 D. 440
    E. 250
49. How many integers, greater than 999 but not greater than 3000, can be formed with the digits 0, 1, 2, 3 and 4, if repetition of digits is allowed?
    A. 376 B. 375
    C. 500 D. 250
    E. 251
50. There are 3 shirts, 4 jeans, 2 socks and 2 skirts. In how many ways can a shopkeeper arrange these things so that all the socks come together and all the skirts come together?
    A. 161280 B. 637480
    C. 582690 D. 479580
    E. 467580

ANSWER KEY:

1) A	11) D	21) C	31) B	41) A
2) C	12) C	22) B	32) C	42) E
3) D	13) E	23) B	33) E	43) B
4) D	14) B	24) D	34) E	44) A
5) E	15) A	25) A	35) C	45) C
6) B	16) D	26) B	36) A	46) C
7) C	17) C	27) E	37) C	47) E
8) E	18) D	28) B	38) E	48) C
9) D	19) A	29) E	39) B	49) E
10) B	20) C	30) D	40) D	50) A

PROBABILITY

- Probability is a number that reflects the chance or possibility of a particular event will occur.
- Probability refers to the extent of occurrence of events.
- When an event occurs like throwing a ball, picking a card from deck, etc., then the must be some probability associated with that event.
- Probability refers to the ratio of wanted outcomes to the total number of possible outcomes.
- $P(A) = \frac{Number\ of\ wanted\ outcomes}{Total\ number\ of\ possible\ outcomes}$

Basic Definition and Formula

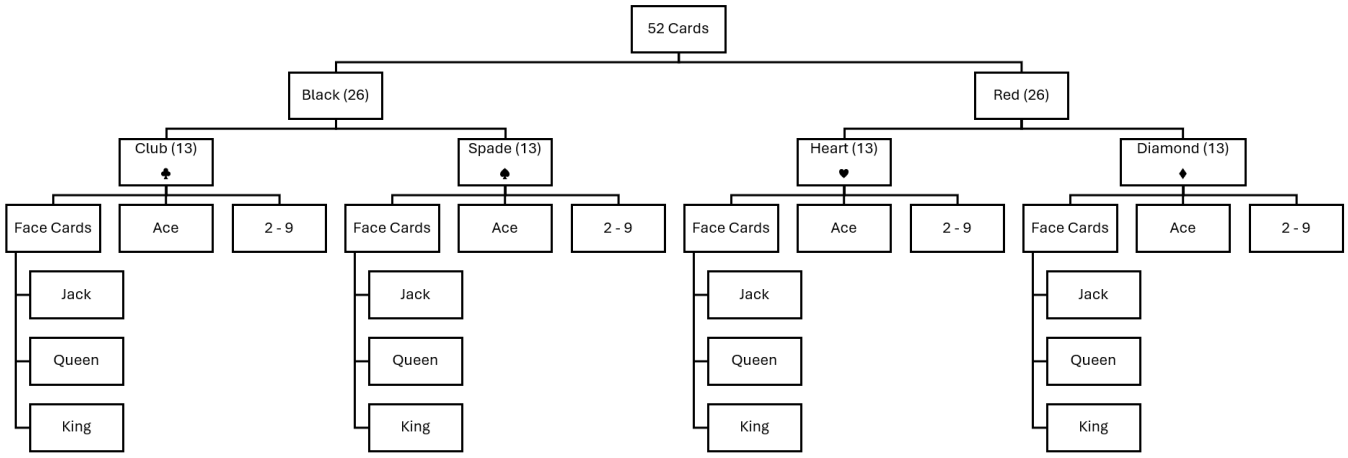
- Random Event:
  - If the repetition of an experiment occurs several times under similar conditions, if it does not produce the same outcome every time but the outcome in a trial is one of the several possible outcomes, then such an experiment is called Random event or a Probabilistic event.
- Elementary Event:
  - The Elementary event refers to the outcome of each random event performed.
  - Whenever the random event is performed, each associated outcome is known as elementary event.
- Sample Space
  - Sample Space refers to the set of all possible outcomes of a random event.
  - Example, when a coin is tossed, the possible outcomes are head and tail.
- Event
  - An event refers to the subset of the sample space associated with a random event.
- Occurrence of an Event
  - An event associated with a random event is said to occur if any one of the elementary event belonging to it is an outcome.

Basic Probability Formulae:

- Probability Range:
  - $0 \le P(A) \le 1$
- Rule of Complementary Events:
  - $P(A^C) + P(A) = 1$
- Rule of Addition:
  - $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- Disjoint Events:
  - Events A and B are disjoint if $P(A \cap B) = 0$
- Conditional Probability:
  - $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$
- Bayes' Formula:
  - $P(A \mid B) = \frac{P(B|A) \times P(A \cap B)}{P(B)}$
- Independent Events:
  - Events A and B are independent if. $P(A \cap B) = P(A) \times P(B)$
- ${}^nC_r = [P(F)]^r \times [P(UF)]^{n-r}$
  F = Favourable Outcomes
  UF = Unfavourable Outcomes
- Odds in favour & against:
  - a:b → Odds in favour
    F:UF
    a = Favourable Outcomes (F)
    b = Unfavourable Outcomes (UF)
    P(A) = a/(a+b) = F/(F+UF)
  - c:d → Odds against
    UF:F
    c = Unfavourable Outcomes (UF)
    d = Favourable Outcomes (F)
    P(A) = d/(c+d) = F/(F+UF)
- While Solving Questions of Probability of an Event, if it is Easy to find Probability of an Event Not happening then Find this and subtract this from 1.
- In any Question "or" is mentioned we use + while Solving Questions using Fundamental Principle of Counting.
- In any Question "and" is mentioned we use x while Solving Questions using Fundamental Principle of Counting.

Types of elements in Sample Space:

Pack of Cards:



Sample Space for:

- 1 Die thrown: {1, 2, 3, 4, 5, 6}
  No. of possible outcomes = 6
- 2 dice thrown:
  {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),
  (2,1), (2,2), (2,3), (2,4), (2,5), (2,6),
  (3,1), (3,2), (3,3), (3,4), (3,5), (3,6),
  (4,1), (4,2), (4,3), (4,4), (4,5), (4,6),
  (5,1), (5,2), (5,3), (5,4), (5,5), (5,6),
  (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}
  No. of possible outcomes = $6^2 = 36$
- Single Coin Toss = {H, T}
  No. of possible outcomes = 2
- Two Coins Tossed = {HH, HT, TH, TT}
  No. of possible outcomes = $2^2 = 4$
- Three Coins Tossed = {HHH, HTT, THT, TTH, HHT, HTH, THH, TTT}
  No. of possible outcomes = $2^3 = 8$

Solved Examples

Question 1:

Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn has a number which is a multiple of 3 or 5?

A. $\frac{1}{2}$ B. $\frac{2}{5}$

C. $\frac{8}{15}$ D. $\frac{9}{20}$

Solution

Here, S = {1, 2, 3, 4, …., 19, 20}.

Let E = event of getting a multiple of 3 or 5 = {3, 6, 9, 12, 15, 18, 5, 10, 20}.

$P(E) = \frac{n(E)}{n(S)} = \frac{9}{20}$

Correct Option: D.

Question 2:

A bag contains 2 red, 3 green and 2 blue balls. Two balls are drawn at random. What is the probability that none of the balls drawn is blue?

A. $\frac{10}{21}$ B. $\frac{11}{21}$

C. $\frac{2}{27}$ D. $\frac{5}{7}$

Solution

Total number of balls = (2 + 3 + 2) = 7.

Let S be the sample space.

Then, n(S) = Number of ways of drawing 2 balls out of 7.

$= {}^7C_2 = \frac{7\times6}{2\times1} = 21$

Let E = Event of drawing 2 balls, none of which is blue.

n(E) = Number of ways of drawing 2 balls out of (2 + 3) balls.

$= {}^5C_2 = \frac{5\times4}{2\times1} = 10$

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{10}{21}$

Correct Option: A.

Question 3:

A bag contains 1100 tickets numbered 1, 2, 3, …, 1100. If a ticket is drawn out of it at random, what is the probability that the ticket drawn has the digit 2 appearing on it?

A. $\frac{291}{1100}$ B. $\frac{292}{1100}$

C. $\frac{290}{1100}$ D. $\frac{301}{1100}$

Solution

Ticket has a maximum of 4 digits on it as Thousands, Hundreds, Tens, Units or TH H T U.

Number of Tickets with 2 in TH place = 0.

Number of Tickets with 2 in H place = From 200 upto 299 = 100.

Number of Tickets with 2 in T place but not in H place = 20 to 29 in T and U places and 00 to 10 except 02 in TH and H places = 10×10 = 100.

Number of Tickets with 2 ONLY in U place but not in TH H or T place

= H or T place both have (0 to 9 excluding 2) + (TH=1 & H=0 & U

= 2 & (T= 0 to 9 excluding 2))

= (9×9) + 9 = 90.

Total Tickets with at least one 2 = 290

Probability = $\frac{290}{1100}$ is answer

Correct Option: C.

Question 4:

In a class there are 60% of girls of which 25% poor. What is the probability that a poor girl is selected is leader?

A. 20% B. 15%

C. 10% D. 25%

Solution

Assume total students in the class = 100

Then Girls = 60% (100) = 60

Poor girls = 25% (60) = 15

So probability that a poor girls is selected leader = $\frac{Poor\ girls}{Total\ Students} = \frac{15}{100} = 15\%$

Correct Option: B.

Questions 5:

What is the probability that the total of two dice will be greater than 8, given that the first die is a 5?

A. $\frac{1}{2}$ B. $\frac{1}{6}$

C. $\frac{1}{9}$ D. None of these

Solution

Let A= first die is 5

Let B = total of two dice is greater than 8

P(A) = 1/6

Possible outcomes for A and B: (5, 4), (5, 5), (5, 6)

P(A and B) = $\frac{3}{36} = \frac{1}{12}$

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{1}{12} \div \frac{1}{6} = \frac{1}{2}$$

Correct Option: A.

Question 6:

A group of 5 friends–Archie, Betty, Jughead, Moose, and Veronica–arrived at the movie theater to see a movie. Because they arrived late, their only seating option consists of 3 middle seats in the front row, an aisle seat in the front row, and an adjoining seat in the third row. If Archie,

Jughead, or Moose must sit in the aisle seat while Betty and Veronica refuse to sit next to each other, how many possible seating arrangements are there?

A. 32 B. 36

C. 48 D. 72

E. 120

Solution

Good = Total – Bad.

Total = arrangements with Archie, Jughead or Moose in the aisle seat.

Number of options for the aisle seat = 3. (Archie, Jughead, or Moose)

Number of ways to arrange the 4 other people = 4 × 3 × 2 × 1.

To combine these options, we multiply: 3 x 4 x 3 x 2 = 72.

Bad = arrangements with Archie, Jughead or Moose in the aisle seat BUT with Betty next to Veronica.

Number of options for the aisle seat = 3. (Archie, Jughead, Moose).

Number of options for the third row seat = 2. (Anyone but Betty and Veronica, since in a bad arrangement they sit next to each other.)

Number of options for the middle of the 3 remaining seats = 2. (Must be Betty or Veronica so that they sit next to each other).

Number of ways to arrange the 2 remaining people = 2 x 1.

To combine these options, we multiply: 3 x 2 x 2 x 2 = 24.

Good arrangements = 72 – 24 = 48.

Correct Option C.

Question 7:

A die is rolled, find the probability that an Odd number is obtained?

A. $\frac{3}{4}$ B. $\frac{1}{2}$

C. $\frac{1}{4}$ D. None of these

Solution

Let us first write the sample space, S of the experiment.

S = {1,2,3,4,5,6}

Let E be the event "an odd number is obtained" and write down.

E = {1,3,5}

We can use the formula of the classical probability.

P(E) = $\frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

Correct Option: B.

Question 8:

Two coins are tossed, find the probability that two Tails are obtained. Note: Each coin has two possible outcomes H (heads) and T (Tails).

A. $\frac{1}{4}$ B. $\frac{1}{2}$

C. $\frac{3}{2}$ D. None of these

Solution

The sample space S is given by.

S = {(H,T),(H,H),(T,H),(T,T)}

Let E be the event "two tails are obtained".

E = {(T,T)}

We use the formula of the classical probability.

P(E) = $\frac{n(E)}{n(S)} = \frac{1}{4}$

Correct Option: A.

Question 9:

Two dice are rolled, find the probability that the sum is

A. equal to 1 B. equal to 4

C. less than 13

Solution

The sample space S of two dice is shown below.

S = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

A. Let E be the event "sum equal to 1". There are no outcomes which correspond to a sum equal to 1, hence

P(E) = $\frac{n(E)}{n(S)} = \frac{0}{36} = 0$

Quickest Way : Sum is always greater than or equal to 1. So it is Impossible Event means Probability will be 0.

B. Three possible outcomes give a sum equal to 4: E = {(1,3), (2,2), (3,1)}, hence.

P(E) = $\frac{n(E)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

C. All possible outcomes, E = S, give a sum less than 13, hence.

P(E) = $\frac{n(E)}{n(S)} = \frac{36}{36} = 1$

Quickest Way: Sum is always less than 13. So it is sure Event means Probability will be 1.

Question 10:

A Speak truth in 25% of cases and B in 30% of cases. In what Percentages of cases are they likely to Contradict to each other in narrating the Same Event?

Options:

A. 40 % B. 41 %

C. 42 % D. None of these

Solution

They contradict each other if one of them Speaks and the other one lies. and vice – versa.

Required Percentages = 0.25 × (1 – 0.30) + (1- 0.25) × 0.30

= 0.40 = 40%

Correct Option B.

Question 11:

Find the odds in favor of throwing a die to get "4 dots".

Solution

Total number of outcomes in throwing a die = 6

Number of favourable outcomes = 1

Number of unfavourable outcomes = (6 - 1) = 5

Therefore, odds in against of throwing a die to get "4 dots" is 5 : 1 or 5/1

Then,

Probability of the event=

$$\frac{No.\ of\ favourable\ outcomes}{No.\ of\ favourable\ \ outcomes + No.of\ unfavourable\ \ outcomes}$$

Question 12:

If odds in favour of A solving a problem are 5 to 3 and odds against B solving the same problem are 2 to 7.

Find probability for:

(i) A solving the problem

(ii) B solving the problem

Solution

Probability of the event =

$$\frac{No.\ of\ favourable\ outcomes}{No.\ of\ favourable\ \ outcomes + No.of\ unfavourable\ \ outcomes}$$

Given odds in favour of A solving a problem are 5 to 3.

Number of favourable outcomes = 5

Number of unfavourable outcomes = 3

(i) A solving the problem

P(A) = P(solving the problem) = 5/(5 + 3)

= 5/8

Given odds against B solving the problem are 2 to 7

Number of favourable outcomes = 7

Number of unfavourable outcomes = 2

(ii) B solving the problem

P(B) = P (solving the problem) = 7/(2 + 7)

= 7/9

Exercise – 1

1. Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn has a number which is a multiple of 4 or 5?
   A. 1/2 B. 3/5
   C. 8/20 D. 8/15
   E. 9/20
2. A bag contains 5 white and 6 black balls. 2 balls are drawn at random. Find the probability that they are of the same colour.
   A. 5/11 B. 7/11
   C. 8/11 D. 1/9
   E. 1/2
3. A bag contains 5 red, 6 yellow and 4 blue balls. Three balls are drawn at random from the bag. The probability that all of them are red, is:
   A. 2/91 B. 1/14
   C. 3/21 D. 2/77
   E. 10/77
4. In a lottery, there are 5 prizes and 15 blanks. A lottery is drawn at random. What is the probability of getting a prize?
   A. 2/4 B. 1/4
   C. 1/5 D. 1/2
   E. 2/5
5. From a pack of 52 cards, two cards are drawn together at random. What is the probability of both the cards being Queens?
   A. 1/225 B. 35/221
   C. 25/57 D. 1/221
   E. 35/256
6. In a box, there are 7 red, 8 blue and 5 green balls. One ball is picked up randomly. What is the probability that it is neither red nor green?
   A. 1/3 B. 3/5
   C. 8/21 D. 7/21
   E. 2/5
7. What is the probability that the card drawn at random from a pack of 52 cards, is a face card?
   A. 3/13 B. 1/13
   C. 3/52 D. 9/52
   E. 7/52
8. A bag contains 8 blue and 6 red balls. One ball is drawn at random. What is the probability that the ball drawn is blue?
   A. 3/7 B. 4/7
   C. 1/8 D. 3/4
   E. 5/7
9. Two cards are drawn together from a pack of 52 cards. The probability that one is a club and one is a diamond, is:
   A. 3/25 B. 29/37
   C. 13/100 D. 13/102
   E. 54/77
10. Three unbiased coins are tossed. What is the probability of getting at most two tails?
    A. 3/4 B. 1/2
    C. 7/8 D. 1/4
    E. 6/8
11. What is the probability of getting a sum 8 from two throws of a dice?
    A. 1/9 B. 3/4
    C. 1/12 D. 2/9
    E. 5/36
12. Three unbiased coins are tossed. What is the probability of getting at least 2 heads?
    A. 1/4 B. 1/2
    C. 3/4 D. 1/3
    E. 2/3

Directions: The following information has been given for questions (13–16). Read the Information carefully and answer the questions given below.

13. In a class, there are 16 boys and 10 girls. Three students are selected at random. The probability that 1 Boy and 2 girls are selected, is:
    A. 21/46 B. 1/5
    C. 45/65 D. 1/50
    E. 18/65
14. Find the probability that both are selected.
    A. 2/63 B. 1/14
    C. 1/63 D. 1/9
    E. 1/7
15. Find the probability that none of them are selected.
    A. 20/63 B. 1/9
    C. 10/63 D. 40/63
    E. 1/7

16. Find the probability that at least one of them is selected.
A. 20/63 B. 23/63
C. 10/63 D. 40/63
E. 1/7

17. From a pack of 52 cards, 3 cards are drawn. What is the probability that one is a 10, one is king, and one is ace?
A. 12/5525 B. 16/5255
C. 16/5525 D. 7/5525
E. 18/5255

18. There are 5 hotels in a town. If 3 men check into the hotels in a day then what is the probability that each check into a different hotel?
A. 12/25 B. 4/5
C. 4/7 D. 13/25
E. 3/5

19. A card is drawn from a pack of 52 cards. The probability of getting a jack of diamond or a ace of club is:
A. 2/13 B. 1/13
C. 1/26 D. 1/52
E. 2/52

20. In a simultaneous throw of a pair of dice. Find the probability of getting the total more than 8.
A. 13/36 B. 5/18
C. 7/15 D. 3/12
E. 5/12

21. A bag contains 3 red, 4 green and 2 blue balls. Two balls are drawn at random. What is the probability that none of the balls drawn is blue?
A. 10/21 B. 11/12
C. 1/12 D. 7/12
E. 7/18

22. If two letters are taken at random from the word TRIGONAL, what is the probability that none of the letters would be vowels?
A. 9/64 B. 2/28
C. 1/3 D. 3/28
E. 1/4

23. A box contains 5 white 6 black and 4 red balls. If 3 balls are drawn one by one without replacement, find the probability of getting all white balls.
A. 5/182 B. 3/91
C. 2/91 D. 13/273
E. None of these

24. Two dice are thrown simultaneously. What is the probability of getting two numbers whose product is odd?
A. 3/4 B. 3/8
C. 5/16 D. 2/7
E. 1/4

25. Three Cars are available in an Automobile shop. Three people applied for the cars. Each applies for one car without consulting others. The probability that all the three apply for the same car is:
A. 2/27 B. 1/27
C. 8/27 D. 7/27
E. 1/9

26. An unbiased die is tossed. Find the probability of getting a multiple of 2.
A. 1/2 B. 1/3
C. 3/4 D. 3/5
E. 2/5

27. A fair six-sided die is rolled twice. What is the probability of getting 5 on the first roll and not getting 3 on the second roll?
A. 1/36 B. 5/36
C. 1/12 D. 1/9
E. 2/36

28. A coin is tossed 5 times. What is the probability that the tail appears an odd number of times?
A. 1/2 B. 1/3
C. 2/3 D. 1/4
E. 1/5

29. A die is thrown three times and the sum of the three numbers is found to be 15. The probability that the first throw was a 5 is
A. 1/6 B. 1/4
C. 3/10 D. 1/10
E. 1/5

30. Three black marketers Somal, Gautam and Salim were selling tickets of a show. The odds in favour of them selling tickets were 1:5, 1:3 and 3:6 respectively. What is the probability that atleast one of them could sell all his tickets?
A. 1/6 B. 7/12
C. 5/12 D. 4/15
E. 3/7

31. In a box, there are 8 blue, 10 white and some black stones. A stone is randomly selected and the probability that the stone is black is ¼. Find the total number of stones in the box?
A. 15 B. 18
C. 20 D. 24
E. 12

32. Out of fifty students, there are 14 who are taking Economics and 29 who are taking Calculus while the rest choose both the subjects. What is the probability that a randomly chosen student from this group is taking only the Calculus class?
A. 8/15 B. 7/15
C. 1/25 D. 4/25
E. 11/25

33. A four-digit number is formed with the digits 1, 2, 3, 4, 5 at random. What is the probability that the number formed is divisible by 5?
A. 1/4 B. 1/5
C. 1/2 D. 3/4
E. 3/5

34. A 4-digit number is formed by the digits 1,2,3, 4 and 5 without repetition. What is the probability that the number formed is a multiple of 4?
A. 1/4 B. 3/5
C. 1/2 D. 1/5
E. 3/4

35. A criminal fires 4 bullets at the policeman. The probability that the policeman will be killed by a bullet is 0.5. What is the probability that the policeman survives?
A. 0.03125 B. 0.06520
C. 0.01250 D. 0.06250
E. None of these

36. You have two identical bags, each containing 5 red marbles and 5 blue marbles. If you choose one bag at random and draw a marble at random, what is the probability that the marble is red?
A. 1/2 B. 1/3
C. 5/6 D. 1/6
E. None of these

37. One bag contains five red and four yellow marbles. Another bag contains seven red and nine yellow marbles. A marble is transferred from the first bag to the second and then a ball is drawn from the second bag. Find the probability that the marble drawn is red.
A. 2/5 B. 4/7
C. 1/3 D. 4/9
E. 8/9

38. There are ten pairs of socks in a cupboard from which four individual socks are picked at random. The probability that there is at least one pair is.
A. 195/323 B. 198/323
C. 99/323 D. 185/323
E. None of these

39. Six blue balls are put in three boxes (in no specific quantity). The probability of putting equal number of balls in all the boxes is:
A. 1/3 B. 10/36
C. 1/6 D. 5/36
E. 1/18

40. Rohan had 4 fifty-rupee notes, 5-hundred-rupee notes and 3 five-hundred-rupee notes in his pocket. Two notes were drawn at random. Find the odds in favor of both being hundred-rupee notes.
A. 5:28 B. 5:33
C. 4:33 D. 1:7
E. 1:11

41. One 5-digit number is to be formed from numbers – 0, 1, 3, 7, and 8 (repetition not allowed). What is the probability that the number formed will be even?
A. 8/15 B. 13/21
C. 7/15 D. 3/10
E. 7/16

42. Red marbles in the ratio 4:1 are transferred to Bag 3 from Bags 1 and 2 respectively. 4 and 8 blue marbles are transferred to Bag 3 from Bags 1 & 2 respectively. The probability of choosing the red marble from bag 3 is 5/11. Find the number of red marbles in bag 3?
A. 12 B. 15
C. 10 D. 9
E. 11

43. A person starting with 256 rupees and making 8 bets, wins four times and loses four times, the wins and losses occurring in random order. The chance for a win is equal to the chance for a loss. If each wager is for half the money remaining at the time of the bet, then the final result is:
A. A gain of Rs. 165B. A loss of Rs. 175

C. A loss of Rs. 165 D. A gain of Rs. 175
E. Cannot be determined.

44. A coin of radius 2 cm is randomly dropped on a square floor full of square-shaped tiles of 10 cm each. What is the probability that the coin will land completely within a tile? In other words, the coin should not cross the edge of any tile.
A. 0.91 B. 0.5
C. 0.49 D. 0.36
E. 0.16

45. A dart is thrown at a board whose dimensions are 4×4 m. If the probability of missing the dart board is 0.25, find the probability of hitting the board at a point that is at maximum distance of 2 m from the centre of board.
A. $3\pi/16$ B. $7\pi/15$
C. $\pi/16$ D. $5\pi/18$
E. $5\pi/16$

46. You have two dice, one is fair, and the other is loaded such that the probability of rolling a 6 is twice as likely as rolling any other number. If you choose one die at random and roll it, what is the probability of rolling a 6?
A. 0.643 B. 0.226
C. 0.647 D. 0.854
E. 0.865

47. If the integers a and b are chosen at random from 1 to 100, then the probability that a number of the form 7^a+7^b is divisible by 5 is
A. 1/4 B. 1/2
C. 1/16 D. 1/6
E. 1/32

48. X speaks truth in 80% of cases and Y in 75% of cases. In what percentage of cases are they likely to contradict each other, narrating the same incident.
A. 30% B. 45%
C. 35% D. 50%
E. 20%

49. In a plane, X lines of lengths 1, 2, 3, 4 and 5 cm are lying. What is the probability that by joining the three randomly chosen lines end to end a triangle cannot be formed?
A. 2/5 B. 3/5
C. 3/10 D. 1/10
E. Cannot be determined.

50. Ten persons are sitting around a circular table facing the centre. What is the probability that three particular people sit together?
A. 3/21 B. 1/12
C. 1/6 D. 3/16
E. 5/12

ANSWER KEY:

1) C	11) E	21) D	31) D	41) B
2) A	12) B	22) D	32) E	42) C
3) A	13) E	23) C	33) B	43) D
4) B	14) A	24) E	34) D	44) D
5) D	15) D	25) E	35) D	45) A
6) E	16) B	26) A	36) A	46) B
7) A	17) C	27) B	37) D	47) A
8) B	18) A	28) A	38) C	48) C
9) D	19) C	29) C	39) E	49) A
10) C	20) B	30) B	40) A	50) B

CLOCKS & CALENDARS

Introduction

Clocks and Calendar is a frequently asked topic. Questions framed for clocks emphasize on significant hands of an analog clock along with questions on calendars emphasize on the calendar mathematics and calculations tangled in determining the day of the week of a particular or given date in the situation.

Clock

A clock is an instrument to measure time. It is meeting the need to measure intervals of time.

The hour hand and the minute hand of a clock move in relation to each other continuously and at any given point of time, they make an angle between 0° and 180° with each other. If the time shown by the clock is known, the angle between the hands can be calculated. Similarly, if the angle between two hands is known, the time shown by the clock can be found out.

Calendar

In general terms, calendar is a series or chart of pages showing the days, weeks, and months of a particular year, or gives specific seasonal information. A calendar is a system of organizing the days in a year.

Important concepts:

Clocks:

Minute Spaces: The face or dial of clock is a circle whose circumference is divided into 60 equal parts, named minute spaces.

Hour hand and minute hand: The smaller hand of a clock is called the hour hand or shorthand and the larger hand (instead of one) is called minute hand or long hand.

- A clock is a complete circle having 360 degrees. It is divided into 12 equal parts i.e. each part is 360/12=30
- As the minute hand takes a complete round in one hour it covers 360 degrees in 60 min. **Minute Hand covers 360/60 = 6 degree /minute**
- Also, as the **hour hand** covers just one part out of the given 12 parts in one hour, this implies **Hour Hand covers 300 in 60 min. i.e. 1/2 degree per minute.**
- Therefore, **the relative speed of the minute hand is 6– ½ =5 ½ degrees**
- Every hour, both the hands coincide once. In 12 hours, they will coincide 11 times. It happens due to only one such incident between 12 and 1'o clock.
- The hands are in the same straight line when they are coincident or opposite to each other.
- When the two hands are at a right angle, they are 15-minute spaces apart.
- In one hour, they will form two right angles and in 12 hours there are only 22 right angles. It happens due to right angles formed by the minute and hour hand at 3'o clock and 9'o clock.
- When the hands are in opposite directions, they are 30-minute spaces apart.
- If a clock indicates 9.15, when the correct time is 9, it is said to be 15 minutes too fast. On the other hand, if it indicates 8.45, when the correct time is 9, it is said to be 15 minutes too slow.
- If both the hour hand and minute hand move at their normal speeds, then both the hands meet after 65 5/11 minutes.
- 22 times in a day, the hands of a clock will be in a straight line but opposite in direction.
- 44 times in a day, the hands of a clock will be straight.
- 44 times in a day, the hands of a clock are at right angles.
- 22 times in a day, the hands of a clock coincide.
- To calculate the angle between the hands of a clock, we use the following formula (where m = minutes and h = hours)

  i) 11/2 m – 30h (when 11/2m > 30h)

  ii) 30h – 11/2m (when 30h > 11/2m)

Calendars:

- A leap year has 366 days.
- Every year divisible by 4 is a leap year, if it is not a century.
- Every 4th century is a leap year and no other century is a leap year.
- Counting odd days
  1 ordinary year ≡ 365 days ≡ (52 weeks + 1 day)
  Hence number of odd days in 1 ordinary year= 1.
  1 leap year = 366 days = (52 weeks + 2 days)
  Hence number of odd days in 1 leap year= 2.
  100 years = (76 ordinary years + 24 leap years)
  = (76 x 1 + 24 x 2) odd days
  = 124 odd days.
  = (17 weeks + 5 days)
  = 5 odd days.
  Hence number of odd days in 100 years = 5.
  Number of odd days in 200 years = (5 x 2) = 10 = 3 odd days
- Any month which has 31 days has 3 odd days.
  (31 / 7 leaves 3 as remainder) and any month which has 30 days has 2 odd days (30 / 7 leaves 2 as remainder).

Solved Examples

1. A clock is started in the afternoon. By 10 minutes past 5, the hour hand has turned through how many degrees?
   Options:
   A. 134° B. 148°
   C. 135° D. 155°
   Solution
   Angle traced by hour hand in 12 hours = 360°.
   Angle traced by hour hand in 5 hrs 10 min
   = 5*60 + 10 = 310 = 310/60 = 31/6 hours
   = 360/12 * 31/6 = 155°
   Correct Option: D
2. How many degrees will the minute hand move, in which the second hand moves 480°?
   Options:
   A. 4° B. 8°
   C. 10° D. 12°
   Solution
   In 480°, the minute hand will cover = 480/60 = 8°
   Therefore, minute hand moves for 8° when the second hand has moved 480.
   Correct Option: B
3. Calculate the angle between the two hands of a clock when the time shown by the clock is 5: 30 a.m.?
   A. 10° B. 15°
   C. 20° D. None of the above
   Solution
   According to the formula
   Angle = (11/2)M – 30H, where M =30, and h =5
   = (11/2) x 30 – 30 x 5
   = |165 – 150|
   taking the mod of the values, we get
   = 15°
   Correct Option: B
4. Calculate the angle between the two hands of a clock when the time shown by the clock is 7: 20 p.m?
   A. 100° B. 115°
   C. 120° D. None of the above
   Solution
   According to the formula
   Angle = (11/2)M – 30H, where M = 20, and h = 7
   = (11/2) x 20 – 30 x 7
   = |110 – 210|
   taking the mod of the values, we get
   = 100°
   Correct Option: A
5. What was the day on 10th November,1581 ?
   A. Thursday B. Monday
   C. Wednesday D. Tuesday
   Solution
   10 November 1581 = (1580 years + Period from 1.1.1581 to 10.11.1581)
   To calculate number of odd days till 1580, we need
   Number of odd days in 1200 years = 0
   Number of odd days in 300 years = 1
   Number of odd days in 80 years = 20 leap years + 60 ordinary years = 2 odd days
   Therefore, 1580 years had 0 + 1 + 2 = 3 odd days
   Now, for calculating odd days for 1.1.1581 to 10.11.1581,
   January (31 days) + February (28 days) + March (31 days) + (April 30 days) + May (31 days) + June (30 days) + July (31 days) + August (31 days) + September (30 days) + October (31 days) + November (10 days) = 314 days
   Total number of odd days in 314 days = (314/7) = 6 odd days
   Total Number of odd days corresponding to the given date = 3 + 6 = 9 days = 9/7 = 2 odd days
   ∴ on 10 November 1581 the day was Tuesday.
   Correct option: D
6. Suppose today is Monday. Now find out what will be the day after 62 days?
   A. Sunday B. Monday
   C. Wednesday D. Tuesday
   Solution
   Each day to the week is repeated after 7 days.
   Therefore, after 63 days, it will be Monday.
   Which means that on 62^{nd} day, it will be Sunday.
   Correct option: A
7. Which year will have the same calendar as that of the year 2019?
   A. 2029 B. 2030
   C. 2031 D. 2024
   Solution
   Count the number of odd days from the year 2019 onwards to get the sum equal to 0 odd day.

Year	Odd Days
2019	1

2020	1
2021	2
2022	1
2023	1
2024	1
2025	2
2026	1
2027	1
2028	1
2029	2

Sum = 14 odd days = 0 odd days

∴ calendar of the year 2030 will have the same calendar as that of the year 2019.

Correct option: B

8. What is the angle between the minute hand the hour hand when the time is 4:30

Options:

A. 35° B. 10°

C. 20° D. None of the above

Solution

It is easy to calculate the angle between the minute and the hour hand by using a simple formula,

Angle = (X*30)-((Y*11)/2)

Multiplying hours with minutes, we get = 4 x 30 = 120

Applying the formula, we get (Yx11)/2

= 30 x 11/ 2 = 165

When we subtract the two values, we get,

= 165 – 130 = 35°

Correct option: A

9. Calculate the time between 6 and 7 o'clock when the hands of a clock are in the same straight line but are not together?

A. 65.45 min past 6

B. 60 minutes past 6

C. 50 minutes past 6

D. Cannot be determined.

Solution

You can use a short formula to calculate the time when the angle is given

Angle = (minutes) -30 (hours)

Using the above formula, we get

180 = (minutes) -30 (hours)

180 = (minutes) – 30 (6)

180 + 180 = minutes

Minutes = 2 x 360/ 11

= 65.45

Correct Option: A

10. What was the day on 26th May 2006?

A. Monday B. Friday

C. Wednesday D. Saturday

Solution

26 May 2006 = (2005 years + time period from 1/1/2006 to 26/5/2006)

To calculate number of odd days till the year 2000, we require

Number of odd days in 1600 years = 0

Number of odd days in 400 years = 0

In the next step, for calculating odd days in the next five years,

5 years = (4 ordinary years + 1 leap year) = 4 +2 = 6 odd days

Now, we have to calculate the number of odd days from 1st January 2006 to 26th May 2006.

January (31 days) + February (28 days because 2006 is not a leap year) + March (31 days) + April (30 days) + May 26 days = 146 days

Total number of odd days in 146 days = (146/7) = 20 weeks + 1 odd day

Total number of odd days in the entire period = 0 (1600 years) + 0 (400 years) + 5 (5 years) + 0 (time from 1/1/2006 to 2/2/2006) = 5 odd days

According to the table, on 26th May 2006, the day was Friday.

Correct Option: B

11. Mary's dad set her clock which shows 2 o'clock in the morning. Through how many degrees will the hour hand rotate when the clock shows 9 o'clock in the evening when Mary is back from office?

A. 210° B. 144°

C. 168° D. 570°

Solution

We know that the angle traced by the hour hand in one hour is 30°.

In 19 hours, the total angle traced by the hour hand (2 am to 9 pm) is 30 x 19 = 570°.

Correct Option: D

12. Every Tuesday 10.00 AM the clock is set by Kevin, doing service in the clock tower which is in Pune. The Clock loses 6 minutes every hour. When the faulty

clock shows 3 P.M. on Saturday what will be the actual time?

A. 10 am B. 12:10 am

C. 1:40 pm D. 12:10 pm

Solution

Let us start from Tuesday 10 am to Saturday 3.00 pm

∴ total hours=4 days 5 hrs =4*24+5=101 hrs. So total time loses=101*6 minutes=606 minutes=606/60=10 hrs 10 minutes so actual time=Saturday 3.00 pm +10 hrs 10 minutes=Sunday 10 am.

Correct Option: A

13. Amar is a curious boy who asks his mathematics teacher that on what day will 1st of the month fall, if 9th of the month falls on the day preceding Sunday?

A. Monday B. Friday

C. Wednesday D. Saturday

Solution

As per the question 9th of the Month=Saturday. (Day preceding Sunday)

Day name repeats after 7 days.

So 9-7=2 which is 2nd of the given month is also Saturday.

Therefore 1st of the given month equates to Saturday Minus one day which will be Friday

Correct Option: B

Exercise – 1

1. The angle between the minute hand and the hour hand of a clock when the time is 4.20 is?
   A. 0 B. 5
   C. 10 D. 20
   E. 25
2. The calendar for the year 2007 will be the same for the year?
   A. 2014 B. 2016
   C. 2018 D. 2017
   E. 2019
3. Which of the following is not a leap year?
   A. 1800 B. 2000
   C. 1700 D. 1200
   E. All of the above
4. On 22nd Nov, 2007 Thursday falls. What day of the week was it on 8th Nov, 2006?
   A. Sunday B. Thursday
   C. Tuesday D. Wednesday
   E. Monday
5. January 1, 2007 was Monday. What day of the week lies on Jan. 1, 2008?
   A. Monday B. Thursday
   C. Tuesday D. Wednesday
   E. Saturday
6. Calculate the angle between the two hands of clock when the clock shows 5:25 p.m.?
   A. 12° B. 12.5°
   C. 15° D. None of these
   E. 18°
7. How many non-leap years are there from 1801 to 1900?
   A. 74 B. 75
   C. 76 D. 77
   E. 83
8. It was Sunday on Jan 1, 2006. What was the day of the week Jan 1, 2010?
   A. Sunday B. Saturday
   C. Friday D. Wednesday
   E. Monday
9. Today is Monday. After 61 days, it will be:
   A. Wednesday B. Saturday
   C. Tuesday D. Thursday
   E. Monday
10. If 6th March, 2005 is Monday, what was the day of the week on 6th March, 2004?
    A. Sunday B. Saturday
    C. Tuesday D. Wednesday
    E. None of these
11. On 8th Feb 2005 it was Tuesday. What was the day of the week on 8th Feb 2004?
    A. Tuesday B. Monday
    C. Sunday D. Wednesday
    E. Friday
12. A clock gains 1 minute per hour. If the clock is set to the correct time at 10:00 PM, what time will it show after 8 hours?
    A. 6:00 AM B. 5:52 AM
    C. 6:08 AM D. 5:44 AM
    E. 6:16 AM
13. A clock with a 24-hour face loses 2 minutes every hour. If the clock is set to the correct time at 12:00 PM, what time will it show after 6 hours?
    A. 5:00 PM B. 5:48 PM
    C. 5:12 PM D. 5:36 PM
    E. 5:24 PM
14. A clock gains 5 seconds per minute. If the clock is set to the correct time at 8:00 AM, what time will it show after 3 hours and 45 minutes?
    A. 11:45 AM B. 12:04 PM
    C. 11:30 AM D. 11:15 AM
    E. 12:15 PM
15. If today is Monday, what day of the week will it be after 59 days?
    A. Tuesday B. Wednesday
    C. Thursday D. Friday
    E. Saturday
16. A clock gains 1 minute per hour. If the clock is set to the correct time at 6:00 PM, what time will it show after 12 hours and 30 minutes?
    A. 6:42 AM B. 5:45 AM
    C. 6:15 AM D. 5:30 AM
    E. 6:45 AM
17. A calendar year has 12 months. What is the probability that a randomly selected month has exactly 31 days?
    A. ½ B. ⅓
    C. 7/12 D. ¼

E. 1/12

18. In a particular year of a non-leap year, January 1st falls on a Wednesday. What day of the week will be the last day of the year?
A. Sunday B. Monday
C. Tuesday D. Wednesday
E. Friday

19. If the day before yesterday was Monday, what will be the day after tomorrow?
A. Thursday B. Wednesday
C. Friday D. Saturday
E. Sunday

20. The calendar year 1800 is not a leap year, but the year 1900 is not a leap year either. However, the year 2000 is a leap year. Which of the following options explains this phenomenon?
A. A leap year occurs every 100 years.
B. A leap year occurs every 200 years.
C. A leap year occurs every 400 years.
D. A leap year occurs every 500 years.
E. A leap year occurs every 1000 years

21. A calendar month has 30 days. If the 15th day of a month is a Wednesday, what will be the day of the week on the 25th day of the same month?
A. Wednesday B. Thursday
C. Friday D. Saturday
E. Sunday

22. What is the angle between the two hands of a clock when the time shown by the clock is 6.30 p.m.?
A. 0° B. 50°
C. 30° D. 15°
E. None of the above

23. At what time between 3 and 4 o'clock will the minute hand and the hour hand are on the same straight line but facing opposite directions.
A. 3:49 B. 3:15
C. $3:39\ ^{1}/_{11}$ D. $3:49\ ^{1}/_{11}$
E. None of the above

24. By how many degrees does the minute hand move in the same time, in which the hour hand move by 28°?
A. 168° B. 336°
C. 196° D. 376°
E. None of the above

25. At what time, between 3 o'clock and 4 o'clock, both the hour hand and minute hand coincide each other?
A. 3:30 B. $3:16\ ^{4}/_{11}$
C. $3:16\ ^{11}/_{4}$ D. $3:16\frac{7}{11}$
E. None of the above

26. What will be the day of the week on 15th August 2010?
A. Sunday B. Monday
C. Tuesday D. Friday
E. Sunday

27. A clock loses 3 minutes every hour. If the clock is set to the correct time at 9:00 AM, what time will it show after 4 hours and 30 minutes?
A. 1:16 PM B. 12:45 PM
C. 12:55 PM D. 1:00 PM
E. 1:15 PM

28. A calendar month has 30 days starting from Monday. What is the probability that a randomly selected day of the month falls on a weekend (Saturday or Sunday)?
A. 2/7 B. 4/15
C. 4/30 D. 1/15
E. 2/15

29. A calendar is set to the correct date on Tuesday, March 1st, 2022. What will be the day of the week on February 28th, 2023?
A. Monday B. Tuesday
C. Wednesday D. Thursday
E. Friday

30. A calendar is set to the correct day on January 1st, 2023. What will be the day of the week on December 31st, 2030?
A. Monday B. Tuesday
C. Wednesday D. Thursday
E. Friday

31. A clock loses 2 minutes every hour. If the clock is set correctly at 12:00 PM, what time will it show after 5 hours and 30 minutes?
A. 5:19 PM B. 5:27 PM
C. 5:33 PM D. 5:37 PM
E. 5:45 PM

32. A clock gains 5 minutes every hour. If the clock shows 3:30 PM, what will be the actual time when the clock shows 5:00 PM?
A. 5:00 PM B. 5:15 PM
C. 5:30 PM D. 5:45 PM
E. 5:08 PM

33. A clock gains 6 minutes every hour. If the clock shows 7:45 AM, what will be the time shown on the clock after 5 hours and 30 minutes?
A. 1:48 PM B. 1:45 PM
C. 1:51 PM D. 1:30 PM
E. 1:39 PM

34. Find the day of the week on 16th July, 1776. (Assuming that 1-1-1 is Monday)
A. Monday B. Tuesday
C. Wednesday D. Thursday
E. Friday

35. March 1st is a Wednesday. Which month of the same year starts with the same day?
A. October B. November
C. December D. None of these
E. September

36. January 7, 1992 was Tuesday. Find the day of the week on the same date after 5 years, i.e., on January 7, 1997?
A. Tuesday B. Wednesday
C. Saturday D. Friday
E. None of the above

37. After 9'O clock at what time between 9 pm and 10 pm will the hour and minute hands of a clock point in opposite direction?
A. 15 minutes past 9
B. 16 minutes past 9
C. $16\frac{4}{11}$ minutes past 9
D. $17\frac{1}{11}$ past 9
E. None of the above

38. My watch gains 5 minutes in every hour. How many degrees the second hand moves in every minute?
A. 375° B. 380°
C. 390° D. 365°
E. 385°

39. How many days are there in x weeks' x days?
A. $7x^2$ B. 7x
C. 14x D. 8x
E. Cannot be determined

40. The first Republic Day of India was celebrated on January 26, 1950. What was the day of the week on that date?
A. Monday B. Wednesday
C. Thursday D. Saturday
E. Friday

41. On what dates of April, 2001 did Wednesday fall?
A. 1st, 8th, 15th, 22nd, 29th
B. 2nd, 9th, 16th, 23rd, 30th
C. 3rd, 10th, 17th, 24th
D. 4th, 11th, 18th, 25th
E. 5th, 12th, 16th, 28th

42. A clock gains 1.5 minutes per hour. If the clock is set to the correct time at 6:00 PM, what time will it show after 3 hours and 45 minutes?
A. 9:12 PM B. 9:20 PM
C. 9:27 PM D. 9:30 PM
E. 9:51 PM

43. A clock has a pendulum that swings back and forth every 1.5 seconds. If the clock is started at 12:00 PM, what time will it show after 2 hours and 15 minutes?
A. 1:30 PM B. 1:15 PM
C. 1:45 PM D. 2:00 PM
E. 3:00 PM

44. A digital clock displays time in the format of HH:MM (hours and minutes). How many possible times can it display a palindrome between 12:00 AM and 11:59 PM?
A. 24 B. 48
C. 36 D. 8
E. 16

45. A clock gains 30 seconds per hour for the first 12 hours of the day, and loses 15 seconds per hour for the next 12 hours of the day. If the clock is set to the correct time at 12:00 AM, what time will it show after 5 days 10 hours and 30 minutes?
A. 12:05 AM B. 12:20 AM
C. 10:50 AM D. 11:20 AM
E. 11:25 AM

46. A clock loses 5 minutes per hour. If the clock is set to the correct time at 10:00 AM on Monday, what time will it show after 4 days, 10 hours, and 30 minutes?
A. 1:08 AM on Friday
B. 1:30 AM on Friday
C. 2:30 AM on Friday
D. 1:08 AM on Thursday
E. 1:30 AM on Thursday

47. How many times does the 29th day of the month occur in 400 consecutive years?
A. 4498 B. 4497
C. 4499 D. 4500

E. 4501

48. In a leap year, what will be the day of the week on March 1st ?
    A. Friday B. Tuesday
    C. Wednesday D. Thursday
    E. Cannot be determined.
49. Jamnalal has two wrist watches – A and B. He set the right time as 6 p.m. on both watches and set an alarm of 6 a.m. for the next day. Watch A gains 30 seconds per 30 minutes and B loses 60 seconds per 30 minutes. What will be the time shown in watch B when the alarm in watch A rings?
    A. 5:35:36 a.m. B. 5:28:30 a.m.
    C. 5:24 a.m. D. 5:24:36 a.m.
    E. None of the above
50. A watch which gains uniformly is 6 minutes slow at 4 pm on a Sunday and $10^2/_3$ minutes fast on the following Sunday at 8 am. During this period (Day and Time) when was the watch correct?
    A. 2:36 am B. 1:36 am
    C. 2:36 pm D. 1:36 pm
    E. None of the above

ANSWER KEY:

1) C	11) C	21) D	31) A	41) D
2) C	12) C	22) D	32) E	42) E
3) C	13) B	23) D	33) A	43) A
4) D	14) B	24) B	34) C	44) E
5) B	15) C	25) B	35) B	45) C
6) B	16) A	26) A	36) A	46) A
7) C	17) C	27) A	37) C	47) B
8) C	18) D	28) B	38) C	48) E
9) B	19) C	29) B	39) D	49) E
10) A	20) C	30) B	40) C	50) B

DATA SUFFICIENCY

Exercise – 1

1. In a school, 48% of the girls aged 16 and above play volleyball then how many of girls play volleyball?
   Statement I: In the school, there are no girls below 16 years who play volleyball.
   Statement II: In the school, girls comprise 50% of the total number of students who play volleyball
   A. The data in statements I alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.
   B. The data in statements II alone is sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.
   C. Either Statement I or Statement II alone is sufficient to answer the question.
   D. The data in both the statements I and II is not sufficient to answer the question.
   E. The data in both the statements I and II together is necessary to answer the question.
2. What will be the cost of painting the four walls and ceiling of a room with length, width and height of 21m, 27 m, and 30 m respectively? The room has two windows and one door.
   Statement I: Cost of painting is Rs. 9 per square meter.
   Statement II: Area of the ground of the room is 567 sq. meter.
   Statement III: Area of one window is 17 sq. meter which is 50% of the area of the door
   A. The data in statements I and III together are sufficient to answer the question, while the data in statement II are not sufficient to answer the question.
   B. The data in statements I and II together are sufficient to answer the question, while the data in statement III are not sufficient to answer the question.
   C. The data in statement I, II and III together are not sufficient to answer the question.
   D. The data in statement only I and II together or only statement III is sufficient.
   E. The data in all the statements I, II and III together are necessary to answer the question.
3. The average of the first four numbers is six times of the fifth number. Find the fifth number.
   Statement I: Average of the first two numbers is equal to the average of the next two numbers.
   Statement II: Average of the first two numbers is four times of the fifth number.
   Statement III: The average of the all the numbers is 5.
   A. The data in statements II and III together are sufficient to answer the question, while the data in statement III are not sufficient to answer the question.
   B. The data in statements I and II together are sufficient to answer the question, while the data in statement III are not sufficient to answer the question.
   C. The data in statement I, II and III together are not sufficient to answer the question.
   D. The data in statement III only are sufficient to answer the question.
   E. The data in all the statements I, II and III together are necessary to answer the question.
4. Can Bheem drive from Gujrat to Lucknow in less than 3 hours?
   Statement I: The average speed of Bheem during the whole journey is less than 66 km/hr
   Statement II: The distance from Gujrat to Lucknow is greater than 235 km.
   Statement III: He is driving a car having maximum speed of 180 km/hr.
   A. The data in statements I and III together are sufficient to answer the question, while the data in statement II are not sufficient to answer the question.
   B. The data in statements I and II together are sufficient to answer the question, while the data

in statement III are not sufficient to answer the question.

C. The data in statements I and II together are sufficient to answer the question, while the data in statement III are not sufficient to answer the question.

D. The data in statement only I and II together or only statement III is sufficient.

E. The data in all the statements I, II and III together are necessary to answer the question.

5. A certain mixture of paint requires violet, green, and yellow in the ratio of 2:3:4, respectively, and no other ingredients. If there is enough violet and green paint available, determine whether there is enough quantity of yellow paint to get the desired amount of mixture?

Statement I: Exactly 540 litres of the mixture are needed.

Statement II: The difference between the quantity of violet paint available and the green paint available is 180 litres.

Statement III: Exactly 540 litres of green paint are available.

A. The data in statements I and III together are sufficient to answer the question, while the data in statement II are not sufficient to answer the question.

B. The data in statements I and II together are sufficient to answer the question, while the data in statement III are not sufficient to answer the question.

C. The data in statement I, II and III together are not sufficient to answer the question.

D. The data in statement only I and II together or only statement III are sufficient.

E. The data in all the statements I, II and III together are necessary to answer the question.

6. How many years old will Mohish be in t years from now?

Statement I: Anurag is 19 years older than Mohish.

Statement II: The sum of the ages of Anurag and Mohish is t years.

A. The data in statements I alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.

B. The data in statements II alone is sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.

C. Either Statement I or Statement II alone is sufficient to answer the question.

D. The data in both the statements I and II is not sufficient to answer the question.

E. The data in both the statements I and II together is necessary to answer the question.

7. At present, the age of P is 1/5th of the age of Q then what are their ages?

Statement I: After 6 years, the ratio of their ages will become 5 : 17.

Statement II: 9 years ago, the age of P was 33.33% of the age of Q.

A. The data in statements I alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.

B. The data in statements II alone is sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.

C. Either Statement I or Statement II alone is sufficient to answer the question.

D. The data in both the statements I and II is not sufficient to answer the question.

E. The data in both the statements I and II together is necessary to answer the question.

8. A person can purchase three articles in Rs. 70. What is the price of costliest article?

Statement I: The cost price of two articles each is Rs. 1 less than the cost price of costliest article.

Statement II: The cost price of two articles is same.

Statement III: The cost price of costliest article is 4.34% more than the cost price of cheapest article.

A. Either statement I alone or statements II and III together are sufficient.

B. Only statement III is sufficient.

C. Only statement I and II together are sufficient.

D. Only statement I and III together are sufficient.

E. None of these.

9. Rajdhani Express leaves Patna at 10:00 am for New Delhi. At what time will it reach New Delhi?

Statement I: For the first 125 km it travels at the speed of 350 km per hour and maintains the same

speed during the entire journey.
Statement II: It has 6 stoppages in between New Delhi and Patna.
Statement III: Before every stoppage, it covers a same distance of 210 km

A. Either statement III alone or statements I and II together are sufficient.
B. Only statement III is sufficient.
C. Only statement I and II together are sufficient.
D. Only statement I, II, and III together are sufficient.
E. None of these

10. What is the sum of the age of Devansh and Soham?
Statement I: The age of Devansh is 6 years more than the age of Soham.
Statement II: 40% of the age of Soham is equal to 30% of the age of Devansh.
Statement III: The ratio between half of the age of Devansh and one third of the age of Soham is 2 : 1.

A. Either statement III alone or statements I and II together are sufficient.
B. Only statement III is sufficient.
C. Only statement I and II is sufficient.
D. Only statement I, II, and III are sufficient.
E. None of these.

11. In a kilometer race, by how many meters Vilas beats Varsha?
Statement I: In a kilometer race, Vilas beats Vedant by 200 meters.
Statement II: The respective ratio of the speed of Vedant and Varsha is 10: 7.
Statement III: In a kilometer race, Vedant beats Varsha by 300 meters

A. Either statement III alone or statements I and II together are sufficient.
B. Only statement III is sufficient.
C. Only statement I and II is sufficient.
D. Only statement I, II, and III are sufficient.
E. Statement I and III together or Statement I and II together are sufficient.

12. How much amount Ratnesh will get at the end of two years if the rate of interest is compounded annually?
Statement I: Kunal invests the same of amount of money under simple interest at the rate of 10% per annum and receives a total amount of Rs.1950 at the end of 3 years.
Statement II: The difference between the simple interest and compound interest on the same sum of money at the same rate of interest at the end of 2 years is Rs 15.

A. The data in statements I alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.
B. The data in statements II alone is sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.
C. Either Statement I or Statement II alone is sufficient to answer the question.
D. The data in both the statements I and II is not sufficient to answer the question.
E. The data in both the statements I and II together is necessary to answer the question.

13. How much time train A will take to cover a distance of 532 km?
Statement I: The ratio of speed at which train A and train B runs is 4 : 5 respectively.
Statement II: The average speed of the train A and train B is 400/9 km per hour.

A. The data in statements I alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.
B. The data in statements II alone is sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.
C. Either Statement I or Statement II alone is sufficient to answer the question.
D. The data in both the statements I and II is not sufficient to answer the question.
E. The data in both the statements I and II together is necessary to answer the question.

14. The population of a city in the year 2020 was 2,92,820 then what was the population of city exactly 4 years ago?
Statement I: The population of city in the year 2020 is exactly 1.4641 times of that in the year 2016.
Statement II: The population of the city increased by 10% each year.

A. The data in statements I alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.
B. The data in statements II alone is sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.
C. Either Statement I or Statement II alone is sufficient to answer the question.
D. The data in both the statements I and II is not sufficient to answer the question.
E. The data in both the statements I and II together is necessary to answer the question.

15. What is the value of the two-digit number?
Statement I: The sum of the digits at unit's place and tenth's place is 15.
Statement II: When the digits are interchanged then the number obtained is 27 larger than the original number.
A. The data in statements I alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.
B. The data in statements II alone is sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.
C. Either Statement I or Statement II alone is sufficient to answer the question.
D. The data in both the statements I and II is not sufficient to answer the question.
E. The data in both the statements I and II together is necessary to answer the question.

16. What is the sum of x and y?
Statement I: $30x + 4y = 216$
Statement II: $y = 54 - 7.5x$
A. The data in statements I alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.
B. The data in statements II alone is sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.
C. Either Statement I or Statement II alone is sufficient to answer the question.
D. The data in both the statements I and II is not sufficient to answer the question.
E. The data in both the statements I and II together is necessary to answer the question.

17. Two friends P and Q together can complete a piece of work in 20 days. In how many days P alone complete the piece of work?
Statement I: Q alone can complete half of the work in 25 days.
Statement II: The efficiency of P is 50% more than that of Q.
A. The data in statements I alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.
B. The data in statements II alone is sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.
C. Either Statement I or Statement II alone is sufficient to answer the question.
D. Neither statement I nor statement II is sufficient to answer the questions.
E. The data in both the statements I and II together is necessary to answer the question.

18. One day I left my home at 9:00 AM and assume that if I run at uniform speed then I will reach my school at 2:00 PM. If my brother left the school for home at 10:00 AM with his uniform speed then at what time will he meet me?
Statement I: The uniform speed of my brother is 14 km per hour.
Statement II: He can reach home at 1:30 PM.
A. The data in statements I alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.
B. The data in statements II alone is sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.
C. Either Statement I or Statement II alone is sufficient to answer the question.
D. Neither statement I nor statement II is sufficient to answer the question.
E. The data in both the statements I and II together is necessary to answer the question.

19. Tejas and Tanmay started working together. After a few days, Sachin joined them and they were able to finish the work in 20 days. All of them together were paid a total of Rs. 1600. Find the share of Sachin.

Statement I: Tejas can do the work in 40 days, while Tanmay can do the same work in 50 days.
Statement II: Tejas can do the work in 120 days, while Sachin can do the same work in 30 days.

A. If the data in statement I alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.
B. If the data in statement II alone is sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.
C. If the data either in statement I alone or in statement II alone is sufficient to answer the question.
D. If the data in both statements I and II together are necessary to answer the question.
E. If the data given in both statements I and II together are not sufficient to answer the question.

20. Two partners, A and B entered in a business, what profit will B get at the end of 2 years?
Statement I: A and B started the business by investing in the ratio 4: 7 and After 2 years, A's share is Rs. 190000.
Statement II: A joined the business with an amount of Rs. 1000000.

A. If the data in statement I alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.
B. If the data in statement II alone is sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.
C. If the data either in statement I alone or in statement II alone is sufficient to answer the question.
D. If the data in both statements I and II together are necessary to answer the question.
E. If the data given in both statements I and II together are not sufficient to answer the question.

21. Find the number of boys in the college, if 50% of the total boys and 45% of the total girls participated in an event.
Statement I: The number of girls participated in the event is 135. There are more than 300 boys in the college.
Statement II: The number of girls in the college is 20% more than the number of boys who participated in the event.

A. If the data in statement I alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.
B. If the data in statement II alone is sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.
C. If the data either in statement I alone or in statement II alone is sufficient to answer the question.
D. If the data in both statements I and II together are necessary to answer the question.
E. If the data given in both statements I and II together are not sufficient to answer the question.

22. Find the total population of Karachi.
Statement I: The ratio of the population of males and females in Karachi is 35 : 27 and the difference between their population is 32000.
Statement II: The population of Karachi is 60% of that of Patna, and the difference between population of Karachi and Patna is 30000.

A. If the data in statement I alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.
B. If the data in statement II alone is sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.
C. If the data either in statement I alone or in statement II alone is sufficient to answer the question.
D. If the data in both statements I and II together are necessary to answer the question.
E. If the data given in both statements I and II together are not sufficient to answer the question.

23. A triangle is circumscribed by a circle. What is the perimeter of the triangle?
Statement I: The circumference of the circle is 56π cm.
Statement II: The largest side of the triangle is diameter of the circle.
Statement III: The triangle is an isosceles triangle.

A. Either statement III alone or statements I and II together are sufficient.
B. Only statement I is sufficient.
C. Only statement I and II together are sufficient.
D. Only statement I, II, and III together are sufficient.
E. None of these

24. What is the respective speed of the two trains, U and V of length 300 meters and 350 meters respectively.
Statement I: They take 10 seconds to cross each other when they are running in opposite direction.
Statement II: They take 130 seconds to cross each other when running in the same direction.
Statement III: The sum of their speed is 130 m per sec.
A. Either Statement I and II together or Statement II and III together is sufficient.
B. Only statement I is sufficient.
C. Only statement I and II together are sufficient.
D. Only statement I, II, and III together
E. are sufficient.
F. None of these

25. A dishonest merchant professes to sell his article at Cost price but uses a faulty scale while selling. What profit percentage does the merchant make when he sells the article at Cost Price?
Statement I: The cost price of the article is Rs. 25 per kg.
Statement II: He uses a scale which weighs less by 20%.
Statement III: If he had sold the articles at 20% profit on cost price then his net profit percentage would have been 50%.
A. Either Statements II or III alone is sufficient.
B. Only statement I is sufficient.
C. Only statement I and II together are
D. sufficient.
E. Only statement I, II, and III together are sufficient.
F. None of these

26. What is the present age of Vedant?
I. Vedant is 4 years older than Arun. The ratio of the age of Arun and Varun is 4 : 5
II. Varun is 1 year older than Vedant and the present age of Arun is 20 years.
III. The present age of Vedant is 5 years less than the age of Mohan.
A. The data in statements I alone or in statement II alone or Statement III alone is sufficient to answer the question.
B. The data in statements II alone is sufficient to answer the question, while the data in statement I and III are not sufficient to answer the question.
C. The data in statements I and II are sufficient to answer the question, while the data in statement III alone is not sufficient to answer the question.
D. The data in all the statements I, II and III are not sufficient to answer the question.
E. The data in all the statements I, II and III together are necessary to answer the question.

27. Find the height of the cylinder.
I. The Curved surface area of the cylinder is 396 cm^2 and the total surface area of the cylinder is 1628 cm^2.
II. The radius of the cylinder is 0.5 cm more than the 3 times of the height of the cylinder.
A. The data in statements I alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.
B. The data in statements II alone is
C. sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.
D. The data in statements I alone or in statement II alone is sufficient to answer the question.
E. The data in both the statements I and II is not sufficient to answer the question.
F. The data in both the statements I and II together is necessary to answer the question.

28. Find the amount of money invested by Jamnalal in the scheme?
Statement I: An increase in simple interest from 88/6% to 116/6% per annum increases his yearly income by 5600.

Statement II: The sum invested get doubled, when invested at 20% per annum for 5 years.

A. If the data in statement I alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.
B. If the data in statement II alone is sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.
C. If the data either in statement I alone or in statement II alone is sufficient to answer the question.
D. If the data in both statements I and II together are necessary to answer the question.
E. If the data given in both statements I and II together are not sufficient to answer the question.

29. Isha, Esha and Rhea work on a project. If each of them is of a different efficiency. Who is the fastest worker among them?
Statement I: Isha and Esha take 10 days to complete the project while Esha and Rhea take 20 days to complete the same project.
Statement II: Isha and Rhea take 60/7 days to complete the project.

A. If the data in statement I alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.
B. If the data in statement II alone is sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.
C. If the data either in statement I alone or in statement II alone is sufficient to answer the question.
D. If the data in both statements I and II together are necessary to answer the question.
E. If the data given in both statements I and II together are not sufficient to answer the question.

30. what is distance between city A and city B?
I. Trishan and Henil started their journey simultaneously from city A to city B with their speeds in the ratio of 3 : 2.
II. Henil takes 4 hours more than Trishan to reach city B.
II. The difference between speeds of Trishan and Henil is 50 km/hr.

A. Only I and II is sufficient.
B. Only II and III is sufficient.
C. Either I or II is sufficient.
D. Neither I nor II is sufficient.
E. All I, II and III are sufficient.

31. Two cycles are travelling towards each other. What is the distance between them just before 30 minutes they meet?
I. They were originally 200 km apart
II. Speed of a cycle A is 60 km / hr and speed of cycle B is 80 km /hr apart.

A. Statement I alone is sufficient.
B. Statement II alone is sufficient.
C. Both statement I and statement II together are sufficient.
D. Both statement I and statement II even together are not sufficient.
E. Either statement I or statement II is sufficient.

32. Average weight of 10 school children is 60 kg. Children M with average weight N is joined the group so the total average weight is increased. Find the new average weight of children.
I. M + N = 65
II. N is prime.

A. Statement I alone is sufficient.
B. Statement II alone is sufficient.
C. Both statement I and statement II together are sufficient.
D. Both statement I and statement II even together are not sufficient.
E. Either statement I or statement II is sufficient.

33. What is P's age?
I. P, Q and K are all of the same age.
II. Total age of Q, K and A is 64 and A is as old as Q and K together.

A. Statement I alone is sufficient.
B. Statement II alone is sufficient.
C. Both statement I and statement II together are sufficient.
D. Both statement I and statement II even together are not sufficient.

E. Either statement I or statement II is sufficient.

34. What is the amount invested by C?
Statement I: Total amount received by D after 3 years is Rs.4800 at compound interest.
Statement II: D and C invested their amount at the rate of 10% per annum.
Statement III: C and D invested their amount at simple interest and compound interest respectively and the difference between the interests received by both after 2 years is Rs.1200.
A. Only I and II are sufficient.
B. Only II and III are sufficient.
C. Either II alone or I and II together are sufficient.
D. All I, II and III are necessary to the answer the question.
E. The question can't be answered even with all I, II and III together.

35. What is the sum of the ages of person 1 and 2?
Statement I: Ratio of the ages of 1 to 3 is 4: 5 and the ratio of the ages of 1 to 4 is 1: 3.
Statement II: Sum of the ages of 2,3 and 4 is 125 years and 5 years ago the ratio of the ages of 1 to 4 is 3: 11.
Statement III: 4's age is 200% more than that of 1's age and the difference between the ages of 1 and 4 is 40 years.
A. Only I and II are sufficient.
B. Only II and III are sufficient.
C. Either II alone or I and II together to sufficient.
D. All I, II and III are necessary to the answer the question.
E. The question can't be answered even with all I, II and III.

36. What is the total surface area of the conical box?
Statement I: Ratio of height of the box to height of the cylindrical box is 2: 1.
Statement II: Height of the cylindrical box is equal to the perimeter of the square whose area is 9 cm^2.
Statement III: Radius of the cone is equal to length of the rectangle whose perimeter is 20 cm.
A. Only I and II are sufficient.
B. Only II and III are sufficient.
C. Either II alone or I and II together is sufficient.
D. All I, II and III are necessary to answer the question.
E. The question can't be answered even with all I, II and III.

37. What is the initial quantity of the Ethanol in vessel ?
Statement I: Ratio of the Ethanol and petrol in vessel X and Y is 3: 2 and 4: 3 respectively.
Statement II: 28 liters of the mixture of Y is poured into X and then the ratio of the Ethanol and petrol in vessel X becomes 17: 12.
Statement III: 10 liters of the mixture from vessel Z is taken out and is poured into vessel X, then the ratio of the Ethanol to petrol becomes vessel X is 5: 4.
A. Only I and II are sufficient.
B. Only II and III are sufficient.
C. Either II alone or I and II together to sufficient.
D. All I, II and III are necessary to the answer the question.
E. The question can't be answered even with all I, II and III.

38. Number of females from B in all the years together is what percent of the total number of employees from B in all the years together?
Statement I: Total number of employees from B in 2000 is 280 and the ratio of the number of females to males from B in 2000 is 4: 3.
Statement II: 60% of the total number of employees from B in 1997 to 2000 is males.
A. Statement I alone is sufficient.
B. Statement II alone is sufficient.
C. Both statement I and statement II together are sufficient.
D. Both statement I and statement II even together are not sufficient.
E. either statement I or statement II is sufficient.

39. Four persons A, B, C and D are hired to do work. A, C and D can complete the task in 24 days while working together. Find the number of days taken by B alone to complete the task. Statement I: C and D can complete the task in 36 days, B is 100% more efficient than A.

Statement II: A alone can complete the work in 72 days and ratio of efficiency of C and D is 1:2 respectively.

A. Statement I alone is sufficient.
B. Statement II alone is sufficient.
C. Both statement I and statement II together are sufficient.
D. Both statement I and statement II even together are not sufficient.
E. either statement I or statement II is sufficient.

40. Find the number of factors of the two-digit number.
Statement I: The two-digit number is 27 more than the two-digit number obtained by reversing the digits of the number.
Statement II: The number is multiple of 9.

A. Statement I alone is sufficient.
B. Statement II alone is sufficient.
C. Both statement I and statement II together are sufficient.
D. Both statement I and statement II even together are not sufficient.
E. either statement I or statement II is sufficient.

41. What is the length of the train A?
I. Two trains A and B running in opposite directions and cross each other in 50 seconds.
II. The length of the train B is 640m and the difference between the speeds of trains A and B is 28 km/h.
III. The ratio of the speed of the trains is 19:26

A. The data in statements I and statements II are sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.
B. The data in statements II alone is sufficient to answer the question, while the data in statement I and III is not sufficient to answer the question.
C. The data in statements I alone or in statement II alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.
D. The data in all the statements I, II and III is not sufficient to answer the question.
E. The data in all the statements I, II and III together is necessary to answer the question.

42. Is the sum of the costs of a Pen and a Sharpener more than the sum of the costs of a pencil and an Eraser?
I. 30% of the cost of the Pen and 20% of the cost of the Sharpener is more than 40% of a cost of the pencil and 60% of the cost of Eraser.
II. 20% of the cost of the Pen and 30% of the cost of the Sharpener is less than 10% of the cost of the pencil and 15% of the cost of the Eraser.

A. The data in statement I alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.
B. The data in statement II alone is sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.
C. The data in statement I alone or in statement II alone is sufficient to answer the question.
D. The data in both the statements I and II is not sufficient to answer the question.
E. The data in both the statements I and II together is necessary to answer the question.

43. A surveyor draws three parallel straight lines on a Paper for a land survey. Two points A and B are marked on the first straight line, points C and D are marked on the second straight line and points E and F are marked on the third straight line. Each of these six points can move to any positions on its respective straight line.
Statement I: The maximum number of triangles that can be drawn by joining these points is 24.
Statement II: The minimum number of triangles that can be drawn by joining these points is zero.

A. The data in statements I alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.
B. The data in statements II alone is sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.
C. The data in statements I alone or in statement II alone is sufficient to answer the question.

D. The data in both the statements I and II is not sufficient to answer the question.
E. The data in both the statements I and II together is necessary to answer the question.

44. A fisherman rows a boat upstream for a certain distance at a speed of 36 km/hr. If the ratio of speed of man in still water to the speed of stream is 5 : 2, find the time taken to row a boat downstream to cover the same distance covered upstream.
Statement I: The total time taken by the man to cover upstream and downstream is 50/14 hours.
Statement II: The time taken to cover the distance upstream is 2.5 hours.
A. If the data in statement I alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.
B. If the data in statement II alone is sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.
C. If the data either in statement I alone or in statement II alone is sufficient to answer the question.
D. If the data in both statements I and II together are necessary to answer the question
E. If the data given in both statements I and II together are not sufficient to answer the question.

45. Suraj mixes water and milk in an empty container A. Find the amount of water mixed by Suraj.
Statement I: The ratio of the amount of water and the amount of milk in the container A is 5:13, respectively after Suraj has mixed milk and water. Suraj sells 72 litres of the mixture and then adds 44 more litres of a mixture of water and milk in the container. After mixing 44 more litres of the mixture, the ratio of the amount of water and the amount of milk becomes 4 : 9, respectively.
Statement II: The ratio of the amount of water and the amount of milk in the container A is 5 : 13, respectively after Suraj has mixed milk and water. Suraj sells 72 litres of the mixture and then adds 44 more litres of a mixture of water and milk in the container in the ratio 5 : 6, respectively.
A. If the data in statement I alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.
B. If the data in statement II alone is sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.
C. If the data either in statement I alone or in statement II alone is sufficient to answer the question.
D. If the data in both statements I and II together are necessary to answer the question
E. If the data given in both statements I and II together are not sufficient to answer the question.

46. Is the average age of the students of a college less than 20 years?
Statement I: The strength of the class XIII is less than 25% of the strength of the college.
Statement II: The average age of the students of class XIII of the college is 21 years and that of the remaining classes is 19 years.
A. If the data in statement I alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.
B. If the data in statement II alone is sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.
C. If the data either in statement I alone or in statement II alone is sufficient to answer the question.
D. If the data in both statements I and II together are necessary to answer the question.
E. If the data given in both statements I and II together are not sufficient to answer the question.

47. Among 30 retired persons, is the number of persons, who are having at least 10 years of service, less than 13?
Statement I: Exactly 22 persons joined the service before 2010 and exactly 19 persons retired after

2020.

Statement II: Exactly 19 persons joined the service after 2010 and exactly 11 persons retired before 2020.

A. If the data in statement I alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.
B. If the data in statement II alone is sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.
C. If the data either in statement I alone or in statement II alone is sufficient to answer the question.
D. If the data in both statements I and II together are necessary to answer the question.
E. If the data given in both statements I and II together are not sufficient to answer the question.

48. The lengths of trains P and Q are in the ratio 2:3. Which of them takes less time to cross the same platform?

Statement I: The time taken by train P to cross train Q when they are moving in opposite directions is half the time taken by it to cross train Q, when moving in the same direction.

Statement II: The ratio of the length of train Q to that of the platform is 4 : 3.

A. If the data in statement I alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.
B. If the data in statement II alone is sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.
C. If the data either in statement I alone or in statement II alone is sufficient to answer the question.
D. If the data in both statements I and II together are necessary to answer the question.
E. If the data given in both statements I and II together are not sufficient to answer the question.

49. What is the value of a two-digit number?

a. The ratio of the value of the number and the sum of the digits of that number is 3 : 1.

b. The number obtained after interchanging the digits is greater than the original number by 45.

c. The sum of the number and its square is 28 times the number itself.

A. All the three together.
B. Either (c) alone or (a) and (b) together
C. Any one of the three
D. All the three together are not sufficient
E. none of these

50. There are four members m1, m2, m3 and m4 partners in the business. What is the profit share of m2?

Statement I: m1 and m2 started the business with investment of Rs.x and Rs.2x respectively and after 6 months m3 and m4 joined them with investment of Rs. (x + 1000) and Rs.3x respectively.

Statement II: At the end of one years and profit Share of m3 is Rs.4000.

Statement III: At the end of one year the profit ratio of m3 and m4 is 2:3.3

A. Only I and II are sufficient.
B. Only II and III are sufficient.
C. Either II alone or I and II together to sufficient.
D. All I, II and III are necessary to the answer the question.
E. The question can't be answered even with all I, II and III.

ANSWER KEY:

1) D	11) E	21) D	31) B	41) E
2) C	12) E	22) C	32) C	42) C
3) D	13) E	23) D	33) C	43) B
4) B	14) C	24) A	34) E	44) C
5) C	15) E	25) A	35) A	45) D
6) D	16) D	26) C	36) E	46) D
7) C	17) C	27) A	37) A	47) A
8) A	18) B	28) A	38) C	48) A
9) E	19) C	29) A	39) A	49) B
10) C	20) A	30) E	40) C	50) D

DATA COMPARISON

Exercise – 1

In each of these questions, two quantities are mentioned, viz Quantity I and Quantity II

To mark the answers to the questions, use the following instructions.

Mark A. if Quantity I is greater than Quantity II

Mark B. if Quantity II is greater than Quantity I.

Mark C. if the two quantities are equal.

Mark D. if it is impossible to determine a relationship.

Mark E. if the greater Quantity cannot be determined but the two quantities are definitely not equal.

1. **Quantity I:** A plane starts full of passengers. At the first destination, it drops two-third of the passengers and takes 4400 more. At the second destination, it drops 3/5 of the new total and takes 7500 more. On arriving at the third station, it is found to have 39000 passengers. Find the number of passengers in the beginning.
   Quantity II: man owns $\frac{7}{8}$ of a property and sells $\frac{8}{11}$ of his shares for Rs. 91,000. What is the value of the property?
2. **Quantity I:** If 5a + 7b = 45 and $\frac{5a}{4b} = \frac{23}{25}$ then, b – a =?
   Quantity II: c + $\frac{1}{d}$ = 1, $d + \frac{1}{a} = 1$ then, acd =?
3. **Quantity I:** 884, 442, 448, 224, ___, 115
   What number should fill the blank?
   Quantity II: 230
4. **Quantity I:** 325, 359, 427, 529, ___, 835
   What number should fill the blank?
   Quantity II: 664
5. **Quantity I:** 5, 18, 57, 174, _.
   Quantity II: 5, 16, 60, 236, _.
6. **Quantity I:** 140, 166, 194, 224, 256, 290, 326, _.
   Quantity II: 365
7. In the question given below, if the given mathematical symbols are changed from '+' to '÷', '–' to '×', '÷', to '–' and from '×' to '+'.
   Quantity I: 304 × 123 – 23 × 10 + 30
   Quantity II: 523 × 250 – 6 + 15 ÷ 12
8. **Quantity I:** The average monthly savings of a family was Rs. 5500 during the first 3 months; Rs. 5250 during the next 4 months and Rs. 5330 during the last 5 months of a year. If the total expense during the year was Rs. 12550, find the average monthly income of the family.
   Quantity II: The smallest perfect square that is divisible by 7!
9. **Quantity I:** The sum of seven consecutive natural numbers is 196. How many of these numbers are prime?
   Quantity II: $111x^2-222x-333=0$. Find the difference of the roots.
10. **Quantity I:** The marks obtained by a student are given below. What is his total percentage in all the subjects?

Subject	Total Marks	Obtained Marks
Database	100	97
C++	80	75
Python	100	79
Java	80	72
Networking	100	68

    Quantity II: What was the percent profit/loss made/incurred by selling an article for Rs. 62000? The ratio of the selling price to the cost price of the article is 17 : 9.
11. **Quantity I:** When an article is sold for Rs. 500, the profit percent is five times as much as when it is sold for Rs. 420. The cost price of the article is:
    Quantity II: 25, 26, 54, __, 664
12. **Quantity I:** $x^2 + 16x = 161$
    Quantity II: $2y^2 + 8y - 42 = 0$
13. **Quantity I:** $43x^2 + 11x + 57 = 13x^2 + 56$
    Quantity II: $y^2 - 16y + 39 = 0$
14. **Quantity I:** $12x^2 + 65x + 97 = 4x^2 + 17x + 33$
    Quantity II: $y^2 - 22y + 117 = 0$
15. **Quantity I:** 60% of 500 + 75% of 320
    Quantity II: In an examination, 80% of the students passed in Java, 85% in Python and 75% in both Java and Python. If 60 students failed in both the subjects, find the total number of students.
16. **Quantity I:** 6 kg of tin and 36 kg of lead are mixed to form an alloy. The percentage of tin in the alloy is:
    Quantity II: What percent of 92 is 15?
17. **Quantity I:** 132 : 156 :: 182: ?
    Quantity II: 7 : 56 : : 14 : ?

18. **Quantity I:** 4, -8, 16, -32, 64, x
    Quantity II: 1, 4, 9, 16, 25, 36, 49, y

In each of these questions, two quantities are mentioned, viz. Quantity I and Quantity II.

To mark the answers to the questions, use the following instructions.

Mark A. if Quantity I is greater than Quantity II
Mark B. if Quantity II is greater than Quantity I.
Mark C. if the two quantities are equal.
Mark D. if it is impossible to determine a relationship.
Mark E. if the greater Quantity cannot be determined but the two quantities are definitely not equal.

19. What is the age of Sujay?
    Quantity I: Sujay is 4 years younger than Akash and the ratio of their ages is 7 : 8.
    Quantity II: Average age of Dhiraj and Sujay is 25 years and Dhiraj is 2 years older than Sujay.
20. What is the volume of the cylinder?
    Quantity I: The curved surface area of the cylinder is 220 cm^2. and the height of the cylinder is 2 cm less than the radius of the cylinder.
    Quantity II: 780 cm^3
21. Find the percentage of boys in the class this year.
    Quantity I: This year the percentage of girls in the class is 65%.
    Quantity II: Last years out of the 300 students, 60% was girls and this year the number of girls increased by 10% but total students remain the same.
22. **Quantity I:** Find the volume of a cylinder with radius 15 cm and height 21 cm.
    Quantity II: Find the volume of a sphere with radius 21 cm.
23. **Quantity I:** The distance travelled by a bus in 4 hours is 240 km. If the speed of bus is increased by 20% then what will be the time taken by bus to cover the triple of distance?
    Quantity II: A bus which is travelling from point A to point B which are 150 km apart, covers half of distance with 25 km/h and rest of distance with 30 km/hr and take rest of 45 minutes after travelling half of distance then what will be the total time taken by bus to reach destination?
24. **Quantity I:** Speed of boat in still water, if a man can travel 72 km downstream in 9 hours and 40 km upstream in 10 hours.
    Quantity II: Speed of boat in still water, if a man can travel 36 km downstream in 9 hours and the speed of stream is 1 kmph.
25. What is the area of the rectangle?
    Quantity I: The area of the rectangle is equal to the area of the square whose side is 26 cm.
    Quantity II: The perimeter of the rectangle is 132 cm, if the ratio of the length and the breadth of the rectangle is 6 : 5.
26. A box contains 7 White balls, 4 Red balls, 5 Black balls and 9 Orange balls.
    Quantity I: If two balls are drawn at random the probability that both the balls are either Black or red
    Quantity II: If four balls are drawn at random the probability that all are of different colors.
27. A 250 metres long train running at the speed of 100 kmph crosses another train running in opposite direction at the speed of 80 kmph in 9 seconds.
    Quantity I: The length of the other train.
    Quantity II: The length of the first train shrinks by 3/4th of that of the other train.
28. In a circle with centre O, PT and PS are tangents drawn to it from point P. If PT = 30 cm and OT = 16 cm
    Quantity I: The length of PO.
    Quantity II: Double the length of the hypotenuse of a right-angled triangle the other two sides of which are 8 cm and 15 cm respectively.
29. The perimeter of a square is equal to twice the perimeter of a rectangle of length 27 cm and breadth 15 cm.
    Quantity I: The perimeter of a semicircle whose diameter is equal to the side of the square.
    Quantity II: The perimeter of another semicircle whose radius is 21 cm.
30. The ratio of the salary of Raj to that of Simran is 5 : 8. If the salary of Raj increases by 60% and that of Simran decreases by 25% then the new ratio of their salaries becomes 4 : 3.
    Quantity I: Due to extra leaves, the salary of Raj gets deducted by 3/4
    Quantity II: Due to incentives, the salary of Simran gets increased by 12%

In each of these questions, two quantities are mentioned, viz. Quantity I and Quantity II

To mark the answers to the questions, use the following instructions.

Mark A. if Quantity I is greater than Quantity II

Mark B. if Quantity II is greater than Quantity I.

Mark C. if the two quantities are equal.

Mark D. if it is impossible to determine a relationship.

Mark E. if the greater Quantity cannot be determined but the two quantities are definitely not equal.

31. **Quantity I:** 853 × 125 + lcm (856, 124) – 56132
    Quantity II: 98467 + 652 × 26
32. **Quantity I:** A number is divided by 29, the remainder is 26. When the same number is divided by 13, the remainder is 5. What is the number?
    Quantity II: Remainder obtained when dividing 3^9 by 264.
33. **Quantity I:** $6x^2 - 101x + 33 = 0$
    Quantity II: $y^2 - 34y = -93$
34. **Quantity I:** $-110x + 27x^2 = 19x^2 - 78$
    Quantity II: $18y^2 - 41y + 21 = 0$
35. **Quantity I:** $5x^2 - 42x = -85$
    Quantity II: $y^2 + 8y - 9 = 0$
36. **Quantity I:** $6x^2 - 11x + 4 = 0$
    Quantity II: $32y^2 - 124y + 16 = 6y^2 - 16y$
37. **Quantity I:** If 80% of (P + Q) = 10% of (P – Q), then find P : Q
    Quantity II: If r : s = 7 : 5, then find the ratio (4r + 5s) : (4r – 5s).
38. **Quantity I:** Four people rented a field. W fed 24 sheep for 3 months; L 10 sheep for 5 months; M 35 sheep for 4 months and N 21 sheep for 3 months. If W's share of rent is Rs. 1440, find the total rent of the workshop.
    Quantity II: P, Q and R are three partners. They altogether invested Rs.14000 in business. At the end of the year, P got Rs. 337.50, Q got Rs. 1125 and R got Rs. 637.50 as profit. The difference between the investments of Q and P was:
39. **Quantity I:** 72 : 90 :: 156 : ?
    Quantity II: 68 : ? :: 222 : 350
40. **Quantity I:** In how many ways can the letters of the word 'vitriolic' be arranged?
    Quantity II: In how many ways a committee, consisting of 5 men and 6 women can be formed from 8 men and 10 women?
41. **Quantity I:** $\log_{10} 5 + \log_{10} (5x + 2) = \log_{10} (3x + 14) + \log_{10} 5$
    Quantity II: If log 2 = 0.30103, find the number of digits in 2^{26}
42. **Quantity I:** There are 12 boys and 8 girls in a class. If 5 students are selected at random, what is the probability that 2 girls and 3 boys are selected?
    Quantity II: A card is drawn from a well shuffled pack of 52 cards. What is the probability of getting jack or spade card?
43. In the multiplication each alphabet represents a different digit.
    $\frac{XYX \times X}{LML}$
    Quantity I: Y
    Quantity II: X

In each of these questions, two quantities are mentioned, viz. Quantity I and Quantity II

To mark the answers to the questions, use the following instructions.

Mark A. if Quantity I is greater than Quantity II

Mark B. if Quantity II is greater than Quantity I.

Mark C. if the two quantities are equal.

Mark D. if it is impossible to determine a relationship.

Mark E. if the greater Quantity cannot be determined but the two quantities are definitely not equal.

44. Ajay can do the half of the work in 12 days and Vijay can do the whole work in 15 days.
    Quantity I: In how many days will Ajay and Vijay do the whole work?
    Quantity II: If Ajay starts the work and works for 4 days, after that Vijay join him, in how many days will the whole work be completed?
45. A box contains 4 Iphone phones, 5 Redmi phones and 3 Motorola phones.
    Quantity I: If two phones are drawn at random, the probability that both the phones are either Motorola or Iphone.
    Quantity II: If two phones are drawn at random, the probability that both the phones are either Redmi or Motorola.

46. The sum of the diameter and the circumference of circle A is 232 cm and the radius of circle B is 14 cm less than the radius of circle A.
Quantity I: Twice the radius of circle A.
Quantity II: The circumference of circle B.
47. **Quantity I:** No. of days in which A will work alone, given A and B can complete work in 8days, B and C can complete work in 12 days, C and A can complete work in 16 days.
Quantity II: No. of days in which A will work alone, given A and B can complete work in 24 days, they started work together and after working for 8 days A left and B completed remaining work in 10 days.
48. **Quantity I:** Kusum is a fruit seller, she sells mango, guava and banana at 16%, 10% and 8% profit respectively. If the ratio of the cost of the fruits is ratio 1 : 3 : 6 and the ratio of the number of fruits of each type sold is 3 : 1 : 1, what is his net profit?
Quantity II: Kusum sells mango, guava and banana at 12%, 8% and 10% profit respectively. If the ratio of the cost of the fruits is ratio 2 : 3 : 5 and the ratio of the number of fruits of each type sold is 4 : 2 : 1, what is his net profit?
49. **Quantity I:** A trader sells two articles X and Y. He sells one at the loss of 16% and other at the profit of 10% but finally he observes that there is no loss or gain. If the total sale price of these two articles is Rs. 39,000, find the difference between their cost prices?
Quantity II: A man buys 5 cows and 6 goats in Rs. 30,000. Instead, if he buys 4 cows and 3 goats, he will have to pay Rs. 12000 less. What is the cost price of five cow?
50. Two persons, P and Q together can complete a piece of work in 40 days. The efficiency of P is 25% more than that of Q.
Quantity I: If P works at 20% of his efficiency, then how many days will he take to complete two – sixth of the work?
Quantity II: If Q works at 25% of his efficiency, then how many days will he take to complete half of the work?

ANSWER KEY:

1) A	11) A	21) A	31) B	41) B
2) A	12) D	22) B	32) A	42) A
3) C	13) B	23) A	33) E	43) D
4) A	14) B	24) A	34) E	44) B
5) B	15) B	25) B	35) E	45) B
6) B	16) B	26) B	36) E	46) B
7) A	17) C	27) A	37) B	47) B
8) B	18) B	28) C	38) A	48) A
9) E	19) A	29) C	39) A	49) B
10) B	20) B	30) D	40) A	50) B

SET THEORY & VENN DIAGRAMS

Sets

Sets are represented as a collection of well-defined objects or elements, and it does not change from person to person. A set is represented by a capital letter. The number of elements in the finite set is known as the cardinal number of a set.

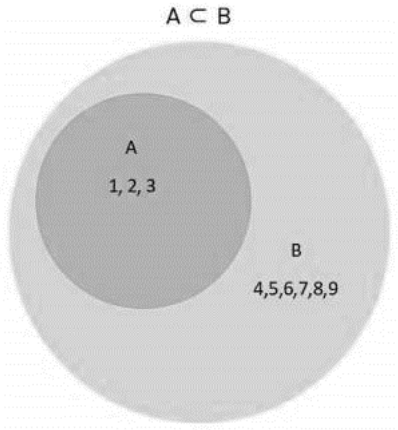
Venn Diagrams

A diagram used to represent all possible relations of different sets. A Venn diagram can be represented by any closed figure, whether it be a Circle or a Polygon (square, hexagon, etc.). But usually, circles are used to represent each set.

Types of Sets:

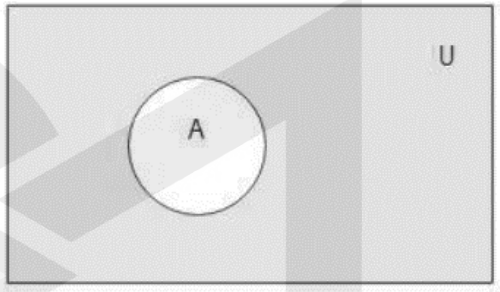
- Empty Set
  - A set which does not contain any element is called an empty set or void set or null set. It is denoted by { } or Ø.
  - A set of apples in the basket of grapes is an example of an empty set because in a grapes basket there are no apples present.
- Singleton Set
  - A set which contains a single element is called a singleton set.
  - Example: There is only one apple in a basket of grapes.
- Finite set
  - A set which consists of a definite number of elements is called a finite set.
  - Example: A set of natural numbers up to 10.
    A = {1,2,3,4,5,6,7,8,9,10}
- Infinite set
  - A set which is not finite is called an infinite set.
  - Example: A set of all natural numbers.
    A = {1,2,3,4,5,6,7,8,9……}
- Equivalent set
  - If the number of elements is the same for two different sets, then they are called equivalent sets. The order of sets does not matter here. It is represented as:
  - n(A) = n(B)
    where A and B are two different sets with the same number of elements.
  - Example: If A = {1,2,3,4} and B = {Red, Blue, Green, Black}
    In set A, there are four elements and in set B also there are four elements. Therefore, set A and set B are equivalent.
- Equal sets
  - The two sets A and B are said to be equal if they have exactly the same elements, the order of elements do not matter.
  - Example: A = {1,2,3,4} and B = {4,3,2,1}
    A = B
- Disjoint Sets
  - The two sets A and B are said to be disjoint if the set does not contain any common element.
  - Example: Set A = {1,2,3,4} and set B = {5,6,7,8} are disjoint sets, because there is no common element between them.
- Subsets
  - A set 'A' is said to be a subset of B if every element of A is also an element of B, denoted as A ⊆ B. Even the null set is considered to be the subset of another set. In general, a subset is a part of another set.
  - Example: A = {1,2,3}
    Then {1,2} ⊆ A.
  - Similarly, other subsets of set A are: {1},{2},{3},{1,2},{2,3},{1,3},{1,2,3},{}.
  - Note: The set is also a subset of itself.
  - If A is not a subset of B, then it is denoted as A⊄B.
  - Example, if all the elements of set A are also members of set B, then we can say that A ⊂ B.
  - If A = {1, 2, 3} and B = {1,2,3,4,5,6,7,8,9}
    Then A ⊂ B i.e. 'A is a subset of B'. For example, if all the elements of set A are also members of set B, then we can say that A ⊂ B.

If A = {1, 2, 3} and B = {1,2,3,4,5,6,7,8,9}
Then A ⊂ B i.e. 'A is a subset of B'.



Venn Diagram Symbols- Set Symbols

- Proper Subset
  - If A ⊆ B and A ≠ B, then A is called the proper subset of B and it can be written as A⊂B.
  - Example: If A = {2,5,7} is a subset of B = {2,5,7} then it is not a proper subset of B = {2,5,7}
    But, A = {2,5} is a subset of B = {2,5,7} and is a proper subset also.

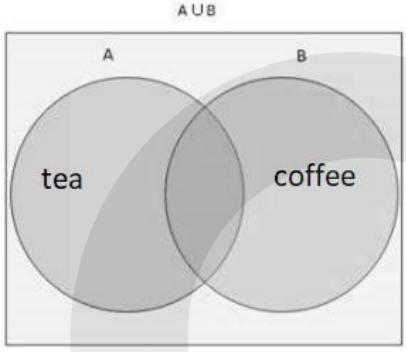


- Superset
  - Set A is said to be the superset of B if all the elements of set B are the elements of set A. It is represented as A ⊃ B.
  - For example, if set A = {1, 2, 3, 4} and set B = {1, 3, 4}, then set A is the superset of B.
- Universal Set
  - A set which contains all the sets relevant to a certain condition is called the universal set. It is the set of all possible values.
  - Example: If A = {1,2,3} and B {2,3,4,5}, then universal set here will be:
    U = {1,2,3,4,5}

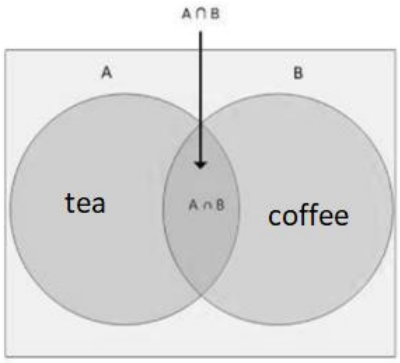
Operations on Sets

The basic operations on sets are:

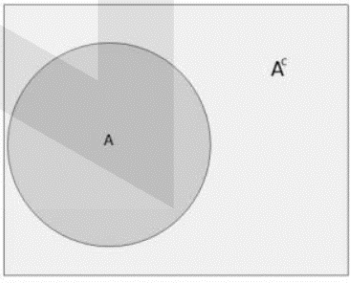
- Union of sets
  - If set A and set B are two sets, then A union B is the set that contains all the elements of set A and set B. It is denoted as A ∪ B.
  - Example: Set A = {1,2,3} and B = {4,5,6}, then A union B is:
    A ∪ B = {1,2,3,4,5,6}
  - For example, set A may represent people who like tea, and set B may portray people who like coffee. So, A ∪ B may represent people who like tea, coffees, or both.



- Intersection of sets
  - If set A and set B are two sets, then A intersection B is the set that contains only the common elements between set A and set B. It is denoted as A ∩ B.
  - Example: Set A = {1,2,3} and B = {4,5,6}, then A intersection B is:
    A ∩ B = { } or Ø
    Since A and B do not have any elements in common, so their intersection will give null set.
  - So, if A represents people who like tea and B has people who like coffee So, the A ∩ B set contains all people who like both tea and coffee.



- A complement of a set
  - The complement of any set, say P, is the set of all elements in the universal set that are not in set P. It is denoted by P'.
  - Properties of Complement sets
    - P ∪ P′ = U
    - P ∩ P′ = Φ
    - Law of double complement: (P')' = P
    - Laws of empty/null set(Φ) and universal set(U),
      Φ′ = U and U′ = Φ.



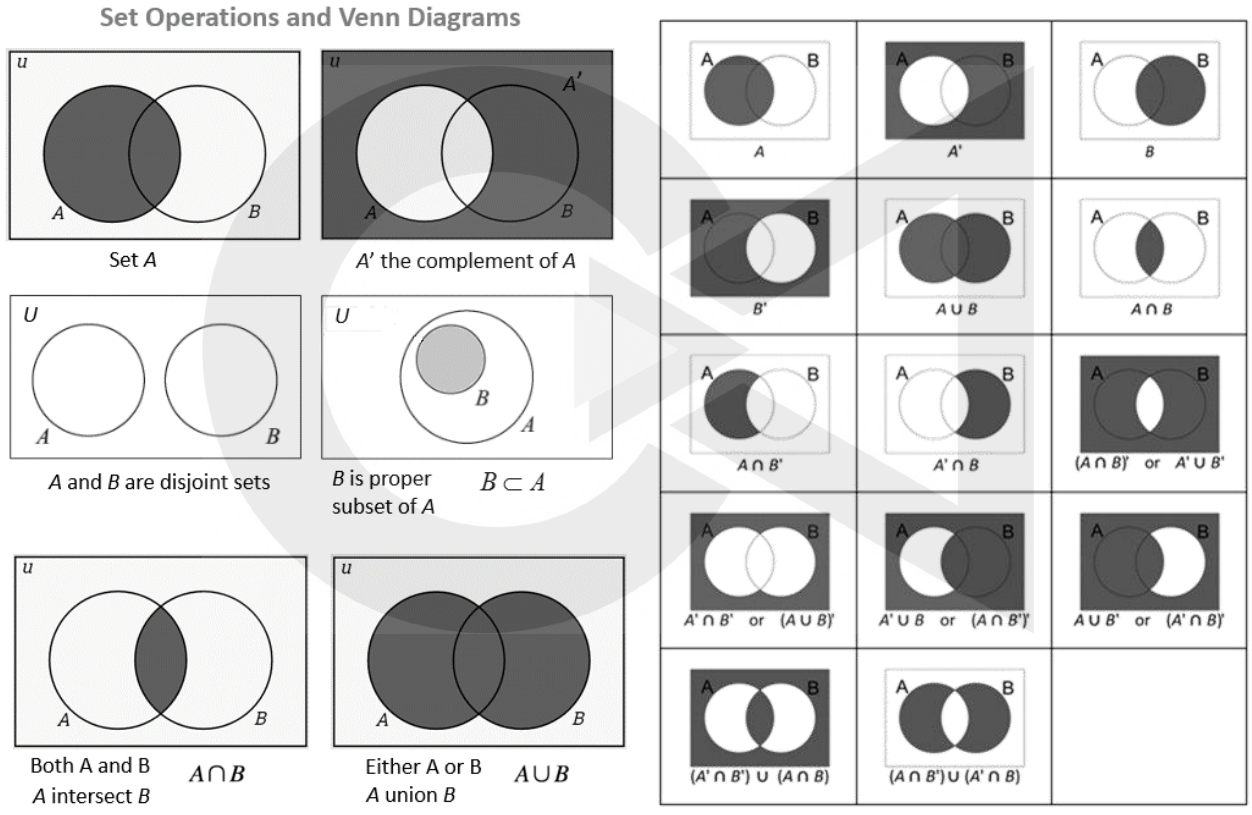
- Cartesian product of sets.
  - If set A and set B are two sets then the cartesian product of set A and set B is a set of all ordered pairs (a, b), such that a is an element of A and b is an element of B. It is denoted by A × B.
  - We can represent it in set-builder form, such as:
    A × B = {(a, b) : a ∈ A and b ∈ B}
  - Example: set A = {1,2,3} and set B = {Bat, Ball}, then;
    A × B = {(1, Bat), (1, Ball), (2, Bat), (2, Ball), (3, Bat), (3, Ball)}

- Difference of Sets
  - If set A and set B are two sets, then set A difference set B is a set which has elements of A but no elements of B. It is denoted as A – B.
  - Example: A = {1, 2, 3} and B = {2,3,4} A – B = {1}

Sets Formulae:

For any three sets A, B and C

- n (A ∪ B) = n(A) + n(B) – n (A ∩ B)
- If A ∩ B = ∅, then n (A ∪ B) = n(A) + n(B)
- n(A – B) + n(A ∩ B) = n(A)
- n(B – A) + n(A ∩ B) = n(B)
- n(A – B) + n(A ∩ B) + n(B – A) = n(A ∪ B)
- n (A ∪ B ∪ C) = n(A) + n(B) + n(C) – n (A ∩ B) – n (B ∩ C) – n (C ∩ A) + n(A ∩ B ∩ C)

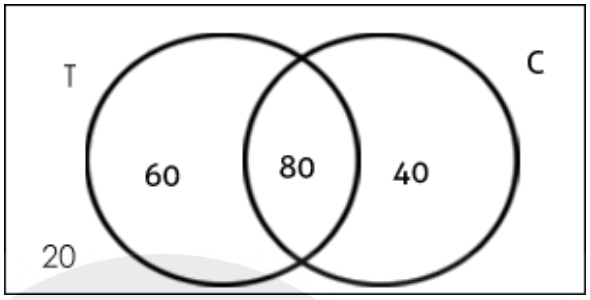


Solved Examples

Example 1: In a college, 200 students are randomly selected. 140 like tea, 120 like coffee and 80 like both tea and coffee.

- How many students like only tea?
- How many students like only coffee?
- How many students like neither tea nor coffee?
- How many students like only one of tea or coffee?
- How many students like at least one of the beverages?

Solution: The given information may be represented by the following Venn diagram, where T = tea and C = coffee.

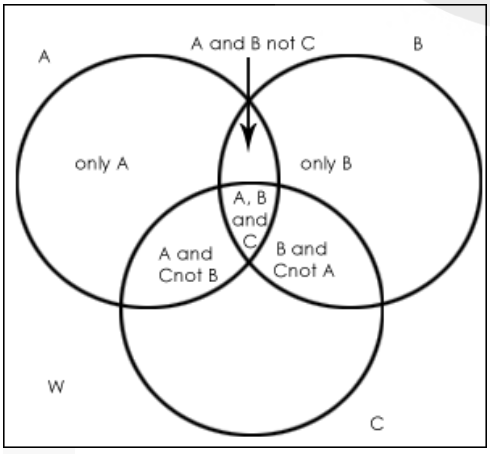


- Number of students who like only tea = 60
- Number of students who like only coffee = 40
- Number of students who like neither tea nor coffee = 20
- Number of students who like only one of tea or coffee = 60 + 40 = 100
- Number of students who like at least one of tea or coffee = n (only Tea) + n (only coffee) + n (both Tea & coffee) = 60 + 40 + 80 = 180

Three or More Sets in a Venn Diagram

Three sets Venn diagrams or 3-circle Venn diagrams consist of three overlapping circles to show the relationship between the elements of three sets. Many connections exist in a three-set Venn diagram. If A, B, and C are three circles in Venn Diagram, then we can observe;

- Members common in A and B = Members in A and B only plus elements in A, B, and C.
- Members common in B and C = Members in B and C only plus elements in A, B, and C.
- Members common in A and C = Members in A and C only plus elements in A, B, and C.



Where,

W = number of elements that belong to none of the sets A, B or C

Example:

In a survey of 500 students of a college, it was found that 49% liked watching football, 53% liked watching hockey and 62% liked watching basketball. Also, 27% liked watching football and hockey both, 29% liked watching basketball and hockey both and 28% liked watching football and basket ball both. 5% liked watching none of these games.

How many students like watching all the three games?

Find the ratio of number of students who like watching only football to those who like watching only hockey.

Find the number of students who like watching only one of the three given games.

Find the number of students who like watching at least two of the given games.

Solution:

n(F) = percentage of students who like watching football = 49%

n(H) = percentage of students who like watching hockey = 53%

n(B)= percentage of students who like watching basketball = 62%

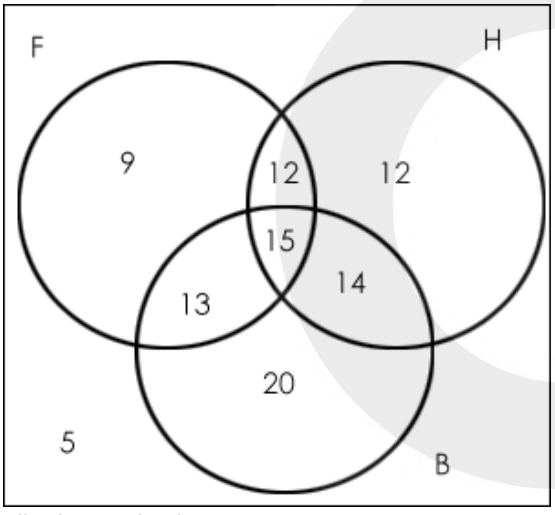
n (F ∩ H) = 27% ; n (B ∩ H) = 29% ; n(F ∩ B) = 28%

Since 5% like watching none of the given games so, n (F ∪ H ∪ B) = 95%.

Now applying the basic formula,

95% = 49% + 53% + 62% –27% – 29% – 28% + n (F ∩ H ∩ B)

Solving, you get n (F ∩ H ∩ B) = 15%.



All values in the diagram are in %.

- Number of students who like watching all the three games = 15 % of 500 = 75.
- Ratio of the number of students who like only football to those who like only hockey = (9% of 500)/(12% of 500) = 9/12 = 3:4.
- The number of students who like watching only one of the three given games = (9% + 12% + 20%) of 500 = 205
- The number of students who like watching at least two of the given games=(number of students who like watching only two of the games) +(number of students who like watching all the three games)= (12 + 13 + 14 + 15)% i.e. 54% of 500 = 270.

NUMBER SYSTEM SOLUTIONS

1. **Answer: B**
   Solution
   Let P = 4q + 3. Then 2P = 8q + 6 = 4(2q + 1) + 2.
   Thus, when 2P is divided by 4, the remainder is 2.
2. **Answer: A**
   Solution
   $= (2 \times 3)^7 \times (5 \times 7)^3 \times 11^{10}$
   $= 2^7 \times 3^7 \times 5^3 \times 7^3 \times 11^{10}$
   Thus, there are (7 + 7 + 3 + 3 + 10) = 30 prime number
3. **Answer: C**
   Solution
   Let the smaller number be x.
   Then, larger number = (2395 + x)
   Therefore 2395 + x = (6x + 15)
   5x = 2380
   x = 476.
4. **Answer: C**
   Solution
   Putting x = 2, we get $2^2 (2^2 - 1) = 12$.
   checking with other integers, the above equation always gives a value which is a multiple of 12,
   So, $x^2 (x^2 - 1)$ is always divisible by 12.
5. **Answer: D**
   Solution
   2, 4, 8, 16, ..., 1024 is a G.P with a =2
   and r = 4/2 =2
   Let the number of terms be n. Then
   $2 \times 2^{n-1} = 1024$
   or $2^{n-1} = 512 = 2^9$
   Thus, n - 1 = 9
   n = 10
6. **Answer: B**
   Solution
   Required digit = Unit digit
   = (1 × 2 × 3 × 4 × 5 × 6 × 7 × 8 × 9)
   = 0.
7. **Answer: A**
   Solution
   Every such number must be divisible by L.C.M of 4,5,6 i.e., 60.
   Such numbers are 240, 300, 360, 420, 480, 540.
   Clearly, there are 6 such numbers
8. **Answer: C**
   Solution
   Required numbers; 1×2×3×4 = 24, which is divisible by 24.
   Also, checking 5×6×7×8 = 1680, which is also divisible by 24
9. **Answer: D**
   Solution
   On dividing 11158 by 77, the remainder is 70
   Thus, Number to be added = (77 - 70) = 7
10. **Answer: A**
    Solution
    On dividing 427398 by 15, we get remainder = 3.
    Least number to be subtracted is 3.
11. **Answer: B**
    Solution
    (2×1 + 1) = 3.
    (2×2 + 1) = 5.
    (2×3 + 1) = 7.
    (2×4 + 1) = 9.
    which is not prime, n=4.
12. **Answer: C**
    Solution
    For a natural number N, expressed as the product of prime numbers by prime factorization method, as-
    $N = X^a + Y^b + Z^c$ (where X, Y, Z are prime numbers and a, b, c are their respective powers)
    The number of factors of N is (a + 1) × (b + 1) × (c + 1)
    Here, 9321 can be expressed as-
    $9321 = 3 \times 13 \times 239 = 3^1 \times 13^1 \times 239^1$
    Hence, number of factors of 9321 = (1 + 1) × (1 + 1) × (1 + 1) = 2 × 2 × 2 = 8.
13. **Answer: D**
    Solution
    Last digit for the power of 6 is 6 (always)
    Power cycle of 7 is 7, 9, 3, 1.
    Now 467/4 gives a remainder of 3
    Then the last digit is $7^3 = 3$
    Last digit is 6 + 3 = 9
14. **Answer: D**
    Solution
    Let the three integers be x, x + 2 and x + 4.
    Then, 3x = 2(x + 4) + 3 $\leftrightarrow$ x = 11.
    Third integer = x + 4 = 15.

15. **Answer: B**

Solution

Let the ten's digit be x and unit's digit be y.

Then, (10x + y) – (10y + x) = 36

→ 9(x – y) = 36

→ x – y = 4.

16. **Answer: B**

Solution

Since the number is greater than the number obtained on reversing the digits, so the ten's digit is greater than the unit's digit.

Let ten's and unit's digits be 2x and x respectively.

Then, (10 × 2x + x) – (10x + 2x) = 36

→ 9x = 36

→ x = 4.

∴ Required difference = (2x + x) – (2x – x) = 2x = 8.

17. **Answer: D**

Solution

Let the ten's digit be x and unit's digit be y.

Then, x + y = 15 and x – y = 3 or y – x = 3.

Solving x + y = 15 and x – y = 3, we get: x = 9, y = 6.

Solving x + y = 15 and y – x = 3, we get: x = 6, y = 9.

So, the number is either 96 or 69.

Hence, the number cannot be determined

18. **Answer: D**

Solution

Let the ten's digit be x and unit's digit be y.

Then, number = 10x + y.

Number obtained by interchanging the digits = 10y + x.

∴ (10x + y) + (10y + x) = 11(x + y), which is divisible by 11.

19. **Answer: B**

Solution

Let the middle digit be x.

Then, 2x = 10 or x = 5. So, the number is either 253 or 352.

Since the number increases on reversing the digits, so the hundred's digits is smaller than the unit's digit.

Hence, required number = 253.

20. **Answer: C**

Solution

Let the number be z. then, z – 36 = 86 – z

2z = 86 + 36 ⇒ 2z = 122.

z = 61.

Hence, the required number is 61.

21. **Answer: A**

Solution

Let, the number be z, Then, 7z – 15 = 2z + 10

5z = 25 ⇔ z = 5.

Hence, the required number is 5.

22. **Answer: C**

Solution

Let the numbers be x and (15 – x).

Then, $x^2+(15–x)^2=113$

$x^2+ 225 + x^2– 30x = 113$

$⇒ 2x^2 – 30x + 112 = 0$

$⇒ x^2 – 15x + 56 = 0$

⇒ (x – 7) (x – 8) = 0

⇒ x = 7 and x = 8.

So, the numbers are 7 and 8.

23. **Answer: D**

Solution

Let the four consecutive even numbers be x, x + 2, x + 4 and x + 6.

Then, sum of these numbers = (27 × 4) = 108.

So, x + (x + 2) + (x + 4) + (x + 6) = 108

or 4x = 96 or x = 24.

So, the largest number = (x + 6) = 30.

24. **Answer: C**

Solution

Let the number be x, x + 2 and x + 4.

Then, $x^2 + (x + 2)^2 + (x + 4)^2 = 2531$

$⇒ 3x^2 + 12x – 2511 = 0$

$⇒ x^2 + 4x – 837 = 0$

⇒ (x – 27) (x + 31) = 0 ⇒ x = 27.

Hence, the required numbers are 27, 29 and 31.

25. **Answer: C**

Solution

Let the number be x and y, such that x > y.

Then, 3x – 4y = 5 .... (1)

and (x + y) – 6(x – y) = 6

⇒ – 5x + 7y = 6 .... (2)

Solving (1) and (2), we get : x = 59 and y = 43.

Hence, the required numbers are 59 and 43.

26. **Answer: C**

Solution

Let the number be x
As per the equation,
X – 2/5 X = 75
3/5 X = 75
X = 125

27. **Answer: C**

Solution

Let the number be x and y, such that x > y.
Then, 4x – 5y = 4 .... (1)
and (x + y) – 8 (x – y) = 4
⇒ – 7x + 9y = 4 .... (2)
Solving (1) and (2), we get : x = 56 and y = 44.
Hence, the required numbers are 56 and 44.

28. **Answer: C**

Solution

x + y = 10 .... (1)
y + z = 19 .... (2)
x + z = 21 .... (3)
Adding (1), (2) and (3),
we get : 2(x + y + z) = 50 or (x + y + z) = 25. Thus, x = (25 – 19) = 6; y = (25 – 21) = 4; z = (25 – 10) = 15.
Hence, the required numbers are 6, 4 and 15.

29. **Answer: A**

Solution

Use the prime number test. Neither 359 nor 323 are divisible by either 2 or 3. The perfect squares closest to the given numbers are 324 and 361.
Here, consider larger of the two numbers.
n^2 = 361 and √(n) = 19
So, test divisibility of the given numbers with numbers of the form 6k±1 up to 19 i.e., 5, 7, 11, 13, 17, 19.
323 is divisible by 17 and 19, whereas 359 is not divisible by any of the numbers.
359 is a prime number, while 323 is not.

30. **Answer: D**

Solution

Let the required number be n.
When n divides 126, the remainder is 6.
∴ n will completely divide 126 – 6 ie. 120.
Similarly, when n divides 149, the remainder is 5.
∴ n will completely divide 149 –5 i.e., 144.
Hence, n is a common factor of 120 and 144.
Since, n is the greatest possible number such that it satisfies the given conditions, n has to be the HCF of 120 and 144.
120 = 2 × 2 × 2 × 3 × 5
and 144 = 2 × 2 × 2 × 2 × 2 × 3 × 3
So, the HCF of 120 and 144 is 2 × 2 × 2 × 3
i.e., 24. Thus, n is 24.

31. **Answer: B**

Solution

Using the first statement, 259 = nq +7
∴ nq = 252
Similarly, 547 = (2n)q + r = 2(nq) + r
547 = 2(252) + r
r= 547 – 504 = 43

32. **Answer: E**

Solution

A number which gives a remainder of 1 on dividing by 13 = (13n+1)
Further, when the same number is divided by 17, the remainder is 11. So, the same number can also be represented as (17m + 11)
∴ 13n + 1 = 17m +11
∴ 13n = 17m + 10
∴ n = $\frac{17m+10}{13}$
Since n and m are both integers, look for values of m that give an integral value of n.
This condition is satisfied for m = 4 and n=6
Thus, the first number that satisfies this condition is:
13n + 1 = (13 × 6) +1 = 79
or, 14m + 5 = (17 × 4) + 11= 79

33. **Answer: C**

Solution

1 is the only common factor between numbers as the number are co-prime.
Also, the middle number is common between the given two products.
∴ middle number = HCF of 851 and 1517=37
∴ First number = 851/37 = 23 and second number 1517/37 = 41
required sum = 23 + 37 + 41 = 101

34. **Answer: B**

Solution

Let the required number be N.

N = 14Q + 3

On squaring, N = $(14Q + 3)^2$ = $196Q^2$+ 84Q + 9.

When this number is divided by 14, the first two terms are divisible but 9 is not!

Hence, the required remainder is 9.

35. **Answer: D**

Solution

Since the number is divisible by 3, sum of digits should be divisible by 3.

x+(2+4+7) should be divisible by 3 i.e., x + 13 should be divisible by 3.

This is satisfied for x = 2, 5, 8.

Hence, option A and C are eliminated.

You can substitute each value of x in 247x and check for divisibility by 7.

2472 = 7(353) +1; 2475 = 7(353) + 4; 2478 = 7(354) + 0;

Hence, 247x is divisible by 7, when x = 8.

36. **Answer: D**

Solution

Dividing 123(n^3 + 32n) by 24 is equivalent to dividing 41(n^3 + 32n) by 8.

Since the given number is divisible by 8, but 41 is not, the other term i.e. (n^3 + 32n) has to be divisible by 8.

Since 32n is already divisible by 8, n^3 also has to be divisible by 8.

Hence, n^3 is an even number.

Since n is prime, the only value where n is prime and n^3 is even is when n = 2.

37. **Answer: C**

Solution

143 = 13 x 11

So, if the number divisible by both the numbers 13 and 11, then the number is divisible by 143 also.

429 → 11, 13

396 → 11

462 → 11

792 → 11

858 → 11, 13

2178 → 11

3432 → 11, 13

7150 → 11, 13

Therefore, the following numbers are divisible by 143 : 429, 858, and 3432

Required number of numbers = 4.

38. **Answer: A**

Solution

Unit digit in $(8654)^{1793}$ = Unit digit in $(4)^{1793}$

= Unit digit in $[(4^2)^{896}$ x 4]

= Unit digit in (6 x 4) = 4

Unit digit in $(245)^{317}$ = Unit digit in $(5)^{317}$ = 5

Unit digit in $(531)^{491}$ = Unit digit in $(1)^{491}$ = 1

Required digit = Unit digit in (4 × 5 × 1) = 0.

39. **Answer: D**

Solution

Let the smaller number be x. Then larger number = (x + 1535).

∴ x + 1535 = 5x + 95

→ 4x = 1440

→ x = 360

∴ Smaller number = 360.

40. **Answer: E**

Solution

Let the required fraction be x. Then $\frac{1}{x} - x = \frac{11}{30}$

$\therefore \frac{1-x^2}{x} = \frac{11}{30}$

∴ 30 – $30x^2$ = 11x

∴ $30x^2$ + 11x – 30 = 0

∴ $30x^2$ + 36x – 25x – 30 = 0

∴ 6x (5x + 6) – 5(5x +6) = 0

$\therefore$ (5x + 6) (6x – 5) = 0 $\therefore x = \frac{5}{6}$

41. **Answer: E**

Solution

Unit digit in 3^4 = 1 → Unit digit in $(3^4)^{17}$ = 1

∴ Unit digit in 3^{65} = Unit digit in [$(3^4)^{17}$ x 3] = (1 x 3) = 3

Unit digit in 6^{75} = 6

Unit digit in 7^4 → Unit digit in $(7^4)^{14}$ is 1.

Unit digit in 7^{59} = Unit digit in $[(7^4)^{14} \times 7^3]$ = (1 x 3) = 3

∴ Required digit = Unit digit in (3 × 6 × 3) = 4.

42. **Answer: B**

Solution

Required numbers are 12, 16, 20, ..., 96

This is an A.P. in which a = 12, d = 4 and l = 96.

t_n = 96

$\therefore \ a + (n - 1)d = 96$

$\therefore \ 12 + (n - 1) \times 4 = 96$

$\therefore \ (n - 1) \times 4 = 84$

$\therefore (n - 1) = 21$
$\therefore n = 22$
Required sum = $\frac{n}{2}(a + l) = \frac{22}{2} \times (12 + 96) = (11 \times 108)$
= 1188

43. **Answer: B**
Solution
Let $S_n = (2 + 4 + 6 + ... + 50)$. This is an A.P. in which a = 2, d = 2 and l = 50
Let the number of terms be n. Then,
a + (n – 1)d = 50
$\Rightarrow$ 2 + (n – 1) × 2 = 50
$\Rightarrow$ n = 25.
Required sum = $\frac{n}{2}(a + l) = \frac{25}{2} \times (2 + 50) = (25 \times 26)$
= 650

44. **Answer: B**
Solution
This is a G.P. in which a = 2, r = $\frac{2^2}{2}$ = 2 and n = 10
$S_n = \frac{a(r^n - 1)}{(r - 1)} = \frac{2 \times (2^{10} - 1)}{(2 - 1)}$ = 2 (1024 – 1)
= 2 × 1023 = 2046

45. **Answer: D**
Solution
Let the two consecutive even integers be 2n and (2n + 2). Then,
$(2n + 2)^2 - 2n^2 = 4n^2 + 8n + 4 - 4n^2$
= 2(4n + 2) = 4(2n + 1), which is divisible by 4.

46. **Answer:** C
Solution
When n is even. ($x^n - a^n$) is completely divisibly by (x + a)
($21^{300} - 1^{300}$) is completely divisible by (21 + 1), i.e., 22.
→ (21^{300} – 1) is completely divisible by 22.
→ On dividing 21^{300} by 22, we get 1 as remainder.

47. **Answer: D**
Solution
We know that,
$(1^2 + 2^2 + 3^2 + ... + n^2) = \frac{1}{6} n (n + 1) (2n + 1)$
Putting n = 10,
Required sum = $(\frac{1}{6} \times 10 \times 11 \times 21) = 385$

48. **Answer: D**
Solution
Given sum = (1 + 1 + 1 + ... to n terms) – $(\frac{1}{n} + \frac{2}{n} + \frac{3}{n})$ + ... to n terms
= $n - \frac{n}{2}\left(\frac{1}{n} + 1\right)$ [Ref: n^{th} terms = (n/n) = 1]
= $n - \frac{n+1}{2}$
= 1/2 (n –1)

49. **Answer: B**
Solution
(x^n – 1) will be divisibly by (x + 1) only when n is even.
(81^{17} – 1) = {(9^2)^{17} – 1} = (9^{34} – 1), which is divisible by (9 +1), i.e., 10

50. **Answer: C**
Solution
Required numbers are 102, 108, 114, ..., 996
This is an A.P. in which a = 102, d = 6 and l = 996
Let the number of terms be n. Then,
a + (n – 1)d = 996
102 + (n – 1) x 6 = 996
6 × (n – 1) = 894
(n – 1) = 149
∴ n = 150.

SURDS AND INDICES SOLUTIONS

1. **Answer: A**

Solution

Concept used are

$a^m \times a^n = a^{m+n}$; $\frac{a^m}{a^n} = a^{m-n}$; $[a^m]^n = a^{m\times n}$

$$\left[\frac{\left(\frac{3}{14}\right)^{-6}}{\left(\frac{3}{14}\right)^{3}}\right]^3 \times \left(\frac{3}{14}\right)^{-9}$$

$$= \left[\left(\frac{3}{14}\right)^{-9}\right]^3 \times \left(\frac{3}{14}\right)^{-9}$$

$$= \left(\frac{3}{14}\right)^{-36}$$

2. **Answer: B**

Solution

Concept used are

$a^m \times a^n = a^{m+n}$; $\frac{a^m}{a^n} = a^{m-n}$; $[a^m]^n = a^{m\times n}$

$5^{-8+3n+4} = 5^{11-3}$

$5^{3n-4} = 5^8$

$3n - 4 = 8$

$3n = 12$

$n = 4$.

3. **Answer: B**

Solution

Concept used are $a^m \times a^n = a^{m+n}$;

$\frac{a^m}{a^n} = a^{m-n}$; $[a^m]^n = a^{m\times n}$

$7^{11+6n} = 7^{25-n}$

$11+6n = 25-n$

$7n = 14$

$n = 2$.

4. **Answer: B**

Solution

$16^2 \times 64^{6n+5} - 4^m = 0$

$(4^2)^2 \times (4^3)^{6n+5} = 4^m$

$4^4 \times 4^{18n+15} = 4^m$

$18n+19 = ms$

5. **Answer: C**

Solution

$27^3 \times 81^{2n+3} - 9^m = 0$

$27^3 \times 81^{2n+3} = 9^m$

$(3^3)^3 \times (3^4)^{2n+3} = (3^2)^m$

$3^9 \times 3^{8n+12} = 3^{2m}$

$8n+21 = 2m$

6. **Answer: A**

Solution

Since $(3 + 4\sqrt{3})^2 = 9 + 48 + 2 \times 3 \times 4\sqrt{3} =$

$57 + 24\sqrt{3}$

$57 + 24\sqrt{3} = a + b\sqrt{3}$

On comparing we can see that, (a, b) = (57,24).

7. **Answer: C**

Solution:

$4^4 = 256$, $5^4 = 625$

$$\left(\frac{4}{5}\right)^x = \frac{256}{625} = \frac{44}{54} = \left(\frac{4}{5}\right)^4$$

$x = 4$

$x^x = 4^4 = 256$

8. **Answer: C**

Solution

$11^3 = 1331$, $15^3 = 3375$

$$\left(\frac{11}{15}\right)^x = \frac{1331}{3375} = \frac{11^3}{15^3} = \left(\frac{11}{15}\right)^3$$

$x = 3$

$x^{-x} = \frac{1}{x^x} = \frac{1}{3^3} = \frac{1}{27}$

9. **Answer: B**

Solution

$625^{0.25} = 5^a \times 5^6$

$(5^4)^{0.25} = 5^{a+6}$

$5^1 = 5^{a+6}$

$a = 1-6 = -5$

10. **Answer: D**

Solution

$$(3\sqrt{5} + 6\sqrt{2}) - (\sqrt{20} - \sqrt{18})$$
$$= 3\sqrt{5} + 6\sqrt{2} - 2\sqrt{5} + 3\sqrt{2}$$
$$= \sqrt{5} + 9\sqrt{2}$$

11. **Answer: A**

Solution

$$\frac{\left(\frac{1}{343}\right)^{-\frac{5}{3}}}{\left(\frac{1}{49}\right)^{\frac{3}{2}}} = \frac{(7^{-3})^{-\frac{5}{3}}}{(7^{-2})^{\frac{3}{2}}} = \left(\frac{7^5}{7^{-3}}\right) = 7^{5+3} = 7^8$$

12. **Answer: C**

Solution

$$\frac{100000^5}{1000^7} + 1 = \frac{(10^5)^5}{(10^3)^7} + 1$$
$$= \frac{10^{25}}{10^{21}} + 1 = 10^4 + 1$$

13. **Answer: B**

Solution

$7^{-35} - 7^{-36} = 7^{-35}(1 - 7^{-1})$

$= 7^{-35}(1 - (\frac{1}{7})) = 7^{-35}(\frac{6}{7}) = 7^{-36} \times 6$

14. **Answer: A**

Solution

$\left(\frac{35}{21}\right)^{1.8} \times 45^{0.9} = 5^x$

$\left(\frac{5}{3}\right)^{1.8} \times (5 \times 9)^{0.9} = 5^x$

$\left(\frac{5}{3}\right)^{1.8} \times 5^{0.9} \times (3^2)^{0.9} = 5^x$

$5^{(1.8+0.9)} = 5^{2.7}$

x = 2.7

15. **Answer: C**

Solution

$3^{4n+3} = \frac{1}{9^{n-2}}$

$3^{4n+3} = (3^{-2})^{n-2}$

4n+3 = -2n +4

6n = 1

n = 1/6

16. **Answer: D**

Solution

$5^{n+4} - 5^{n+2} = 24$

$5^{n+2}(5^2 - 1) = 24$

5^{n+2} x 24 = 24

$5^{n+2} = 1 = 5^0$ [$\because 5^0 = 1$]

n + 2 = 0

n = -2.

17. **Answer: A**

Solution

As $256 = 2^8$; $576 = 24^2$; $64 = 2^6$; $27 = 3^3$

$\left(\frac{256}{576}\right)^{\frac{1}{4}} \times \left(\frac{64}{27}\right)^{-\frac{1}{3}} \times \left(\frac{216}{8}\right)^{-1}$

$\left(\frac{2^8}{24^2}\right)^{\frac{1}{4}} \times \left(\frac{3^3}{2^6}\right)^{\frac{1}{3}} \times \left(\frac{8}{216}\right)$

$= \left(\frac{2^2}{\sqrt{24}}\right) \times \left(\frac{3}{4}\right) \times \left(\frac{1}{27}\right) = \frac{1}{2\sqrt{6} \times 9} = \frac{1}{18\sqrt{6}}$

18. **Answer: B**

Solution

$\left(\sqrt[3]{0.000729}\right)^{\frac{1}{2}} = (0.000729)^{\frac{1}{2} \times \frac{1}{3}}$

$= \left(\frac{3}{10}\right)^{6 \times \frac{1}{6}} = 0.3$

19. **Answer: C**

Solution

$[\sqrt{x}^{(-2/5)}]^{25} = x^{\left(\frac{1}{2} \times \left(-\frac{2}{5}\right) \times 25\right)}$

$= x^{[\left(-\frac{1}{5}\right) \times 25]} = x^{(-5)}$

$= (1/x)^5 = 1/32$

20. **Answer: B**

Solution

$$\frac{0.0036^{\frac{1}{2}} + 0.000027^{\frac{1}{3}} + 1}{0.0081^{\frac{1}{2}} - 0.0064^{\frac{1}{2}}}$$

$$= \frac{(0.06 + 0.03 + 1)}{(0.09 - 0.08)} = 109$$

21. **Answer: A**

Solution

$\sqrt{8} + 2\sqrt{32} - 3\sqrt{128} + 4\sqrt{50}$

$= 2\sqrt{2} + 8\sqrt{2} - 3 \times 8\sqrt{2} + 4 \times 5\sqrt{2}$

$= (2 + 8 - 24 + 20)\sqrt{2}$

$= 6\sqrt{2} = 6 \times 1.414 = 8.484$

Hence, option A is correct.

22. **Answer: C**

Solution

$(1331)^{\left(\frac{8}{3}\right)\left(\frac{1}{4}\right)} = 1331^{\frac{2}{3}}$

$=11^{(3)(2/3)} = 11^2 = 121$

23. **Answer: A**

Solution

$3^{6(0.3)} \times (3^3)^{(-0.1/3)} \times 3^{4(0.2)} = 3^{2.5}$

24. **Answer: C**

Solution

$2^{-2^{-2}} = \frac{1}{\sqrt[4]{2}}$

25. **Answer: D**

Solution

$(0.04)^{-1.5} = (0.04)^{-(3/2)} = (0.2)^{2(-3/2)}$

$= \frac{1}{(0.2)^3} = \frac{1}{0.008} = \frac{1000}{8} = 125$

26. **Answer: C**

Solution

$(0.000064)^{(0.25)(4/3)} = (0.04^3)^{(0.25)(4/3)} =$

$(0.04^3)^{(1/4)(4/3)} = 0.04$

27. **Answer: C**

Solution

Using indices formulas,

$a^m \times a^n = a^{m+n}$;

$\frac{a^m}{a^n} = a^{m-n}$; $8^{11.36}$ x $8^a = 8^{15.85}$

$8^a = \frac{8^{15.85}}{8^{11.36}}$

$8^a = 8^{15.85 - 11.36}$

$8^a = 8^{4.49}$

a = 4.49

28. **Answer: B**

Solution

$\left(\frac{a}{b}\right)^{2y+3} = \left(\frac{b}{a}\right)^{y-2}$

$\left(\frac{a}{b}\right)^{2y+3} = \left(\frac{a}{b}\right)^{2-y}$

2y+3 = 2–y

3y = –1

$y = -\frac{1}{3}$

29. **Answer: B**

Solution

$x^z = y^3$

$5^{10.2z} = 5^{(1.5 \times 3)}$

$z = \frac{4.5}{10.2}$

z = 0.44

30. **Answer:** E

Solution

$2^a = 4096 = 2^{12}$

a = 12

$2^{a-5} = 2^{12-5} = 2^7 = 128$

31. **Answer:** C

Solution

$5^{(x+y)} = 125 = 5^3$

x + y = 3 ---------(1)

$5^{(x-y)} = 15625 = 5^6$

x – y = 6 ---------(2)

Solving equ (1) & (2) we get

x = 4.5 ; y = –1.5

32. **Answer: B**

Solution

Concept used are $a^m \times a^n = a^{m+n}$;

$\frac{a^m}{a^n} = a^{m-n}$; $[a^m]^n = a^{m \times n}$

$3^x \times 3^{-1} + 3^x \times 3^3 + 3^x \times 3^{-2} = 741$

$3^x [3^{-1} + 3^3 + 3^{-2}] = 741$

$3^x [\frac{1}{3} + 27 + \frac{1}{9}] = 741$

$3^x \left(\frac{247}{9}\right) = 741$

$3^x = 27 = 3^3$

x = 3.

33. **Answer: D**

Solution

$(a+b)^2 = a^2+b^2+2ab$

$\sqrt{15 + 2\sqrt{56}} = \sqrt{7 + 8 + 2\sqrt{56}}$

$\sqrt{(\sqrt{7}^2 + \sqrt{8}^2 + 2\sqrt{(7 \times 8)}} = \sqrt{(\sqrt{7} + \sqrt{8})^2}$

$= \sqrt{7} + \sqrt{8}$

34. **Answer: D**

Solution

$(a+b)^2 = a^2+b^2+2ab$

$\sqrt{12 + 2\sqrt{35}} = \sqrt{7 + 5 + 2\sqrt{35}}$

$\sqrt{((\sqrt{7})^2 + (\sqrt{5})^2 + 2\sqrt{7 \times 5})} =$

$\sqrt{(\sqrt{7} + \sqrt{5})^2}$

$= \sqrt{7} + \sqrt{5}$

35. **Answer:** C

Solution

$2^3 \times 5^6 \times \frac{2240}{28} = 10^x$

$2^3 \times 5^6 \times 80 = 10^x$

$2^3 \times 5^6 \times 2^4 \times 5 = 10^x$

$2^7 \times 5^7 = 10^x$

$10^7 = 10^x$

x = 7.

36. **Answer: B**

Solution

$2^5 \times 3^3 \times \frac{2700}{50} = 6^x$

$2^5 \times 3^3 \times 54 = 6^x$

$2^5 \times 3^3 \times 2 \times 3^3 = 6^x$

$2^6 \times 3^6 = 6^x$

$6^6 = 6^x$

x = 6

37. **Answer: B**

Solution

$(4 + 6\sqrt{5})^2 = 16 + 180 + 2 \times 4 \times 6\sqrt{5}$

$= 196 + 48\sqrt{5}$

$= 196 + 48\sqrt{5} = 196 + K\sqrt{5}$

On comparing we can see that, K = 48.

38. **Answer: B**

Solution

$\sqrt{13 - 2\sqrt{36}}$

$= \sqrt{(\sqrt{9})^2 - 2\sqrt{9 \times 4} + (\sqrt{4})^2}$

$= \sqrt{(\sqrt{9} - \sqrt{4})^2}$

$= \sqrt{9} - \sqrt{4}$

= 3 – 2 = 1.

39. **Answer: D**

Solution

$\sqrt{23-4\sqrt{33}} = \sqrt{23-2\sqrt{33\times 4}}$

$= \sqrt{23-2\sqrt{132}}$

$= \sqrt{\left(\sqrt{12}\right)^2 - 2\sqrt{12\times 11} + \left(\sqrt{11}\right)^2}$

$= \sqrt{12} - \sqrt{11}$

40. **Answer: D**

Solution

$256^{1.20} \times 16^{2.60} = 4^{2a} \times 16^2$

$(4^4)^{1.20} \times (4^2)^{2.60} = 4^{2a} \times 4^4$

$4^{4.80} \times 4^{5.20} = 4^{2a+4}$

$4^{10} = 4^{2a+4}$

2a + 4 = 10

a = 3.

41. **Answer: B**

Solution

$2^x = 16^y = 32^z$

$2^x = 2^{4y} = 2^{5z}$

x = 4y = 5z

$\frac{1}{x} + \frac{1}{8y} + \frac{1}{12z} = 46$

$\frac{1}{x} + \frac{1}{2x} + \frac{1}{\frac{12x}{5}} = 46$

$\frac{23}{12x} = 46$

$\frac{1}{x} = 24$

$x = \frac{1}{24}$

42. **Answer: B**

Solution

$3^{2x} = 81^y = 27^{3z}$

$3^{2x} = 3^{4y} = 3^{9z}$

2x = 4y = 9z

$\frac{4}{x} + \frac{1}{12y} + \frac{5}{18z} = 13$

$\frac{4}{x} + \frac{1}{6x} + \frac{5}{4x} = 13$

$\frac{65}{12x} = 13$

$\frac{5}{12x} = 1$

$\frac{1}{x} = \frac{12}{5}$

43. **Answer: A**

Solution

$25^x = 125^{2y} = 5^{3z}$

$5^{2x} = 5^{6y} = 5^{3z}$

2x = 6y = 3z

$\frac{3}{8x} + \frac{1}{3y} + \frac{4}{18z} = \frac{82}{36}$

$\frac{3}{24y} + \frac{1}{3y} + \frac{4}{36y} = \frac{82}{36}$

$\frac{41}{72y} = \frac{82}{36}$

$\frac{1}{2y} = 2$

y =$\frac{1}{4}$

44. **Answer: C**

Solution

$216^{(-1/3)} + 64^{(2/3)} + 32^{(-1/5)}$

$= 6^{3(-1/3)} + 4^{3(2/3)} + 2^{5(-1/5)}$

$= 6^{-1} + 4^2 + 2^{-1}$

= 1/6 + 16 +1/2

= 100

45. **Answer: A**

Solution

$\sqrt{3} + \sqrt{8-2\sqrt{15}} = ?$

$\sqrt{8 - 2\sqrt{15}} = \sqrt{\{\left(\sqrt{5}\right)^2 - 2\times\sqrt{5}\times\sqrt{3} + \left(\sqrt{3}\right)^2\}}$

$= \sqrt{\left(\sqrt{5}-\sqrt{3}\right)^2} = \sqrt{5} - \sqrt{3}$

$\sqrt{3} + \sqrt{8-2\sqrt{15}} = \sqrt{3} + \sqrt{5} - \sqrt{3} = \sqrt{5}$

46. **Answer: C**

Solution

The given problem is in the form of

$(a-b)^3$ = (a-b) (a^2+ab+b^2)

a = 216, b=125

$$\frac{216\times 216\times 216 - 125\times 125\times 125}{216\times 216 + 216\times 125 + 125\times 125}$$

$= \frac{(a-b)^3}{a^2+ab+b^2}$

= (a-b) (a^2+ab+b^2)/(a^2+ab+b^2)

= a-b = 216-125 = 91

47. **Answer: B**

Solution

$\sqrt{\left[\left(35+\sqrt{16}\right)\left(18-\sqrt{25}\right)\right]}$

$= \sqrt{[(35+4)(18-5)]}$

$= \sqrt{[(39)(13)]}$

$= 13\sqrt{3}$

48. **Answer: C**

Solution

$\sqrt{\left[\left(58+\sqrt{36}\right)\left(11-\sqrt{81}\right)\right]}$

$= \sqrt{[58+611-9]}$

$= \sqrt{64\times 2} = 8\sqrt{2}$

49. **Answer: B**

Solution

To check whether the above cases are correct or not, just

simplifying by squaring on both sides

Case I. $4\sqrt{3} > 6\sqrt{2}$

Squaring on both sides,

$(4\sqrt{3})^2 > (6\sqrt{2})^2$

$16 \times 3 > 36 \times 2$

48 > 72 which is not true, as we know that 72 > 48.

So, statement I is not true.

Case II. $2\sqrt{5} < 3\sqrt{4}$

Squaring on both sides,

$(2\sqrt{5})^2 < (3\sqrt{4})^2$

$4 \times 5 < 9 \times 4$

20 < 36 which is true, as we know that 36 > 20.

So, statement II is true.

So, the answer is Only II.

50. **Answer: C**

Solution

To check whether the above cases are correct or not,

just simplifying by squaring on both sides

Case I. $8\sqrt{3} > 7\sqrt{2}$

Squaring on both sides,

$(8\sqrt{3})^2 > (7\sqrt{2})^2$

$64 \times 3 > 49 \times 2$

192 > 98 which is true.

So, statement I is true.

Case II. $6\sqrt{2} < 5\sqrt{3}$

Squaring on both sides,

$(6\sqrt{2})^2 < (5\sqrt{3})^2$

$36 \times 2 < 25 \times 3$

72 < 75 which is true.

So, statement II is true.

So, the answer is Both I & II.

LINEAR EQUATIONS SOLUTIONS

1. **Answer: A**

Solution

⇒ P = Q/2

⇒ Q = R/2

⇒ P + Q + R = 1400

⇒ (Q/2) + Q + (2Q) = 1400

⇒ Q = 400

⇒ P = Q/2 = 400/2 = 200

⇒ R = 2Q = 2 × 400 = 800

So, R has Rs 800.

2. **Answer: D**

Solution

5x – 4 = 2x + 17

5x – 2x = 17 + 4

3x = 21

x = 7.

3. **Answer: B**

Solution

Let the original list price of the book be = Rs x

The original number of books purchased = 200/x

When the price of the book is reduced by 5, the new number of books purchased will be = 200 / (x – 5)

New number of books purchased –Original number of books purchased = 2

⇒ 200 / (x – 5) – 200 / x = 2

⇒ (200x – 200x + 1000) / (x – 5)x = 2

⇒ $2x^2$ – 10x = 1000

On solving this equation, we get:

x = 25 or x = –20

x can't be negative. So the original cost of book is Rs 25

4. **Answer: D**

Solution

Let the cost of 1 pencil, 1 pen and 1 eraser are respectively x, y, z. Then according to the given condition

2x + 3y + 6z = 22

6x + 5y + 2z = 42

On adding these two equations we get

8x + 8y + 8z = 64

2x + 2y + 2z = 16

The two pencils, two pens and two erasers cost will cost Rs 16.

5. **Answer: B**

Solution

Let price of Lemon and Apple is L and A respectively

As given,

=> L+A=16 ...(1)

=> L+T=6 ...(2)

=> A = 4+L

=> or L– A= – 4 ...(3)

=> A=4+T. ...(4)

=> Simply adding eq. 1 and 3

We get,

=> 2L=12

=> L=6

=> So, A = 10

6. **Answer: C**

Solution

Let the cost of 1 saree = Rs. x

Let the cost of 1 shirt = Rs. y

A/Q

2x + 5y = 2600 ⋯(1)

And x = 4y ⋯(2)

2(4y) + 5y = 2600

8y + 5y = 2600

13y = 2600

y = 200

Cost of 8 shirts = 8 X 200 = Rs. 1600

7. **Answer: B**

Solution

Let cost of 1 chair = Rs. x

cost of 1 table = Rs. y

A/Q 4x + 3y = 1200 ⋯(1)

7x + 6y = 1800 ⋯(2)

(2) – (1), we get

3x + 3y = 600

The cost of 3 chairs and 3 tables is Rs 600

8. **Answer: A**

Solution

Let the breadth of the rectangle be x.

Then, its length = (x + 6) cm

∴ Perimeter of rectangle = 2 [x + (x + 6)] = 2[2x + 6]

= 4x + 12

∴ 4x + 12 = 72

4x = 72 – 12 = 60

x = 15

Breadth of rectangle = 15cm and length = 15 + 6 = 21cm

9. **Answer: D**

Solution

3x + 6 = 93

3x = 93 – 6 = 87

x = 29

10. **Answer: B**

Solution

Let numbers be x and 13 – x

x(13 – x) = 42

(x – 7) (x – 6) = 0

x = 7, 6

11. **Answer: A**

Solution

Let numbers be x and y, then

x + y = 441 ⋯(i)

x – y = 35 ⋯(ii)

on adding eqns. (i) and (ii)

2x = 476

x = (476/2) = 238.

12. **Answer: D**

Solution

Let numbers be x and y, then

x + y = 53 ⋯(i)

x – y = 13 ⋯(ii)

on adding eqns. (i) and (ii)

2x = 66

x = (66/2) = 33.

13. **Answer: A**

Solution

Let cost of one horse be Rs. x and Cost of one cow be Rs. y

So, x + 3y = 520 ⋯(i)

x – y = 60 ⋯(ii)

subtracting eqn. (i) and (ii)

4y = 460 => y = 115

∴ cost of one horse = 115 + 60 = Rs 175

14. **Answer: B**

Solution

Let cost of one table be Rs. x and Cost of one chair be Rs. y

So, x + 2y = 380 ⋯(i)

x – y = 110 ⋯(ii)

subtracting eqn. (i) and (ii)

3y = 270 => y = 90

∴ cost of one table = 90 + 10 = Rs 200

15. **Answer: A**

Solution

Let the cost of the tie be x.

Then, the cost of the shirt would be (x + 560), as it costs Rs. 560 more than the tie.

The total cost of the shirt and tie can be expressed as:

(x + 560) + x = 900

Simplifying this equation, we get:

2x + 560 = 900

2x = 340

x = 170

Therefore, the cost of the tie is Rs. 170, and the cost of the shirt is:

x + 560 = 170 + 560 = 730

Hence, the cost of the shirt is Rs. 730.

16. **Answer: D**

Solution

Let the number be x.

Hence, the two consecutive numbers are x and x+1.

According to the given statement, the equation becomes

⇒ x + x + 1 = 241

⇒ 2x + 1 = 241

⇒ 2x = 240

⇒ x = 240/2 = 120

If x = 120, then x + 1 = 120 + 1 = 121

Hence, the two numbers are 120 and 121.

17. **Answer: B**

Solution

Given, 6x – 10y = 10 .... (i)

and x/(x + y) = 5/7

⇒ 7x = 5x + 5y

⇒ 2x – 5y = 0 .... (ii)

On multiplying Eqn. (ii) by 2 and subtracting from Ed.(i), we get

$$\begin{array}{r} 6x - 10y = 10 \\ -\ 4x - 10y = 0 \\ \hline 2x = 10 \end{array}$$

∴ x = 5

Putting the value of x in Eqn. (i), we get

30 – 10y = 10

⇒ 10y = 20

⇒ y = 2

∴ (x – y) = 5 – 2 = 3

18. **Answer: B**

Solution

Let the cost of one chair be ₹ x

and cost of one table be ₹ y.

By given condition,

10x + 6y = 6200 ..(i)

and 3x + 2y = 1900

⇒ 9x + 6y = 5700 ...(ii)

On subtracting Eqn. (ii), we get

x = ₹ 500

From Eq (i),

5000 + 6y = 6200

⇒ 6y = 1200

∴ y = ₹ 200

The cost of 4 chair and 5 tables

= 4x + 5y

= 2000 + 1000

= ₹ 3000

19. **Answer: C**

Solution

Let the incomes of two persons be 8x and 5x and their expenditure be 2y and y , respectively.

∵ Saving = Income – Expenditure

∴ 1000 = 8x – 2y ...(i)

and 1000 = 5x – y ...(ii)

On multiplying Eqn. (ii) by 2 and subtracting from Eqn. (i), we get

$$\begin{array}{r} 8x - 2y = 1000 \\ -\,10x - 2y = 2000 \\ \hline -2x = -1000 \end{array}$$

∴ x = 500

∴ Monthly incomes are

8x = ₹ 4000 and

5x = ₹ 2500

∴ Difference = ₹ 4000 – 2500 = ₹ 1500

20. **Answer: C**

Solution

Let the fraction be x/y,

According to the question,

(x + 5) / (y + 5) = 7/8

⇒ 8x + 40 = 7x + 35

∴ 8x – 7y = -5 ..(i)

Again, according to the question,

(x + 3)/(y + 3) = 6/7

⇒ 7x + 21 = 6y + 18

∴ 7x – 6y = -3 ..(ii)

On multiplying Eqn. (i) by 6 and Eqn. (ii) by 7 and subtracting, we get

$$\begin{array}{r} 48x - 42y = -30 \\ -\,49x - 42y = -21 \\ \hline -x = -9 \end{array}$$

∴ x = 9

On putting the value of x in Eqn. (i) , we get

72 – 7y = -5

⇒ -7y = -5 – 72

⇒ y = -5 – 72

⇒ y = (-77) / (-7) = 11

∴ Required fraction = 9/11

21. **Answer: A**

Solution

Let the ten's digits be x and unit's digit be y.

Then,

(10x + y) – (10y + x) =36 ⇔9(x-y) = 36

x – y = 4

22. **Answer: C**

Solution

Let us assume the ten's digit of the number is n. Then the unit's digit will be 3n.

According to question,

The sum of the digits is equal to 8.

sum of the digit = 8

n + 3n = 8

⇒ 4n = 8

⇒ n = 2

So, ten's digit is 2 and unit's digit is 6.

So the number = 10n + 3n = 10 x 2 + 3 x 2 = 20 + 6 = 26.

23. **Answer: C**

Solution

Let us assume the number be N.

According to question,

A number when subtracted by 1/7 of itself gives the same value as the sum of all the angles of a triangle,

N – N × 1/7 = 180 (As we know sum of all angels of a triangle is 180^o .)

⇒ N – N/7 = 180

⇒ (7N – N)/7 = 180

⇒ (7N – N) = 180 × 7

⇒ 6N = 180 × 7
⇒ N = 180 × 7/6
⇒ N = 30 x 7
⇒ N = 210

24. **Answer: B**
Solution
Let us assume the ten's place digit be x.
Then, the number = 10x + 3
and sum of digits = x + 3
According to question,
So, (x + 3) = 1/7(10x + 3)
⇔7x + 21 =10x +3
⇔ 3x = 18
⇔ x = 6
The number = 10x + 3 = 10 × 6 + 3 = 60 + 3 = 63

25. **Answer: D**
Solution
Let us assume the two parts be y and (54 – y).
Then, according to question
10(54 – y) + 22y = 780
⇒ 540 – 10y + 22y = 780
⇒ 12y = 780 – 540
⇒ 12y = 240
⇔ y = 20.
∴ Bigger part = (54 – y) = (54 – 20) = 34

26. **Answer: D**
Solution
The present ages of two persons are 36 and 50 years respectively
After n years the age of both persons will be 36 + n and 50 + n respectively.
According to question,
After n years the ratio of their ages will be 3:4,
(36 + n)/(50 + n) = 3/4
⇒ (36 + n) × 4 = 3 × (50 + n)
⇒ 36 × 4 + 4 × n = 50 × 3 + 3 x n
⇒ 144 + 4n = 150 + 3n
⇒ 4n – 3n =150 –144
⇒ n = 6

27. **Answer:** B
Solution
Let total bill would be ₹ P
Each one will to pay = ₹ P/11
10 friends could pay 10 x 60 = ₹ 600
According to question
600 + P/11 + 50 = P
⇒ 650 = P – P/11
⇒ 650 = (11P – P)/11
⇒ 650 x 11 = (11P – P)
⇒ 650 x 11 = 10P
⇒ P = 11 x 65
⇒ P = 715
Amount paid by 11th friend = 715/11 + 50 = ₹ 115

28. **Answer: E**
Solution
Solution: Let pigeons = P, sheep = S
According to the question,
P + S = 90 ...(i)
2P + 4S = 248. ...(ii)
On multiplying Eq. (i) by 2 and subtracting from Eq. (ii), we get

$$\begin{array}{rrrr} & 2P & + & 2S = 180 \\ - & 2P & + & 4S = 248 \\ \hline & -2S & = & -68 \end{array}$$

∴ S = 34

29. **Answer: C**
Solution
6x + 1 > 7 – 4x
⇒ x > 3/5
∴ 3/5 < x ≤ 2

30. **Answer: A**
Solution
Let cost of 1 plate and 1 clay be p and c, respectively.
Now, according to the question,
21p + 9c = ₹ 819
⇒ 3(7p + 3c) = ₹ 819
₹ 7p + 3c = ₹ 273
Cost of 7 plates and 3 clays = ₹ 273

31. **Answer: A**
Solution
Let there be x hens and y cows. Then,
x + y = 32 and 2x + 4y = 90
Solving x + y = 32 and x + 2y = 45, we get: x = 19.

32. **Answer: D**
Solution
Let there be x peacock and y horses. Then,
x + y = 52 and 2x + 4y = 140
Solving x + y = 52 and x + 2y = 70, we get: y = 18.

33. **Answer: B**
Solution

Let there be x peacock and y horses. Then,
x + y = 84 and 2x + 4y = 200
Solving x + y = 84 and x + 2y = 100, we get: x = 68.

34. **Answer: B**
Solution
Let the larger number be x and the smaller be y.
Then, 2x – 30 = y and y + 10 = x
∴ 2x – y = 30 & x – y = 10.
by solving these equations, we get: x = 20 & y = 10.
Hence, the larger number = 20.

35. **Answer: A**
Solution
Let the larger number be x and the smaller be y.
Then, 3x – 16 = y and 2y – 3 = x
∴3x – y = 16 & – x + 2y = 3.
by solving these equations, we get: x = 7 & y = 5.
Hence, the larger number = 7.

36. **Answer: B**
Solution
Let the parts be A, B and C.
Acc. To question–
(A / 2) = (B / 3) = (C / 4); C = 2A, B = (3 / 2)A.
A + B + C = 513
A + (3 / 2) A + 2A = 513
2A + 3A + 4A = 1026.
9A = 1026; A = 114
B = (3 / 2) × 114 = 171
C = 2A = 114 × 2 = 228.

37. **Answer: C**
Solution
Let the parts be A, B and C.
Acc. To question–
(A / 4) = (B / 5) = (C / 3); C = (3/4)A, B = (5/4)A.
A + B + C = 816
A + (5 / 4) A + (3/4)A = 816
4A + 5A + 3A = 3264.
12A = 3264; A = 272
B = (5 / 4) × 272 = 340
C = (3/4)A = 272 × (3/4) = 204.

38. **Answer: C**
Solution
Let a two-digit number be (10x + y) and reversing number be (10y + x)
∴ Required sum = 10x + y + 10y + x = 11x + 11y = 11(x + y)
Hence, it's divisible by 11.

39. **Answer: B**
Solution
Let the greater number be x and smaller number be y
∴ x – y = 35 ...(i)
and x = 2y + 5 ...(ii)
From eqs. (i) and (ii)
2y + 5 – y = 35
⇒ y = 30
On putting the value of y in Eq. (ii), we get
x = 2 × 30 + 5 = 65
Hence, required sum = x + y = 65 + 30 = 95.

40. **Answer:** C
Solution
Let the fare from India to Dubai be ₹ x and the fare from Dubai to New York be ₹ y.
By given condition,
x + y = 96000 ...(i)
and 4x + 5y = 443000 ...(ii)
On solving Eqs. (i) and (ii), we get
x = 37000 and y = 59000
Hence, fare from Dubai to New York is ₹ 59000.

41. **Answer: B**
Solution
8x – 3y = 18 ...(i)
7x + 4y = 29 ...(ii)
By equation (i) × 4 & (ii) × 3,
32x – 12y = 72 ...(iii)
21x + 12y = 87 ...(iv)
by solving this equation we get
x = 3, y = 2
∴ 5x + 8y = 5 × 3 + 8 × 2 = 15 + 16 = 31.

42. **Answer: D**
Solution
9x – 5y = 11 ...(i)
4x + 2y = 26 ...(ii)
By equation (i) × 2 & (ii) × 5,
18x – 10y = 22 ...(iii)
20x + 10y = 130 ...(iv)
by solving this equation we get
x = 4, y = 5
∴ 6x + 10y = 6 × 4 + 10 × 5 = 24 + 50 = 74.

43. **Answer: D**
Solution
Let x be the number of tickets bought by Shruti.

Total money spent = 35x + 65x + 85x = 185x = 3145
x = 3145 / 185 = 17
She buys 17 tickets of each type.

44. **Answer:** B
Solution
Let x be the number of shares bought by Veer.
Total money spent = 101x + 108x + 120x = 329x = 8225
x = 8225 / 329 = 25
He buys 25 shares of each type.

45. **Answer: B**
Solution
When the maximum marks were 800 the secured marks would be 360
% of marks out of 800 = 360/800 × 100 = 45%
Now 540 is 45% of maximum marks 'A'
540 = 45/100 × A
A = 1200
The maximum marks of the test was 1200.

46. **Answer: C**
Solution
When the maximum marks were 480 the secured marks would be 312
% of marks out of 480 = 312/480 x 100 = 65%
Now 455 is 65% of maximum marks 'A'
455 = 65/100 x A
A = 700
The maximum marks of the test is 700.

47. **Answer: A**
Solution
Let the 2nd number be x and 3rd number be y.
First number =420
first number = (2/3)rd × third number
y = 3/2 × 420
y = 630
4/5 of y = 30% of x
4/5 x 630 = (3/10)x
504 = (3/10)x
x = 504 × (10/3)
x = 1680
25% of the second number = 25/100 x 1680 = 420

48. **Answer: B**
Solution
Number of pencils < erasers < scales
Sum must be 32
To get the maximum erasers, pencils should be as less as possible
Pencils = 9
erasers > pencils = 10 or 11
scales > pencils = 11 or 12
maximum erasers = 11

49. **Answer: C**
Solution
Number of Ice-creams < Cookies < Pastries
Sum must be 38
If ice-creams = 11
Cookies = 12
Pastries = 15
So, minimum cookies he can buy is 12.

50. **Answer: A**
Solution
Aryan's present age = 8 + 5 = 13
After 6 years Aryan's age = 19
Father's age after six years = Twice of Aryan's age = 2 × 19 =38
Father's present age = 32
Surya's age = 1/4th of 32 = 8

PROBLEMS ON AGES

1. **Answer: B**
Solution
Ramu + Rinu + Sonia = 102
Rinu = Sonia + 16
Ramu + Rinu = 75
Ramu + Sonia + 16 = 75
Ramu = 59 – Sonia
59 – Sonia + Sonia + 16 + Sonia = 102
Sonia = 27 years
Ramu = 864/27 = 32 years.

2. **Answer: A**
Solution
(A – 5)/(B – 5) = 2/1
A – 5 = 2B – 10
A – 2B = -5
A = B + 10
B + 10 – 2B = -5
B = 15
A = 15 + 10 = 25
C = 120/100 × (25 + 15)
= 48 years
After 8 years C's age = 48 + 8 = 56

3. **Answer: E**
Solution
(7x + 5)/(2x + 5) = 3/1
7x + 5 = 6x + 15
x = 10
The age of B 10 years hence = 2 × 10 + 5 + 10 = 35 years.

4. **Answer: A**
Solution
Rajiv's present age = 22 + 3 = 25 years
After 6 years, Lagan's mother's age = 2*Rajiv's age after 6 years
→ 2 × 31 = 62 years
Present age of Lagan's mother = 56 years
Lagan's present age = (1/4)*Lagan's mother's age, after 4 years
→ J = (1/4) × 60
→ J = 15
Lagan's present age = 15 years

5. **Answer: B**
Solution
Rajesh = 7x and Pravin = 6x
Anil = 6x – 4 and Shweta + 7x = 54
Shweta = 54 – 7x
$(6x + 6x – 4 + 54 – 7x)/3 = 23\frac{1}{3}$ → ∴ x = 4
Shweta = 54 – 7x = 26 years
Anil = 6x – 4 = 20 years
Required difference = 26 – 20 = 6 years.

6. **Answer: C**
Solution
Present age of A = 7x
Present age of C = 9x
Present age of B
= 3 + (7x + 9x)/4 = 3 + 4x
9x – 4x – 3 = 12
5x = 15 → ∴ x = 3 years

7. **Answer: E**
Solution
Let the present age of Olaf = 3x-3
And the present age of Moana = 2x-3
The present age of Luna
= (3x-3)/3 = x-1
3x-3+2x-3+x-1 = 77
6x-7 = 77
6x = 84 → ∴ x = 14
The present age of Moana = 2 × 14 - 3 = 25 years
The age of Moana 5 years ago = 25-5 = 20 years.

8. **Answer: A**
Solution
Let A's age after 4 years be $\mathcal{x}$
Then,
x – (4/5 × $\mathcal{x}$) = 10
x = 50
A's present age is 46.
Required age = 46 + 12 = 58.

9. **Answer: D**
Solution
The ratio of the age of Vinit, Sohail and Rudra = 6:1:10
Given,
→ 10x - x = 45
→ 9x = 45
→ x = 5
Vinit's present age = 6×5 = 30
After 6 years, age will be = 30+6 = 36

10. **Answer: E**
Solution

Let the present age of B = 5x
And the present age of A = 5x × 120/100 = 6x
And the present age of C = 5x × 3/2 = 15x/2
6x + 15x/2 = 48 + 3 × 2
12x + 15x = 54 × 2
27x = 108
x = 4
The present age of B = 5 × 4 = 20 years
Therefore, 8 years hence, age of B = 20 + 11 = 31 years.

11. **Answer: B**
Solution
The ratio of ages of A, B and C = 1: 4: 11 (x, 4x, 11x)
11x – 4x = 56
x = 8
Required average = (4x + 11x)/2 = 15x/2 = 15 * 8/2 = 60.

12. **Answer: D**
Solution
Let the present age of C = 40-5 = 35 years.
Age of A = 35 × 5/7 = 25 years.
And age of B = [25+7]-2 = 30 years.
So, required average age= [35+25+30]/3 = 30.

13. **Answer: B**
Solution
Average age of A,B and C = 26
A+B+C = 78
Sum of the present age of B and C = 65
Difference between the present age of A and C = 30
The present age of A = A + B + C – (B + C)
= 78-65 = 13
The present age of C = 13 + 30 = 43
Average age of A and C = (43 + 13)/2 = 56/2 = 28 years.

14. **Answer: A**
Solution
Average age of 7 midfield players = 27
Sum of 7 midfield players age = 27 × 7 = 168
Sum of 6 midfield players age = 25 × 6 = 150
Age of the 7th player = 168 - 150 = 18

15. **Answer: B**
Solution
(2x-4)/(5x+3)=7/24
48x-96=35x+21
13x=117
x=9
A=2 × 9-4=14 years
B=5 × 9+3=48 years
C=48 × 100/160=30 years
Required average=(48+30)/2=78/2=39 years.

16. **Answer: A**
Solution
Total age of the class
= 5 × 17 + 2 × 20 + 14 × 3= 167
Required average = 167/10 = 16.7 years.

17. **Answer: B**
Solution
A+B+C=68 years
(3x+6)/(4x+6)=4/5
15x+30=16x+24
x=6 years
B=3x=18 years
C=4x=24 years
A=68-18-24=26 years.

18. **Answer: D**
Solution
x + y =106 ⋯(1)
(x+4)/(y+2)=7/9
9x+36=7y+14
9x-7y=-22 ⋯(2)
(2)-(1) × 7
16x=720
x=45
Y=61
x:y=45:61

19. **Answer: E**
Solution
Let the son's present age be x years .
Then, (40 - x) = x → x= 20.
Son's age 5 years back = (20 - 5) = 15 years.

20. **Answer: C**
Solution
Let B's present age = x years. Then, A's present age = (x + 8) years.
(x + 8) + 10 = 2(x - 10)
→ x + 18 = 2x - 20
→ x = 38.

21. **Answer: B**
Solution

Let the son's age be x years and the father's age be 5x years

3x – 5 = 8(x – 5)

∴ 5x = 35

∴ x = 7

Age of the father = 5x = 5×7 = 35

22. **Answer: A**

Solution

Let the present ages of daughter and mother be x and (74 – x) years respectively.

Then, (74 – x) – 4 = 5(x – 4)

70 – x = 5x – 20

6x = 90 → x = 15.

23. **Answer: D**

Solution

Total age of 12 students = 216 years

Total age of 20 students = 380 years

Total age of 8 new students = 380 – 216 = 164 years

Therefore, Average age of 8 new students = 164/8 = 21 years.

24. **Answer: E**

Solution

Let the Father's present age be x years.

Then, the person's present age = 5/12 x years.

(5x/12 + 8) = 1/2 (x + 8)

2(5x + 96) = 12(x + 8) → x = 48.

25. **Answer: B**

Solution

Krishnan's age = 3A and Venkat's age = 5A

(3A+9)/(5A+9) = 3/4

→ 4 (3A + 9) = 3 (5A + 9)

→ A = 3

Therefore, Venkat's age = 15 years.

26. **Answer: D**

Solution

Present total age of the members = 30(5) = 150 years.

Present age of the youngest member = 10 years

Present total age of the remaining four members = 150 – 10 = 140 years

Their average age at the time of the birth of the youngest member = [140–(4×10)]/5 = 20 years.

27. **Answer: E**

Solution

Let current ages of X and Y correspondingly, is 5A & 8A

Given: 8A + 5A = 104

→ A = 8

Proportion of ages after 0.8 decades will be

5A + 8 : 8A + 8 48:72 (or) 2:3

28. **Answer: D**

Solution

Let A's age 10 years ago = x years.

Then, B's age 10 years ago = 2x years.

(x + 15) / (2x+ 15) = 4/5

→ x = 5.

So, the total of their present ages =(x + 15 + 2x + 15) = (3x + 30) = 45 years.

29. **Answer: C**

Solution

Let x = the youngest child. Each of the other four children will then be x+3, x+6, x+9, x+12.

We know that the sum of their ages is 50 .

so, x+(x+3)+(x+6)+(x+9)+(x+12) = 150

→ x = 24

The third child is 30 years old.

30. **Answer: E**

Solution

Tom's age when he was born = 0 years

→ His brother's age = 6 year

→ His father's age = 6+33 = 39

→ His mother's age = 39 – 3 = 36

So, his sister's age = 36–27 = 9 years.

31. **Answer: D**

Solution

Let Varun's age = x yrs.

From the given conditions,

x+1 = 3(x–11)

x = 17 years.

32. **Answer: E**

Solution

Current average of the couple = 21×2 + 7×2 = 56

Average of the family = 20 x 3 = 60

Age of the child = 60 – 56 = 4

Therefore, the age of the child is 4 years.

33. **Answer: E**

Solution

6 years ago, the ratio of ages of A and B = 2: 3 (2x, 3x)

Present ages of A and B = 2x + 5, 3x + 5

After 6 years, the ratio of ages of A and B = 3: 4

According to the question,
(2x + 10)/(3x + 10) = (3/4)
8x + 40 = 9x + 30
x = 10
Sum of the present ages of A and B = 2x + 3x + 10 = 60 years.

34. **Answer: D**
Solution
2x + 16/5x + 16 = 4/7
14x + 112 = 20x + 64
x = 8
Present age of A = 16
Present age of B = 5 * 8 = 40
Required ratio = 12:36 = 1:3.

35. **Answer: B**
Solution
Total age of (A + B + C) 4 years ago = 42 x 3 = 126
Total of present age of (A + B + C) = 126 + 12 = 138
But, according to the question (A + B) after 6 yrs = 102
So, sum of present age of (A + B) = 102 – 12 = 90 yrs
Thus, the present age of C = 138 – 90 = 48 yrs.

36. **Answer: C**
Solution
The ratio of the ages of A to B
= 2: 1 (2x, x)
(2x + 12)/(x + 12) = 8/5
10x + 60 = 8x + 96
x = 18
A = 18 × 2 = 36 years
B = 18 years
10 years ago, A= 40 – 10 = 30
B = 18 – 10 = 8
Required ratio = 30 : 8 = 15 : 4

37. **Answer: B**
Solution
The ratio of Ravi : son = 3 : 1
Ratio of Ravi : father = 1:2
Son = 12 years
Ratio of Ravi : son : Father = 3 : 1 : 6
6x = 12 × 6 = 72
Required answer = 72+5 = 77 years

38. **Answer: D**
Solution
Let the present age of C = x
The present age of A = x + 10
The present age of B = x + 4
(x + 10)/(x + 4) = 6/5
x = 26
The present age of C = 26 years
The present age of A = 26 + 10 = 36 years
The present age of B = 26 + 4 = 30 years
Required total = 26 + 36 + 30 + 10 * 3 = 122 years.

39. **Answer: E**
Solution
Shital's present age = 44 – 9 = 35 years
Tara's present age = 35 – 2 = 33 years
From the ratio of the present ages of Disha to that of Tara is 9: 11, we get Disha's age
= 27 years
Present age of Disha's father
= 27 + 26 = 53 years

40. **Answer: A**
Solution
The ratio of present age of Kapil and Pratik = 3 : 7
(3x, 7x)
Given,
= > (3x + 6) / (7x – 8) = 7/9
= > x = 5
The age of Pratik, 10 years hence = 7x + 10 = 45 years.

41. **Answer: E**
Solution
Anshu = (Ranjan + 5) + 10
= Ranjan + 15 -------- (I)
Ranjan = (Renu - 4) + 7
= Renu + 3 ------- (II)
Ranjan's age = 20 + 3 = 23 years
After 5 Anshu's age = 23 + 15 + 5 = 43 years

42. **Answer: A**
Solution
As given Kushal's 8th birthday was celebrated 2 years ago,
Kushal's present age = 15 + 2 = 17 years.
Mother's age is twice of Kushal's age after 10 years.
→ M + 10 = 2 (K + 10) = 2 (17 + 10)
M = 44 years,
Aisha's age = 1/4(M) = 44/4 = 11 years.

43. **Answer: E**
Solution

Present age of B = 32 – 7 = 25 years
Age of B before 4 years = 25 – 4 = 21 years
Age of A after 3 years = 2 × 21 = 42 years
Present age of A = 42 – 5 = 37 years
Required average = (37 + 25)/2 = 31 years.

44. **Answer: E**
Solution
The age of Q, 2 years hence = 3 * The age of P, 10 years ago
The ratio of age of Q, 2 years hence and the age of P, 10 years ago = 3: 1
The present age of P and Q = x + 10, 3x – 2
4 years hence, the age of P = The age of Q, 8 years ago
x + 10 + 6 = 3x – 2 – 10
x = 14
The present age of Q = 3x – 2 = 40 years.

45. **Answer: D**
Solution
Manish = x years and Aruna = x + 4 years
(x + 4) + 9 = 3(x – 9)
→ x = 20 years

46. **Answer: A**
Solution
Vithal – 4 = 3 × Raj
13x – 10x = 9
x = 3
Present age of Raj = 10 * 3 – 6 = 24 years
Present age of Vithal = 2 * 24 + 4 = 52 years

47. **Answer: E**
Solution
Let the present age Harish and Bharani be 13x and 9x.
According to the question,
(13x + 6)/ (9x + 6) = 4/3
→ x = 2
Present age of Harish = 26 years
Present age of Bharani = 18 years
Required ratio = 36: 28 = 9: 7

48. **Answer: B**
Solution
(x + 11)/(3x + 11) = 1/2
3x + 11 = 2x + 22
x = 11
Required ratio = (1 * 11 – 5):(3 * 11 – 5)
= 6:28
= 3:14

49. **Answer: C**
Solution
(5x – 5) × 125/100 = (6x – 5)
x = 5
Required average = (25 + 4 + 30 + 4)/2
= 31.5 years.

50. **Answer: E**
Solution
Let the age of Somya be x and
the age of her daughter is y
Then five years before x–5=p5(y–5) ⋯(1)
Five years hence x+5 = 3(y+5)–8 ⋯(2)
By solving (1) & (2), we get
5y – 15 = 3y + 7
y = 11 → x = 35
Therefore, the age of Somya = 35 and her daughter = 11.

QUADRATIC EQUATIONS SOLUTIONS

1. **Answer: C**

Solution

The given polynomial or quadratic equation is

$x^2 + 5x + 6 = 30$

Solving by factorization method,

$x^2 + 5x + 6 - 30 = 0$.

$x^2 + 5x - 24 = 0$

$x^2 - 3x + 8x - 24 = 0$

$x(x - 3) + 8(x - 3) = 0$

$(x - 3)(x + 8) = 0$

$(x - 3) = 0, (x + 8) = 0$

$x = +3, -8$.

Hence option C.

2. **Answer: A**

Solution

$5x^2 - 14x + 8 = 0$

$5x^2 - 10x - 4x + 8 = 0$

$5x(x-2) - 4(x-2) = 0$

$(5x-4)(x-2) = 0$

So,

$5x-4 = 0$; $x = +4/5$

$(x-2) = 0$; $x = +2$

Therefore, 4/5 and 2 are the roots of the given equation.

Hence option A.

3. **Answer: E**

Solution

$2x^2 + x - 400 = 0$

$a = 2, b = 1, c = -400$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(2)(-400)}}{2(2)}$$

$$x = \frac{-1 \pm \sqrt{1 + 3200}}{4}$$

$$x = \frac{-1 \pm \sqrt{3201}}{4} = \frac{-1 \pm 56.57}{4}$$

$$\therefore x = \frac{-1+56.57}{4} \text{ or } x = \frac{-1-56.57}{4}$$

$$\therefore x = \frac{55.57}{4} \text{ or } x = \frac{-57.57}{4}$$

$$\therefore x = 13.89 \text{ or } x = -14.39$$

$2x^2 - 25x + 32x - 400 = 0$

$x(2x - 25) + 16(2x - 25) = 0$

$(x + 16)(2x - 25) = 0$

So,

$x + 16 = 0$; $x = -16$

$(2x - 25) = 0$; $x = 25/2 = 12.5$

Therefore, -16 and 12.5 are two roots of the given equation. Hence option D.

4. **Answer: E**

Solution

In the given equation $a = 2$, $b = -19$ and $c = 44$

Hence $ac = 2 \times 44 = 88 = 8 \times 11$

Thus, the factors of 88 whose sum is equal to -19 are -11 and - 8.

$2x^2 - 8x - 11x + 44 = 0$

$2x(x-4) - 11(x-4) = 0$

$(x-4)(2x - 11) = 0$

$x-4 = 0$ or $2x - 11 = 0$

$x = 4$ or $11/2$

Hence, the roots of the equation are +4 and +11/2.

Hence option E.

5. **Answer: A**

Solution

Equation 1:

$3x^2 - 44x + 96 = 0$

$3x^2 - 36x - 8x + 96 = 0$

$3x(x - 12) - 8(x - 12) = 0$

$(3x-8)(x-12) = 0$

$x = 8/3, 12$

Equation 2:

$y^2 + 14y + 48 = 0$

$y^2 + 8y + 6y + 48 = 0$

$y(y +8) + 6(y + 8) = 0$

$(y +6)(y +8) = 0$

$y = -6, -8$

$x > y$

Hence option A.

6. **Answer: B**

Solution

Since 5 is one of the roots of $x^2 + 4x + a = 0$, $x = 5$ satisfies the given equation.

i.e., substitute x = 5 in the equation to get the value of a.

$\therefore 5^2 + 4(5) + a = 0$

$\therefore 25 + 20 + a = 0$ i.e., $a = -45$

Hence option B.

7. **Answer: A**

Solution

If a quadratic equation has equal roots, the value of the discriminant i.e., $b^2 - 4ac = 0$

$\therefore 256 - (4)(4)c = 0$

$\therefore 256 - 16c = 0$

$C = 16$

Hence option A.

8. Answer: B

Solution

Since the equation $x^2 + ax + 36 = 0$ has equal roots,

$b^2 - 4ac = 0$

$\therefore a^2 - 4(1)(36) = 0$

$a^2 = 144$ i.e. $a = \pm 12$

Since $a > 0$; $a = 12$

Hence option B.

9. Answer: C

Solution

$(x - 6)(x-8) = 3$

$\therefore x^2 - 14x + 48 = 3$

$\therefore x^2 - 14x + 45 = 0$

$\therefore x^2 - 9x - 5x + 45 = 0$

$x(x-9)-5(x-9) = 0$

$(x-9)(x-5) = 0$

$x = 9$ or $x = 5$

Hence option C.

10. Answer: A

Solution

Since the roots of the equation are reciprocals of each other, their product is 1.

Product of roots = $c/a = p/23$

$p/23 = 1$ i.e. $p = 23$

Hence option A.

11. Answer: E

Solution

both the equations are of the form $ax^2 + bx - c = 0$.

Thus, each equation has one positive and one negative root.

Hence, the relation between x and y cannot be determined.

Hence option E.

12. Answer: E

Solution

$27x^2 + 43x = 14$ i.e., $27x^2 + 43x - 14 = 0$

$5y^2 + 16y = 7$ i.e., $5y^2 + 16y - 7 = 0$

Both the equations are of the form $ax^2 + bx - c = 0$.

Thus, each equation has one positive and one negative root.

Hence, the relation between x and y cannot be determined.

Hence option E.

13. Answer: E

Solution

$x^2 - 25x + 156 = 0$

$x^2 - 12x - 13x + 156 = 0$

$x(x - 12) - 13(x - 12)$

$(x-13)(x - 12) = 0$

$x = 13$ or $x = 12$

$y^2 - 19y + 78 = 0$

$y^2 - 13y - 6y + 78 = 0$

$(y-13)(y - 6) = 0$

$y = 13$ or $y = 6$

When $x = 13$, $x \geq y$

When $x = 12$ and $y = 13$, $x < y$

Hence, the relation between x and y cannot be determined.

Hence option E.

14. Answer: D

Solution

$x^2 + 28x + 195 = 0$

Since this equation is of the form $ax^2 + bx + c = 0$, both roots are negative i.e $x < 0$

$y^2 - 25x + 126 = 0$

Since this equation is of the form $ax^2 - bx + c = 0$, both roots are positive i.e $y > 0$

Hence, $x < y$.

Hence option D.

15. Answer: E

Solution

Both the equations are of the form $ax^2 + bx - c = 0$.

Thus each equation has one positive and one negative root.

Hence, the relation between x and y cannot be determined. Hence option E.

16. Answer: D

Solution

$x^2 + 45x + 296 = 0$

Since this equation is of the form $ax^2 + bx + c = 0$, both roots are negative i.e $x < 0$

$y^2 - 39y + 224 = 0$

Since this equation is of the form $ax^2 - bx + c = 0$, both roots are positive i.e $y > 0$

Hence, $x < y$.

Hence option D.

17. Answer: C

Solution

$x^2 - 13x + 12 = 0$

$x^2 - 12x - x + 12 = 0$

$(x-12)(x - 1) = 0$

$x = 12$ or $x = 1$

$y^2 + 6y - 7 = 0$

$\therefore y^2 + 7y - y - 7 = 0$

$(y + 7)(y - 1) = 0$

$y = -7$ or $y = 1$

When $x = 12$, $x > y$ and when $x = 1$, $x \geq y$

Hence, $x \geq y$.

Hence option C.

18. Answer: E

Solution

$8y^2 + 54y = 29$ i.e., $8y^2 + 54y - 29 = 0$

Both the equations are of the form $ax^2 + bx - c = 0$.

Thus, each equation has one positive and one negative root.

Hence, the relation between x and y cannot be determined.

Hence option E.

19. Answer: A

Solution

$11x^2 - 2 = 3(10x - 6)$

$\therefore 11x^2 - 30x + 16 = 0$

Since this equation is of the form $ax^2 - bx + c = 0$, both roots are positive i.e $x > 0$

$44y + 21 = -6y^2 + 21y$

$\therefore 6y^2 + 23y + 21 = 0$

Since this equation is of the form $ax^2 + bx + c = 0$, both roots are negative i.e $y < 0$

Hence, $x > y$.

Hence option A.

20. Answer: E

Solution

Both the equations are of the form $ax^2 + bx - c = 0$.

Thus, each equation has one positive and one negative root.

Hence, the relation between x and y cannot be determined.

Hence option E.

21. Answer: A

Solution

I. $4x^2 - 16x + 12 = 0$

$\Rightarrow 4x^2 - 12x - 4x + 12 = 0$

$\Rightarrow 4x(x - 3) - 4(x - 3) = 0$

$\Rightarrow (4x - 4)(x - 3) = 0$

$\Rightarrow x = 1$ or $x = 3$

II. $15y^2 - 14y + 3 = 0$

$\Rightarrow 15y^2 - 9y - 5y + 3 = 0$

$\Rightarrow 3y(5y - 3) - 1(5y - 3) = 0$

$\Rightarrow (5y - 3)(3y - 1) = 0$

$\Rightarrow y = 1/3, 3/5$

While comparing the root values of x and y, we find that both the values of x are greater than y's.

Hence option A.

22. Answer: A

Solution

I. $\frac{19}{\sqrt{x}} + \frac{12}{\sqrt{x}} = 5\sqrt{x}$

$\frac{31}{\sqrt{x}} = 5\sqrt{x}$

$X = 31/5$

II. $2y^2 - 11y + 14 = 0$

$2y^2 - 7y - 4y + 14 = 0$

$y(2y - 7) - 2(2y - 7) = 0$

$(2y - 7)(y - 2) = 0$

$y = 2, \quad 7/2$

$x > y$

Hence option A.

23. Answer: E

Solution

As $x = 4$ and -4 and $y = 5$ and -5

While comparing the values of x and y, one root value of x lies between the two root values of y.

Hence option E.

24. Answer: C

Solution

$2x^2 - 11x + 12 = 0$

$2x^2 - 8x - 3x + 12 = 0$

$2x(x-4) - 3(x-4) = 0$

$(2x-3)(x - 4) = 0$

$x = 3/2, 4$

$8y^2 - 18y + 9 = 0$

$8y^2 - 12y - 6y + 9 = 0$

$4y(2y - 3) - 3(2y - 3) = 0$

$(2y - 3)(4y - 3) = 0$

$y = 3/2, \ 3/4$

$x \geq y$

Hence option C.

25. Answer: C

Solution

$4x^2 + 8x + 3 = 0$

$4x^2 + 6x + 2x + 3 = 0$

$2x(2x + 3) + (2x + 3) = 0$

$(2x + 3)(2x + 1) = 0$

$x = -3/2, \ -1/2$

$2y^2 + 7y + 6 = 0$

$2y^2 + 4y + 3y + 6 = 0$

$2y(y + 2) + 3(y + 2) = 0$

$(y + 2)(2y + 3) = 0$

$y = -2, \ -3/2$

$x \geq y$

Hence option C.

26. Answer: E

Solution

$4x^2 + 13x = 105$ i.e. $4x^2 + 13x - 105 = 0$

Hence, both the equations are of the form $ax^2 + bx - c = 0$.

Thus, each equation has one positive and one negative root.

Hence, the relation between x and y cannot be determined.

Hence option E.

27. Answer: D

Solution

$x^2 + 32x + 255 = 0$

$x^2 + 15x + 17x + 255 = 0$

$\therefore (x + 15)(x + 17) = 0$

$x = -15$ or $x = -17$

$y^2 + 22y + 117 = 0$

$y^2 + 13y + 9y + 117 = 0$

$(y + 13)(y + 9) = 0$

$y = -13$ or $y = -9$

In both the cases, $x < y$

Hence option D.

28. Answer: A

Solution

$15x^2 - 63x + 434 = 14x^2 - 24x + 66$

$x^2 - 39x + 368 = 0$

Since this equation is of the form $ax^2 - bx + c = 0$, both roots are positive i.e. $x > 0$

$39y + 8 = -22y^2 - 6$

$\therefore 22y^2 + 39y + 14 = 0$

Since this equation is of the form $ax^2 + bx + c = 0$, both roots are negative i.e., $y < 0$

Hence, $x > y$.

Hence option A.

29. Answer: D

Solution

$x^2 + 105x + 1424 = 0$

Since this equation is of the form $ax^2 + bx + c = 0$, both roots are negative i.e $x < 0$

$y^2 - 67y + 252 = 0$

Since this equation is of the form $ax^2 - bx + c = 0$, both roots are positive i.e $y > 0$

Hence, $x < y$.

Hence option D.

30. Answer: D

Solution

$2x^2 + 46x + 63 = 17x - 6$

$2x^2 + 29x + 69 = 0$

Since this equation is of the form $ax^2 + bx + c = 0$, both roots are negative i.e $x < 0$

$7y^2 - 42y + 158 = 35y - 52$

$7y^2 - 77y + 210 = 0$

Since this equation is of the form $ax^2 - bx + c = 0$, both roots are positive i.e $y > 0$

Hence, $x < y$.

Hence option D.

31. Answer: D

Solution

$4x^2 + 48x + 108 = 0$

Since this equation is of the form $ax^2 + bx + c = 0$, both roots are negative i.e., $x < 0$

$3y^2 - 61y + 240 = 0$

Since this equation is of the form $ax^2 - bx + c = 0$, both roots are positive i.e., $y > 0$

Hence, $x < y$

Hence option D.

32. Answer: E

Solution

$x^2 + 15x = 216$ i.e. $x^2 + 15x - 216 = 0$

$y^2 + 16y = 336$ i.e. $y^2 + 16y - 336 = 0$

Hence, both the equations are of the form $ax^2 + bx - c = 0$.
Thus each equation has one positive and one negative root.
Hence, the relation between x and y cannot be determined.
Hence option E.

33. Answer: E

Solution

Both the equations are of the form $ax^2 + bx - c = 0$.
Thus, each equation has one positive and one negative root.
Hence, the relation between x and y cannot be determined.
Hence option E.

34. Answer: E

Solution

Both the equations are of the form $ax^2 + bx - c = 0$.
Thus, each equation has one positive and one negative root.
Hence, the relation between x and y cannot be determined.
Hence option E.

35. Answer: D

Solution

$2x^2 + 42x = -76$
$\therefore 2x^2 + 42x + 76 = 0$
$\therefore 2(x^2 + 21x + 38) = 0$ i.e. $x^2 + 21x + 38 = 0$
Since this equation is of the form ax2 + bx + c = 0, both roots are negative i.e $x < 0$
$5y^2 + 32 = 44y$
$5y^2 - 44y + 32 = 0$
Since this equation is of the form $ax^2 - bx + c = 0$, both roots are positive i.e $y > 0$
Hence $x < y$.
Hence option D.

36. Answer: C

Solution

$6x + \frac{6}{x} = 15$
$6x^2 + 6 = 15x$
$6x^2 - 15x + 6 = 0$
$6x^2 - 12x - 3x + 6 = 0$
$6x(x - 2) - 3(x-2)=0$
$(x-2)(6x - 3) = 0$
$x = 2$ or $1/2$
Hence option C.

37. Answer: E

Solution

Equation 1:
$x^2+4x- 45 = 0$
$x^2 + 9x-5x - 45 = 0$
$(x + 9)(x-5) = 0$
$x=-9$ or $x = 5$
Equation 2:
$4y^2-19y+28 = 0$
$4y^2-16y-7y + 28 = 0$
$(4y-7)(y-4)=0$
$y = 7/4 = 1.75$ or $y = 4$
When $x = 5$, $x>y$ and when $x = -9$, $x<y$
Thus, the relationship between x and y is not known.
Hence option E.

38. Answer: A

Solution

The first equation has 5 sign changes i.e. number of sign changes = degree of equation. In such a case, all roots are positive i.e. $x > 0$.
The second equation has 0 sign changes. In such a case, all roots are negative. i.e.
$y<0$. $x > y$
Hence option A.

39. Answer: D

Solution

$x^2 + 12x + 4 = 0$
$\alpha + \beta = -12$ and $\alpha \beta = 4$
The equation with roots $\alpha /2$ and $\beta /2$ is
$x^2 - (\alpha + \beta / 2)x + \alpha \beta /4 =0$
$x^2 + 6x + 1 =0$
Hence option D.

40. Answer: C

Solution

Let the numbers be x and y, such that $x > y$.
$x-y=4$ and $x^2 + y^2 = 80$
Now, $(x - y)^2 = x^2 - 2xy + y^2 = (x^2 + y^2) - 2xy$
$16=80-2xy$
$\therefore xy = 32$
Hence, option C.

41. Answer: D

Solution

$(x-7)(x - 5) = (x - 5)(5 - x)$

∴ (x–7)(x – 5) – (x – 5)(5 – x) = 0
(x – 5)(x–7–5 + x) = 0
(x – 5)(2x – 12) = 0
2(x – 5)(x–6)= 0
∴ (x – 5)(x–6)= 0
x = 5 and x = 6
Hence option D.

42. Answer: D

Solution

Here, instead of actually finding the roots, the property of sum of roots and product of roots should be used.
Sum of roots = $\alpha + \beta$ =–(–9)=9 and product of roots = $\alpha\beta = 14$
Now, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
$= (9)^2–2(14)$
= 81–28 = 53
Hence option D.

43. Answer: B

Solution

$9x^7+ 2x^5 - 10x^4 + x^3 – 2x – 14 = 0$ can also be written as
$9x^7+ 0x^6 + 2x^5– 10x^4 + 1x^3 – 2x – 14 = 0$
Since the equation has power 7, it has seven roots.
Also, since it has three sign changes, it has three positive roots.
Hence, it has three positive and four negative roots.
Hence option B.

44. Answer: C

Solution

If the roots of a quadratic equation are real, the discriminant of the equation must be greater than 0 i.e., $b^2 - 4ac > 0$
Here, a = a, b = 8, c = 4
$8^2– 4(a)4 > 0$
∴ 16a ≤ 64 i.e., a ≤ 4
Hence, the maximum possible value of a is 4.
Hence option C.

45. Answer: E

Solution

Since 4 is one of the roots of $x^2+bx+36=0$, x= 6 satisfies this equation.
$(6)^2 + 6b +36 = 0$
6b + 72 = 0 i.e., b = –12
Hence, the second equation is $x^2–12x+c=0$
Since this equation has equal roots, $b^2 - 4ac = 0$
$(–12)^2 - 4(1)(c) = 0$
4c = 144 i.e., c = 36
Hence option E.

46. Answer: A

Solution

$x^2 + 8x + a = 0$ and a < 4
To find the nature of the roots of the equation, find the value of $b^2 - 4ac$.
$b^2 - 4ac = (8)^2–4(1)(a) = 64 - 4a$
As a < 4, 4a < 16
∴64 – 4a > 16
Since the discriminant of the equation is positive, the roots of the equation must be real.
Hence option A.

47. Answer: E

Solution

$x^4 - 19x^2 + 60 = x^4 - 15x^2– 4x^2 + 60$
$= x^2(x^2 - 15)–4(x^2 - 15)$
$= (x^2 - 4)(x^2 - 15)$
$= (x - 2)(x + 2)(x^2 - 15)$
Hence option E.

48. Answer: A

Solution

Since the roots are positive integers, they are real and may or may not be equal.
∴ $b^2 - 4ac \geq 0$
$(–t)^2 - 4(1)(t) \geq 0$
$∴t^2– 4t \geq 0$
$t(t–4) \geq 0$
Case 1: t(t–4)= 0
Hence, t = 0 or t = 4
When t = 0, the equation becomes $x^2 = 0$
Roots of this equation are x = 0 (which are not positive integers).
When t = 4, the equation becomes $x^2 - 4x + 4 = 0$
Roots of this equation are x = 2 (which are positive integers).
Case 2: t(t–4) > 0
Hence, either t > 0 and t –4>0 i.e. t > 4 or t <0 and t – 4< 0ie. t < 0
Among the options, only t = 4 satisfies the given conditions.
Hence option A.

49. Answer: E

Solution

$9x^2 - 102x + 33 = 0$

$\therefore 9x^2 - 3x - 99x + 33 = 0$

$3x(3x - 1) - 33(3x - 1) = 0$

$(3x - 1)(3x - 33) = 0$

$x = 1/3$ (i.e. 0.33) or $x = 33/3$ (i.e. 11)

$3y^2 + 15y - 42 = 0$

$\therefore 3(y^2 + 5y - 14) = 0$ i.e. $y^2 + 5y - 14 = 0$

$\therefore y^2 - 2y + 7y - 14 = 0$

$y(y - 2) + 7(y - 2) = 0$

$(y - 2)(y + 7) = 0$

$\therefore y = 2$ or $y = -7$

When $x = 1/3$ and $y = 2$, $x < y$

When $x = 33/2$, $x > y$

Hence the relation between x and y cannot be determined.

Hence option E.

50. Answer: B

Solution

$x^2 + 14x = 207$

$x^2 + 14x - 207 = 0$

$x^2 + 23x - 9x - 207 = 0$

$\therefore (x+23)(x-9) = 0$

$x = -23$ or $x = 9$

$y^2 - 26y = -153$

$\therefore y^2 - 26y + 153 = 0$

$\therefore y^2 - 9y - 17y + 153 = 0$

$y(y - 9) - 17(y - 9)$

$(y - 9)(y - 17) = 0$

$\therefore y = 9$ or $y = 17$

$\therefore x \leq y$

$\therefore$ Hence option B.

51. Answer: A

Solution

$43x^2 + 17x + 35 = 13x^2 + 33$

$\therefore 30x^2 + 17x + 2 = 0$

$\therefore 30x^2 + 5x + 12x + 2 = 0$

$5x(6x + 1) + 2(6x + 1) = 0$

$(5x + 2)(6x + 1) = 0$

$\therefore x = -2/5$ or $x = -1/6$

$14y^2 + 62y + 103 = 6y^2 + 14y + 39$

$8y^2 + 48y + 64 = 0$

$\therefore 8(y^2 + 6y + 8) = 0$ i.e. $y^2 + 6y + 8 = 0$

$y^2 + 4y + 2y + 8 = 0$

$(y + 4)(y + 2) = 0$

$y = -4$ or $y = -2$

for both values $x > y$

Hence option A.

52. Answer: E

Solution

$6x^2 + 4x - 32 = 0$

$\therefore 6x^2 + 16x - 12x - 32 = 0$

$2x(3x + 8) - 4(3x + 8) = 0$

$\therefore (3x + 8)(2x - 4) = 0$

$x = -8/3$ (i.e. -2.66) or $x = 4/2$ (i.e. 2)

$26y^2 - 41y + 12 = 0$

$26y^2 - 24y - 13y + 12 = 0$

$2y(13y - 12) - (13y - 12) = 0$

$(2y - 1)(13y - 12) = 0$

$y = 1/2$ (i.e. 0.5) or $y = 12/13$ (i.e. 0.92)

When $x = -2.66$, $x < y$ and when $x = 2$, $x > y$

Hence, the relation between x and y cannot be established.

Hence option E.

53. Answer: A

Solution

$5x^2 - 40x = -75$

$\therefore 5x^2 - 40x + 75 = 0$

$\therefore 5x^2 - 25x - 15x + 75 = 0$

$5x(x - 5) - 15(x - 5) = 0$

$\therefore (x - 5)(5x - 15) = 0$

$x = 5$ or $x = 15/5$ (i.e. 3)

$11y^2 + 21y - 36 = 0$

$11y2 + 33y - 12y - 36 = 0$

$2.(y + 3)(11y - 12) = 0$

$y = -3$ or $y = 12/11$ (i.e. 1.09)

For both values, $x > y$

Hence option A.

54. Answer: A

Solution

$6x^2 + 27 = -27x$

$\therefore 6x^2 + 27x + 27 = 0$

$\therefore 6x^2 + 18x + 9x + 27 = 0$

$\therefore 6x(x + 3) + 9(x + 3) = 0$

$(6x + 9)(x + 3) = 0$

$\therefore x = -9/6$ (i.e. -0.66) or $x = -3$

$2y^2 + 21y = -52$

$2y^2 + 21y + 52 = 0$

$\therefore 2y^2 + 8y + 13y + 52 = 0$

∴ 2y (y +4) + 13 (y + 4) =0

(y+ 4)(2y + 13) = 0

y = – 13/2 (i.e. –6.5) or y = –4

For both values, x > y

Hence option A.

55. Answer: D

Solution

$\frac{x^2}{\sqrt{x}} - \frac{6}{\sqrt{x}} = -5\sqrt{x}$

Multiply each term by √x.

∴ x^2 – 6 =–5x

∴ x^2 + 5x –6 = 0

x^2 + 6x – x–6 = 0

∴ (x + 6)(x – 1) = 0

x = – 6 or x = 1

$3y^2$– 25y + 38 = 0

$3y^2$ – 6y – 19y+ 38 = 0

3y(y – 2) – 19 (y – 2) =0

(y–2) (3y – 19) = 0

y = 2 or y = 19/3 (i.e. 6.33)

For both values, x < y

Hence option D.

56. Answer: C

Solution

x^2– 51x+ 644 = 0

x^2– 23x – 28x + 644 = 0

(x–23)(x–28) = 0

x = 23 or x = 28

∴y^2 – 35y + 276 = 0

∴ y^2 – 23y – 12y + 276 = 0

(y–23)(y – 12) = 0

y= 23 or y = 12

When y = 12, x > y and when y = 23, x ≥ y

Hence, x ≥ y

Hence option C.

57. Answer: B

Solution

x^2 + 64x + 141 = 21x – 111

∴ x^2 + 43x + 252 = 0

∴ x^2+36x + 7x + 252 = 0

(x +36) (x + 7) = 0

x = –36 or x = –7

5 y^2 + 55 = –41y +13

∴ 5 y^2 + 41y + 42 = 0

∴ 5 y^2 + 35y + 6y + 42 = 0

∴ (y + 7)(5y + 6) = 0

y = –7 or y = –6/5 (i.e. –1.2)

When x = –36, x < y

When x =–7, x ≤ y

Hence x ≤ y

Hence option B.

58. Answer: D

Solution

$21x^2$ + 14x – 54 = – 51 + $4x^2$

∴ $17x^2$ + 14x – 3=0

∴ $17x^2$ + 17x – 3x – 3 = 0

(x + 1)(17x – 3) = 0

x = –1 or x = 3/17 i.e. < 1

2y + $\frac{16}{y}$= 12

∴ $2y^2$ + 16 = 12y i.e. $2y^2$ – 12y + 16 = 0

$2y^2$ – 8y –4y + 16 = 0

2y(y –4) – 4 (y –4) = 0

(y–4)(2y – 4) = 0

y = 4 or y = 4/2 i.e. 2

For both values, x < y

Hence option D.

59. Answer: D

Solution

$44x^2$ + 18x + 133 = $43x^2$ – 8x

x^2 +26x + 133 = 0

x^2 + 19x + 7x + 133 = 0

(x+19) (x + 7) = 0

x = –19 or x = –7

$21y^2$ + 17y + 72 = $15y^2$ – 25y

∴ $6y^2$ + 42y + 72 = 0

∴ 6(y^2 + 7y + 12) = 0 i.e. y^2 + 7y + 12 = 0

∴y^2 + 4y + 3y + 12 = 0

(y + 4) (y + 3) = 0

y = –4 or y=–3

In both the cases , y > x

Hence option D.

60. Answer: A

Solution

$20x^2$ + 76x + 102 = 19x + 62

∴ $20x^2$ + 57x + 40 = 0

∴ $20x^2$ + 32x + 25x + 40 = 0

∴ (5x + 8)(4x + 5) = 0

x = –8/5 (i.e. –1.6) or x = –5/4 (i.e. –1.25)

$–17y^2$ + 27y +44 = – $21y^2$

∴ $4y^2$ + 27y + 44 = 0

∴ $4y^2 + 16y + 11y + 44 = 0$

$(y + 4)(4y + 11) = 0$

$y = -4$ or $y = -11/4$ (i.e. -2.75)

In both the cases , $x > y$

Hence option A.

61. Answer: B

Solution

In the equation $a = 1$, $b = -8$, $c = 13$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \times 1 \times 13}}{2 \times 1}$$

$$\frac{8 \pm \sqrt{12}}{2} = 4 \pm \sqrt{3}$$

Hence option B.

62. Answer: A

Solution

$x^2 + 12x - 20 = 0$

$x^2 + 2(6)(x) = 20$

$x^2 + 2(6)(x) + (6)^2 = 20 + (6)^2$

$(x + 6)^2 = 20 + 36 = 56$

$x + 6 = \pm\sqrt{56} = +2\sqrt{14}$

Hence, the roots of the equation are $(-6 + 2\sqrt{14})$ and $(-6 - 2\sqrt{14})$

Hence option A.

63. Answer: D

Solution

In the quadratic equation, $2x^2 - 9x + k = 0$

$a=2$, $b = -9$, $c=k$

Let α and β be the roots of the equation, and $\beta = 8\alpha$

Now, $\alpha + \beta = (-b/a) = (9/2)$

$\therefore \alpha + 8\alpha = 9/2$

$9\alpha = 9/2$

Hence $\alpha = (1/2)$

Now, $\alpha\beta = (c/a) = (k/2)$

Hence $8\alpha^2 = (k/2)$

$8(1/2)^2 = k/2$

$k = 4$

64. Answer: E

Solution

Sum of coefficients = $1+27-10-18=28-28=0$

Hence, try to express the given equation in terms of $(x - 1)$, so that we can take it out as a common term.

$x^3 - 10x^2 + 27x - 18 = (x^3 - 2x^2 + x) - 8x^2 + 26x - 18$

$= x(x^2 - 2x + 1) - (8x^2 - 16x + 8) + (10x - 10)$

$= x(x-1)^2 - 8(x^2 - 2x + 1) + 10(x - 1)$

$=(x-1)[x(x-1) - 8(x-1) + 10]$

$=(x-1)(x^2 - 9x + 18)$

$=(x-1)(x-3)(x-6)$

Thus, 1, 3 and 6 are the roots of this equation.

Hence option E.

65. Answer: E

Solution

$(x^2/2) + 8x + 28 = 0$

$x^2 + 16x + 58 = 0$

Since the equation is of the form $ax^2 + bx + c = 0$, both roots of the equation are negative.

Now, x can be found using the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(16) \pm \sqrt{(16)^2 - 4 \times 1 \times 58}}{2 \times 1}$$

$$\frac{-16 \pm \sqrt{24}}{2} = -8 \pm \sqrt{6}$$

Hence option E.

66. Answer: C

Solution

Since the equation has equal roots, the discriminant, i.e. $b^2 - 4ac = 0$.

$(3q+4)^2 - 4(1)(8q + 9) = 0$

$\therefore 9q^2 + 24q + 16 - 32q - 36 = 0$

$\therefore 9q^2 - 8q - 20 = 0$

$\therefore 9q^2 - 18q + 10q - 20 = 0$

$\therefore 9q(q-2) + 10(q - 2) = 0$

$q=2$ or $q = -10/9$

Among these two values, only $-10/9$ is given among the options.

Hence option C.

67. Answer: D

Solution

$X^4 - 36x^2 + 275 = 0$

Let $x^2 = a$

$\therefore a^2 - 36a + 275 = 0$

$a^2 - 11a - 25a + 275 = 0$

$(a-11)(a - 25) = 0$

$\therefore a = 11$ or $a = 25$ i.e. $x^2 = \sqrt{11}$ or $x^2 = 25$

$\therefore x = \sqrt{11}$ or $x = \pm 5$

$\therefore$ Hence option D.

68. Answer: B

Solution

A non-positive root implies that the root can be either negative or zero.
$415x^6-28x^5-32x^4+115x^3-x^2+11x=0$
$x(415x^5-28x^4-32x^3+115x^2-x+11)=0$
$x=0$ or $(415x^5-28x^4-32x^3+115x^2-x+11)=0$
Thus, one root of the equation is zero.
Since the other part of the equation has power 5, it has five roots. Also, since it has four sign changes, it has four positive roots.
Hence, the equation has four positive roots and one negative root. Hence, the overall equation has four positive roots and two non-positive roots (i.e. one zero and one negative).
Hence option B.

69. Answer: C

Solution

$x^4-8x^3+x^2+78x-72=0$
One way of solving this equation is to substitute one value each from each option and see if the equation is satisfied.
The general way is to first check if $x = 1$ is a root of the equation.
For this, check if: sum of coefficients of equation = 0
Here, sum of coefficients=1-8+1+78-72=0
Hence, $x = 1$ is one root of the equation.
Hence, options 2 and 3 can be eliminated. Here, you can try and substitute values from options 1 and 3, but because one option is 'None of the above', you would have to check each number in atleast one of the two options to be sure..
Hence, try to express the given equation in terms of $(x-1)$.
$(x^4-2x^3+x^2)-6x^3+6x^2-6x^2+6x+72x-72=0$
$(x^2)(x^2-2x+1)-6x^2(x-1)-6x(x-1)+72(x-1)=0$
$\therefore (x^2)(x-1)^2-6x^2(x-1)-6x(x-1)+72(x-1)=0$
$(x-1)(x^3-x^2-6x^2-6x+72)=0$
$(x-1)(x^3-7x^2-6x+72)=0$
Consider $(x^3-7x^2-6x+72)=0$. See if $x = 1$ is a factor of this equation as well.
Sum of coefficients=1-7-6+72 = 60 ≠ 0. Hence, $x = 1$ is not a factor of the cubic equation.
Since this equation has two sign changes, it has two positive roots and one negative root.
For a cubic equation of the form $ax^3+bx^2+cx+d=0$, let the roots be p, q and r.
$\therefore$ Sum of roots $=p+q+r=(-b/a)$ and product of roots $=pqr=(-d/a)$
For $x^3-7x^2-6x+72=0$; $a = 1$, $b = -7$, $c = -6$ and $d = 72$
$pqr=-72$ and $p+q+r=7$
Try to express 72 as a product of 3 numbers (none of which is 1, as $x = 1$ is not a factor).
Both the equations above are satisfied when the values are -3, 4 and 6
i.e. $(-3)(4)(6)=-72$ and $(-3)+4+6=7$
Hence, the roots are -3, 4 and 6 i.e. $x = -3$ or $x = 4$ or $x = 6$
Hence option C.

70. Answer: E

Solution

$-56x+31x^2=23x^2-66$
$\therefore 8x^2-56x+66=0$
$\therefore 2(4x^2-28x+33)=0$ i.e. $4x^2-28x+33=0$
$\therefore 4x^2-22x-6x+33=0$
$2x(2x-11)-3(2x-11)=0$
$(2x-11)(2x-3)=0$
$x = 11/2$ (i.e., 5.5) or $x = 3/2$ (i.e., 1.5)
$-10y+11y^2+51=26y+23$
$\therefore 11y^2-36y+28=0$
$\therefore 11y^2-22y-14y+28=0$
$(y-2)(11y-14)=0$
$\therefore y = 2$ or $y = 14/11$ (i.e. 1.27)
When $x = 5.5$, $x>y$ and when $x = 1.5$, $x < y$
Hence, the relation between x and y cannot be determined.
Hence option E.

71. Answer: A

Solution

$7x(x+9)=33x-27$
$\therefore 7x^2+63x-33x+27=0$
$\therefore 7x^2+30x+27=0$
$\therefore 7x^2+21x+9x+27=0$
$(x+3)(7x+9)=0$
$x = -3$ or $x = -9/7$ (i.e. between -2 and -1)

$10y^2 + 31y = 6y^2 - 60$

$\therefore\ 4y^2 + 31y + 60 = 0$

$4y^2 + 15y + 16y + 60 = 0$

$y(4y + 15) + 4(4y + 15) = 0$

$(4y+15)(y + 4) = 0$

$y = -15/4$ or $y = -4$ (i.e., -3.75 and -4)

For both values, $x > y$

Hence option A.

72. Answer: E

Solution

I. $143x^2 + 43x - 66 = 0$

$143x^2 + (121 - 78)x - 66 = 0$

$143x^2 + 121x - 78x - 66 = 0$

$11x(13x + 11) - 6(13x + 11) = 0$

$(13x + 11)(11x - 6) = 0$

$x = 6/11, -11/13$

II. $33y^2 - 37y + 10 = 0$

$33y^2 - (15 + 22)y + 10 = 0$

$33y^2 - 22y - 15y + 10 = 0$

$11y(3y - 2) - 5(3y - 2) = 0$

$(11y - 5)(3y - 2) = 0$

$y = 2/3, 5/11$

While comparing the values of x and y, one root value of x lies between the two root values of y.

Hence option E.

73. Answer: A

Solution

$18x^2 - 74x + 60 = 0$

$2(9x^2 - 37x + 30) = 0$ i.e. $9x^2 - 37x + 30 = 0$

$9x^2 - 27x - 10x + 30 = 0$

$(x - 3)(9x - 10) = 0$

$\therefore x = 3$ or $x = 9/10$ (i.e. 0.9)

$48y^2 - 28y + 4 = 0$

$4(12y^2 - 7y + 1) = 0$ i.e. $12y^2 - 7y + 1 = 0$

$\therefore 12y^2 - 4y - 3y + 1 = 0$

$(3y - 1)(4y - 1) = 0$

$y = 1/3$ (i.e. 0.33) or $y = 1/4$ (i.e. 0.25)

For both the values $x > y$

Hence option A.

74. Answer: D

Solution

$56x^2 - 30x + 4 = 0$

$\therefore 56x^2 - 14x - 16x + 4 = 0$

$\therefore 14x(4x - 1) - 4(4x - 1) = 0$

$(14x - 4)(4x - 1) = 0$

$\therefore x = 4/14$ (i.e. 0.28) or $x = 1/4$ (i.e. 0.25)

$45y^2 - 36y + 7 = 0$

$\therefore 45y^2 - 15y - 21y + 7 = 0$

$15y(3y - 1) - 7(3y - 1) = 0$

$\therefore\ (15y - 7)(3y - 1) = 0$

$y = 7/15$ (i.e. 0.46) or $y = 1/3$ (i.e. 0.33)

For both the values $x < y$

Hence option D.

75. Answer: A

Solution

$6x^2 - 50x + 104 = 0$

$\therefore\ 6x^2 - 24x - 26x + 104 = 0$

$6x(x - 4) - 26(x - 4) = 0$

$(6x - 26)(x - 4) = 0$

$x = 26/6$ (i.e. 4.33) or $x = 4$

$15y^3 - 57y^2 + 36y = 0$

$\therefore\ 3y(5y^2 - 19y + 12) = 0$

$\therefore\ y = 0$ or $5y^2 - 19y + 12 = 0$

$\therefore\ 5y^2 - 15y - 4y + 12 = 0$

$(y - 3)(5y - 4) = 0$

$y = 3$ or $y = 4/5$ (i.e. 0.8)

$y = 0$ or $y = 3$ or $y = 0.8$

For all values of x, $x > y$

Hence, option A.

NUMBER SERIES SOLUTIONS

Exercise – 1

1. **Answer: D**

Solution

Let the first term of an AP = a and the common difference = d

3^{rd} term of AP = a_3 = a + 2d = −6 ⋯(1)

9^{th} term = a_9 = a + 8d = 12 ⋯(2)

Subtracting equation (1) from (2), we get :

8d − 2d = 12 − (−6)

∴ 6d = 18 → d = 18/6 = 3

Substituting it in equation (2),

⇒ a = 12 − 8(3) = 12 − 24 = −12

∴ 16^{th} term = a_{16} = a + 15d

= −12 + 15(3) = −12 + 45 = 33

2. **Answer: D**

Solution

Arithmetic mean of 8, 4, 12, x and 10 = 12

$\frac{8+4+12+x+10}{5}$ = 12

34+x = 12 × 5 = 60 → x = 60 − 34 = 26

3. **Answer: B**

Solution

Sum of AP = $\frac{n}{2}$ (a + l) = $\frac{9}{2}$ (8 + 54) = $\frac{9}{2}$ (62) = 279

4. **Answer: C**

Solution

t_7 = a + 6d = −14 ⋯(1)

t_{12} = a + 11d = 6 ⋯(2)

on solving (1) and (2)

d = 4 & a = −38

t_{16} = a + 15d = −38 + 15(4) = −38 + 60 = 22

5. **Answer: A**

Solution

t_n = a + (n − 1) × d

⇒ t_{25} = a + 24d

and t_{20} = a + 19d

$t_{25} - t_{20}$ = 55

a + 24d − a − 19d = 55

∴ 5d = 55 → d = 11

6. **Answer: B**

Solution

t_n = a + (n − 1) × d

t_{17} = a + 16d

t_7 = a + 6d

$t_{18} - t_8$ = 16d − 6d = 10d

7. **Answer: D**

Solution

t_n = a + (n − 1) × d where d = −3, Let t_n = 0

0 = 27 + (n − 1) × −3

0 = 27 − 3n + 3

∴ 3n = 30 → n = 10

∴ 11^{th} term will be negative.

8. **Answer: A**

Solution

t_n = a + (n − 1) × d

where,

d = x − 3 − x + 8 = 5,

a = x − 8

t_{15} = (x − 8) + (15 − 1) × 5

t_{15} = (x − 8) + 14 × 5 = x − 8 + 70 = x + 62

9. **Answer: C**

Solution

Sum of an AP is given by:

= Number of terms × $\frac{first\ term+last\ term}{2}$

462 = n × $\frac{(1+21)}{2}$

462 = n × $\frac{22}{2}$

n = $\frac{462*2}{22}$ = 42

No. of terms = 42

10. **Answer: B**

Solution

S_n = $\frac{n}{2}$ [2a + (n−1) d]

S_{15} = $\frac{15}{2}$ [2(4) + (15−1) 4] = $\frac{15}{2}$ [8 + 56] = 480

11. **Answer: D**

Solution

t_n = a + (n − 1) × d

4 = a + (27 − 1) × −4

4 = a − 104

∴ a = 4 + 104 = 108

12. **Answer: D**

Solution

t_n = a + (n − 1) × d where d = −6, Let t_n = 0

0 = 92 + (n − 1) × −6

0 = 138 − 6n + 6

6n = 144 → n = 24

13. **Answer: B**

Solution

A sequence is called geometric progression if p_{n+1} = p_n × r where p_1 is the first term and r is common ratio.

14. **Answer: C**

Solution

Since every term of a G.P. is r times the previous term. $t_n = a \times r^{n-1}$

15. **Answer: A**

Solution

Given, a = 40 and r = 4.

We know, $t_n = ar^{n-1}$

$\therefore t_5 = 40 \times 4^4 = 40 \times 256 = 10240$.

16. **Answer: B**

Solution

If $t_n = 2 \times 5^n$ then

$t_1 = 10$, $t_2 = 50$, $t_3 = 250$.

This is a geometric progression with first term 10 and common ratio 5.

17. **Answer: D**

Solution

3^{rd} term $ar^2 = 1/4$, 8^{th} term $ar^7 = 256$

$r^5 = 1024$, Gives us: r = 4.

Hence, the second term will be given by (3^{rd} term/r) = 1/4 × 1/4 = 1/16. Option (D) is correct.

[Note: To go forward in a G.P. you multiply by the common ratio, to go backward in a G.P. you divide by the common ratio.]

18. **Answer: B**

Solution

Let the cost of an equipment is Rs. 100

Now the percentages of depreciation at the end of 1^{st}, 2^{nd}, 3^{rd} years are 15, 13.5, 12, which are in A.P., with a = 15 and d = – 1.5

Hence, percentage of depreciation in the tenth year = a + (10 – 1) d = 15 + 9 (–1.5) = 1.5

Also total value depreciated in 10 years = 15 + 13.5 + 12 + ... + 1.5 = 82.5

Hence, the value of equipment at the end of 10 years = 100 – 82.5 = 17.5

The total cost being

= Rs.12,00,000/100 × 17.5 = Rs.2,10,000

19. **Answer: B**

Solution

First term is –65. The common difference is –2. The last term is –101.

Sum of the first n terms of an AP

$= \frac{n}{2}[2a + (n-1)d]$

To compute the sum, we know the first term a = –65 and the common difference d = –2.

We do not know the number of terms n. Let us first compute the number of terms and then find the sum of the terms.

$t_n = a + (n - 1)d$

$-101 = -65 + (n - 1)(-2)$

Therefore, n = 19

Sum = $\frac{19}{2}[2(-65) + (19-1)(-2)]$

= $\frac{19}{2}[-130 - 36]$ = 19(–83) = –1577

20. **Answer: D**

Solution

The positive integers, which are divisible by 5 are 5, 10, 15, ..., 2000

Out of these 10, 20, 30, ..., 1000 are divisible by 2

Thus, we have to find the sum of the positive integers 5, 15, 25, ..., 1995

If n is the number of terms in it the sequence then

1995 = 5 + 10(n – 1)

$\Rightarrow$ 2000 = 10n

$\therefore$ n = 200

Thus the sum of the series

= $\frac{n}{2}$ (a + l) = $\frac{200}{2}$ (5 + 1995) = $\frac{200}{2}$ (2000) = 2,00,000

21. **Answer: B**

Solution

$t_n = \frac{a}{1-r} \Rightarrow 1 - r = r$ or r = 1/2

a + ar = 48 or a = 32

Required sum = $\frac{32}{1-\frac{1}{2}}$ = 64.

22. **Answer: D**

Solution

The side of the first equilateral triangle being 12 units, the first area is $36\sqrt{3}$ square units. The second area would be 1/4 of area of largest triangle and so on. $36\sqrt{3}$, $9\sqrt{3}$, $9\sqrt{3}/4$, $9\sqrt{3}/16$, $9\sqrt{3}/64$, ⋯. The infinite sum of this series =

$= \frac{36\sqrt{3}}{1-\frac{1}{4}} = 48\sqrt{3}$ square units.

23. **Answer: A**

Solution

Let the series be a, ar, ar^2, ar^3, ⋯. According to the question $t_n = 4ar/(1 - r)$ or r = 1/5. The series would be 5, 5/5, 5/25, ⋯. and so on. The product of first three terms of the series would be 5 × 1 × (1/5) = 1.

24. **Answer: B**

Solution

2log y = log x + log z or log y^2 = log xz or y^2 = xz, so x, y, z are in GP. Option (B) is correct.

25. **Answer: E**

Solution

$\frac{a}{1-r}$ = 32

a + ar = 30 → a(1+r) = 30 → a = $\frac{30}{1+r}$

Thus, $\frac{\frac{30}{1+r}}{1-r}$ = 32 → $\frac{30}{(1+r)(1-r)}$ = 32

→ $\frac{1}{1-r^2} = \frac{32}{30} = \frac{16}{15}$

→ $1 - r^2 = \frac{15}{16}$

→ $r^2 = 1 - \frac{15}{16} = \frac{1}{16}$ → r = $\frac{1}{4}$

Also, a = $\frac{30}{1+r}$ => a = $\frac{30}{\frac{5}{4}}$ = 6 × 4 = 24

Third term = ar^2 = 24/16 = 3/2

26. **Answer: B**

Solution

The expression can be written as $3^{1/3+1/6+1/12+\cdots}$ = $3^{\text{INFINTE SUM OF THE GP}} = 3^{2/3}$.

27. **Answer: A**

Solution

From the facts given in the question it is self-evident that the common ratio of the GP must be 3 (as the sum of the 2nd and 4th term is thrice the sum of the first and third term).

a + ar^2 = 100 or a = 100/(1 + 9) = 10

Largest term = 10 × $(3)^3$ = 270.

28. **Answer: D**

Solution

Suppose, we take the sum to 1 (n = 1) term of the first series and the sum to 2 terms (2n = 2) of the second series we would get B/A as 4/3

For n = 2 and 2n = 4 we get A = 10/3 and B = 40/9 and B/A = 40/9 / 10/3 = 4/3.

Thus, we can conclude that the required ratio is always constant at 4/3.

29. **Answer: D**

Solution

For 4 digit numbers

Thereby numbers forming an AP would be: 1234, 1357, 2345, 2468, 3210, 3456, 3579, 4321, 4567, 5432, 5678, 6543, 6420, 6789, 7654, 7531, 8765, 8642, 9876, 9753, 9630.

A total of 21 numbers.

If we count the GPs we get: 1248, 8421-a total of 2 numbers. Hence, we have a total of 23, 4-digit numbers where the digits are either APs or GPs.

30. **Answer: B**

Solution

The first perimeter is 320, the second one is $\frac{320}{\sqrt{2}}$, the third one would be 160 and so on till infinite terms:

$\frac{a}{1-r} = \frac{320}{1-\frac{1}{\sqrt{2}}} = \frac{320\sqrt{2}}{\sqrt{2}-1}$

$= \frac{320\sqrt{2}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{320\sqrt{2}(\sqrt{2}+1)}{2-1} = 320(2+\sqrt{2})$

31. **Answer: D**

Solution

Let the no. of vada pav be x.

x = ar^{n-1} = ½ × 2^{12} = 2^{11} = 2048

Exercise – 2

1. **Answer: C**

Solution

To solve this problem, we need to find the pattern in the series. One way to do this is to look at the differences between each consecutive pair of numbers-

8 - 3 = 5

15 - 8 = 7

24 - 15 = 9

35 - 24 = 11

We can see that the differences are increasing by 2 each time. Therefore, the next difference should be 13. To find the next number in the series, we add this difference to the previous number-

35 + 13 = 48

2. **Answer: B**

Solution

To find the pattern in this series, we can look at the differences between each consecutive pair of numbers-

3 - 1 = 2

7 - 3 = 4

15 - 7 = 8

31 - 15 = 16

We can see that the differences are increasing by multiples of 2 each time- 2, 4, 8, 16. Therefore, the next difference should be 32. To find the next number in the series, we add this difference to the previous number-

31 + 32 = 63

3. **Answer: A**

Solution

To find the pattern in this series, we can look at the differences between each consecutive pair of numbers-

10 - 3 = 7

21 - 10 = 11

36 - 21 = 15

55 - 36 = 19

78 - 55 = 23

We can see that the differences are increasing by 4 each time, then by odd numbers. Therefore, the next difference should be 27.

To find the next number in the series, we add this difference to the previous number

78 + 27 = 105

4. **Answer: E**

Solution

This is a Fibonacci sequence, where each number is the sum of the two previous numbers. Therefore, the next number should be

55+89 = 144

Therefore, the answer is not given as an option, and the correct answer is not listed.

5. **Answer: B**

Solution

Given series- 15, 26, 48, 92, 180, ?.

26 - 15 = 11

48 - 26 = 22

92 - 48 = 44

180 - 92 = 88

The logic used here is that the difference between two successive numbers is doubled consecutively.

So, if 88 is doubled, we get 176.

Hence, 180 + 176 = 356.

Thus, the missing term in the sequence is 356.

Therefore, the complete series is 15, 26, 48, 92, 180, 356.

6. **Answer: A**

Solution

Given the number series- 7, 8, 11, 16, 23, ?.

7 + 1 = 8

8 + 3 = 11

11 + 5 = 16

16 + 7 = 23

23 + 9 = 32

Hence, the missing number here is 32.

Here, the series follows a pattern of the sum of consecutive odd numbers.

So, the complete number series is 7, 8, 11, 16, 23, 32.

7. **Answer: B**

Solution

Here, $6^2 = 36$ and $(6+1)^2 + 1 = 50$

Similarly, $7^2 = 49$ and $(7+1)^2 + 1 = 65$.

Hence, the missing number is 65.

8. **Answer: E**

Solution-

Here, $13^2 = 169$

So, $15^2 = 225$

So, the missing number is 225.

I.e., 13 : 169 -- 15 : 225.

9. **Answer: C**

Solution

This is a simple division series; each number is one-half of the previous number.

In other terms to say, the number is divided by 2 successively to get the next result.

4/2 = 2

2/2 = 1

1/2 = 1/2

(1/2)/2 = 1/4 and so on.

10. **Answer: A**

Solution

This is a simple alternating addition and subtraction series. In the first pattern, 3 is added; in the second, 2 is subtracted.

11. **Answer: D**

Solution

The given number series is 7 , 7 , 8 , 6 , 9 , ?

Here in this we have two number sequences, they are 7, 8, 9, ... and 7, 6, 5, ...

Hence, the next number in the given number series is 6 - 1 = 5.

12. **Answer: B**

Solution

Given number series 531 + 17 = 548

548 + 19 = 567

567 + 23 = 590

590 + 29 = 619

619 + 37 = 656

Here addition is based on 17 + 2 = 19, 19 + 4 = 23, 23 + 6 = 29, 29 + 8 = 37

Hence the required term is 619

13. **Answer: D**

Solution

The pattern is ×2, ×5/2, ×2, ×5/2, ×2, ⋯

So missing term is 50×(5/2)=625

14. **Answer: D**

Solution

The correct pattern is ×(1+1), ×(2+2), ×(3+3), ×(4+4), ⋯

So,24 is wrong term and must be replaced by

(6 × 3 + 3) = 21

15. **Answer: C**

Solution

It follows the rule that

1

2×2

$3 \times 3 \times 3$

$4 \times 4 \times 4 \times 4$

$5 \times 5 \times 5 \times 5 \times 5$

So, the missing number is 4 × 4 × 4 × 4 = 256.

16. **Answer: B**

Solution

Each term in the series is obtained by subtracting from the preceding term the number formed by the first three digits of preceding term

So, missing term is 5742 – 574 = 5168

17. **Answer: B**

Solution

The pattern is +0.05 ,+0.1 , +0.15, ⋯

so missing term = 5.8 + 0.20 = 6.0

18. **Answer: B**

Solution

The given series is 9 , 81 , 729 , ? , 59049

It follows a pattern that

9

81 = 9 × 9

729 = 9 × 9 × 9

6561 = 9 × 9 × 9 × 9

59049 = 9 × 9 × 9 × 9 × 9

Hence, the missing number in the series is 6561.

19. **Answer: C**

Solution

The pattern is 2 × 27 = 54, 3 × 23 = 69,

4 × 19 = 76

The next number = 5 × 15.6 = 78

20. Answer: C

Solution

The given sequence is a combination of two series

1) 51, 55, 59, 63, 67

2) 35, 37, 39, 41, 43

So, the missing term is 39

21. Answer: B

Solution

The logic is 7 × 3 – 5 = 16,

16 × 3 – 5 = 43,

43 × 3 – 5 = 124,

124 × 3 – 5 = 367

So, in place of 369, it should be 367.

22. Answer: C

Solution

Starting from the back

109 – 73 = 36,

73 – 48 = 25,

48 – 32 = 16,

32 – 23 = 9,

23 – 19 = 4,

19 – 18 = 1.

Every difference is perfect square starting from 36 and decreasing. Thus the wrong number is 21, it should be 19

23. Answer: D

Solution

Logic is 99 + 1 = 100

100 – 4 = 96,

96 + 9 = 105,

105 – 16 = 89,

89 + 25 = 114.

Thus, the wrong number is 88, it should be 89.

24. Answer: E

Solution

75 – 59 = 16,

59 – 51 = 8,

51 – 47 = 4,

47 – 45 = 2,

45 – 44 = 1,

44 – 43.5 = 0.5

The difference is halved every time. Thus, the wrong number is 44.5, it should be 44.

25. Answer: B

Solution

The pattern is 1 × 2 = 2

2 × 3 = 6,

3 × 4 = 12,

4 × 5 = 20,

5 × 6 = 30,

6 × 7 = 42,

7 × 8 = 56.

8 × 9 = 72.

So, the required number is 6 × 7 = 42

26. Answer: B

Solution

Given series follows the following pattern

$2 = 1^2 + 1$

$3 = 2^2 - 1$

$10 = 3^2 + 1$

$99 = 10^2 - 1$

$9802 = 99^2 + 1$

27. Answer: C

Solution

Given series follows the following pattern

7

7 × 2 – 1 = 13

13 × 2 – 2 = 24

24 × 2 – 3 = 45

45 × 2 – 4 = 86

28. Answer: D

Solution

The given sequence is a combination of three series-

i) 1^{st}, 4^{th}, 7^{th} terms i.e., 2, 4, 8

ii)2^{nd}, 5^{th}, 8^{th} terms i.e., 3, 6, 11

iii)3^{rd}, 6^{th}, 9^{th} terms i.e., 4, 8, 16

In each one of i, ii, iii, each term is twice the preceding term.

So, 11 is wrong and must be replaced by (6 × 2) = 12

29. Answer: B

Solution

Here the number follows the given rule

$1 \times 1 + (1^2) = 2$

$2 \times 2 + (2^2) = 8$

$8 \times 3 + (3^2) = 33$

$33 \times 4 + (4^2) = 148$

$148 \times 5 + (5^2) = 765$

Hence the answer is 148

30. Answer: A

Solution

The series follow the rule,

$21 = 5^2 - 4$

$77 = 9^2 - 4$

$165 = 13^2 - 4$

$285 = 17^2 - 4$

$437 = 21^2 - 4$

Hence the missing term is 165

31. Answer: B

Solution

13

$13 + (3^2 - 3) = 19$

$19 + (5^2 - 3) = 41$

$41 + (7^2 - 3) = 87$

$87 + (9^2 - 3) = 165$

Hence, the next number in the given sequence is 165.

32. Answer: E

Solution

Clearly, alternatively we add and subtract 10% of a term to obtain the next term of the series.

Thus, 10000 + (10% of 10000) = 11000;

11000 - (10% of 11000) = 9900;

9900 + (10% of 9900) = 10890;

10890 - (10% of 10890) = 9801.

9801 + (10% of 9801) = 9801+980 = 10781.

So, missing term = 9900 + (10% of 9900)= 10890

33. Answer: D

Solution

Here the given number series 725, 725, 700, ?, 575, 975 follows a pattern that

725

$725 + (0 \times 0) = 725$

$725 - (5 \times 5) = 725 - 25 = 700$

$700 + (10 \times 10) = 700 + 100 = 800$

$800 - (15 \times 15) = 800 - 225 = 575$

$575 + (20 \times 20) = 575 + 400 = 975$

Hence, the missing number in the given number series is 800.

34. Answer: E

Solution

Clearly, the numerators of the fractions in the given sequence follow the series of 1, 3, 5, 7, and the denominator follows 3, 9, 27, 81, ⋯

Then, the next term in the given sequence is

$9/3^5 = 9/243 = 1/27$

35. Answer: D

Solution

Let first no. be "k" and multiple be "l".

start value of y with 3 then,

$[(k \times y) + \{l \times (l-1)\}]$

Thus, we get

$13 \times 3 + 3 \times 2 = 45$

$45 \times 4 + 4 \times 3 = 192$

$192 \times 5 + 5 \times 4 = 980$

$980 \times 6 + 6 \times 5 = 5910$

$5910 \times 7 + 7 \times 6 = 41412$

Therefore, the missing number is 5910.

36. Answer: D

Solution

The pattern is ×1+3, ×2, ×1+3, ×3, ×1+3, ×4

The next number = $48 \times 4 = 192$

37. Answer: E

Solution

The pattern of given series is-

5

$14 = (5 \times 3) - 1$

$45 = (14 \times 3) + 3$

$(?) = (45 \times 3) - 5 \Rightarrow (?) = 130$

$397 = (130 \times 3) + 7$

$1182 = (397 \times 3) - 9$

Thus, the missing number is 130.

38. Answer: C

Solution

Two series alternate here, with every third number following a different pattern.

In the main series, 4 is added to each number to arrive the next.

In alternate series, 5 is subtracted from each number to arrive the next number.

Here in the main series

5 + 4 = 9

9 + 4 = 13

13 + 4 = 17

17 + 4 = 21

In alternate series

24 – 5 = 19

19 – 5 = 14...

So, the next term will be 21+4 =25

39. Answer: D

Solution

ten = 3(number of letters)

four = 4(number of letters)

sixty = 5(number of letters)

eighty = 6(number of letters)

seventy = 7(number of letters)

sixty-six = 8(number of letters)

eighty-six=9(number of letters)

Then, likewise choose the number which has 9 letters in its Spelling.

40. Answer: A

Solution

Given series is 354, 832, 446, 285, 64?, 7?5

Here the series follow the rule that-

354 = 3 + 5 + 4 = 12

832 = 8 + 3 + 2 = 13

446 = 4 + 4 + 6 = 14

285 = 2 + 8 + 5 = 15

64? = 6 + 4 + ? = 16 → ? = 6

7?5 = 7 + ? + 5 = 17 → ? = 5

So, the missing numbers are 6 and 5

41. Answer: E

Solution

The given number series is 6, 11, 19.5, 33.25, ?

It follows a pattern that,

6

6 × 1.5 + 2 = 11

11 × 1.5 + 3 = 19.5

19.5 × 1.5 + 4 = 33.25

33.25 × 1.5 + 5 = 54.875

42. Answer: E

Solution

The terms of the given series is 1×1, 2×2 + 1, 3×3, 4×4 +1, 5×5, 6×6 + 1, ⋯

So, 36 is wrong and must be replaced by (6×6 + 1) = 37

43. Answer: C

Solution

The given series is 1, 2, 6, 21, 88, 445, ?

1 × 1 + 1 = 2

2 × 2 + 2 = 6

6 × 3 + 3 = 21

21 × 4 + 4 = 88

88 × 5 + 5 = 445

445 × 6 + 6 = 2676

Hence, the next number in the given number sequence is 2676.

44. Answer: B

Solution

The given series follows a pattern of +19, ×19, +17, ×17, +15, ×15, ...

Here the number which is to be replaced in? is 40987+15=41002.

45. Answer: C

Solution

The given sequence is a combination of two series-

i) 1, 4, 9, 16, 25, 36 ii) 3, 6, 11, 19, 27

The pattern in both i, ii is +3, +5, +7, ...

So, in ii) 19 is wrong and must be replaced by (11 + 7) = 18

46. Answer: A

Solution

The given number series is 2, 5, 16, 65, 326, ?

It follows a pattern that,

2

2 × 2 + 1 = 5

5 × 3 + 1 = 16

16 × 4 + 1 = 65

65 × 5 + 1 = 326

326 × 6 + 1 = 1957

Hence, the next number in the given number series is 1957.

47. Answer: C

Solution

The given number series is 20, 11, 13, 22.5, ?

20

20 × 0.5 + 1 = 11

11 × 1 + 2 = 13

13 × 1.5 + 3 = 22.5

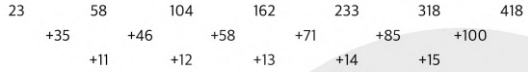
22.5 × 2 + 4 = 49

Hence, the next number in the given series is 49.

48. Answer: D

Solution

The given number series is 23, 58, 104, 162, 233, 318, ?



Hence, the next number in the given number series is 318 + 100 = 418.

49. Answer: B

Solution

The given number series is 2, 11, 48, ? ,1460, 10221. It follows a pattern that,

2

2 × 3 + 5 = 11

11 × 4 + 4 = 48

48 × 5 + 3 = 243

243 × 6 + 2 = 1460

1460 × 7 + 1 = 10221

Hence, the missing number in the given series is 243.

50. Answer: E

Solution

In the given series,

2^{nd} term = (1^{st} term −8)/2 = (988−8)/2=490

3^{rd} term = (2^{nd} term −8)/2= (490−8)/2=241

and so on ⋯

Here 6^{th} term = (5^{th} term −8)/2= (54.25−8)/2 = 23.125

Therefore, 22.125 is wrong.

LOGARITHMS SOLUTIONS

1. **Answer: C**

   Solution

   $\log_{32} x = 0.6 \Leftrightarrow x = (32)^{0.6} = (2^5)^{3/5} = 2^3 = 8$

2. **Answer: C**

   Solution

   $720 = (2 \times 2 \times 2 \times 2) \times (3 \times 3) \times 5.$

   So, $\log 720 = \log (2^4 \times 3^2 \times 5)$

   $= \log 2^4 + \log 3^2 + \log 5$

   $= 4 \log 2 + 2 \log 3 + \log 5.$

3. **Answer: C**

   Solution

   Given expression $= \left(\frac{log4}{log6} \times \frac{log1296}{log4}\right)$

   $= \frac{log1296}{log6} = \frac{loglog\ 6^4}{loglog\ 6} = \frac{4loglog\ 6}{log6} = 4$

4. **Answer: D**

   Solution

   $\log_2 10 = 1/\log_{10} 2 = 1/0.3010 = 1000/301$

5. **Answer: D**

   Solution

   $\log (2^{64}) = 64 \times \log 2 = (64 \times 0.30103) = 19.26592.$

   Its characteristic is 19. Hence, the number of digits in 2^{64} is 20.

6. **Answer: B**

   Solution

   A. Since $\log_a a = 1$, so $\log_{10} 10 = 1$.

   B. $\log (2 + 3) = \log 5$ and $\log (2 \times 3) = \log 6 = \log 2 + \log 3$

   $\therefore \log (2 + 3) \neq \log (2 \times 3)$

   C. Since $\log_a 1 = 0$, so $\log_{10} 1 = 0$.

   D. $\log (1 + 2 + 3) = \log 6 = \log (1 \times 2 \times 3) = \log 1 + \log 2 + \log 3$

7. **Answer: C**

   Solution

   $\log_{27} x + \log_3 x = 4$

   $\log_{3^3} x + \log_3 x = 4$

   $\frac{1}{3}\log_3 x + \log_3 x = 4$

   $\frac{4}{3}\log_3 x = 4$

   $\frac{1}{3}\log_3 x = 1$

   $\log_3 x = 3$

   $x = 3^3 = 27$

8. **Answer: C**

   Solution

   Let $\log_8 512 = x$. Then, $8^x = 512 = 8^3 \Rightarrow x = 3$.

   Let $\log_2 (\log_8 512) = y$.

   Then $\log_2 3 = y$.

   or $2^y = 3 \Rightarrow y = \log_2 3$

   $\therefore \log_2(\log_8 512) = \log_2 3.$

9. **Answer: D**

   Solution

   Let $\log_6 (1 / 216) = n$ Then, $6^n = (1 / 216)$

   $\Rightarrow 6^n = 6^{-3} \Rightarrow n = -3.$

   $\therefore \log_6 (1 / 216) = -3$

10. **Answer: C**

    Solution

    Let $\log_3 243 = n$, then, $3^n = 3^5 \Rightarrow n = 5.$

    $\therefore \log_2 243 = 5.$

11. **Answer: E**

    Solution

    $\log 2^7 \Rightarrow 7 \log 2 = 7 \times 0.3010 \Rightarrow 2.107 = 2.1.$

12. **Answer: A**

    Solution

    $\log_6 36 + \log_6 6 = x \Rightarrow \log_6 (36 \times 6) = x.$

    $\therefore x = \log_6 (216) = \log_6 (6^3) = 3 \log_6 6 = 3 \times 1 = 3.$

13. **Answer: E**

    Solution

    $\log_{2401} x = -(1 / 4)$ then,

    $x = (2401)^{-1/4} \Rightarrow (7^4)^{(-1/4)} = 1 / 7$

14. **Answer: A**

    Solution

    $\log_{10} 25 \Rightarrow \log_{10} (100 / 4) = \log_{10} 100 - \log_{10} 4$

    $\Rightarrow 2 - \log_{10} 4$

    $2 - \log_{10} (2)^2 \Rightarrow 2 - 2(\log_{10} 2)$

    $\Rightarrow 2 - 2(0.3010) \Rightarrow 2(1 - 0.3010) = 1.398.$

15. **Answer: B**

    Solution

    $\log_4 x = -2 \Rightarrow x = 4^{-2} = (1 / 4^2) = 1 / 16$

16. **Answer: B**

    Solution

    We know that: $a^{\log_a x} = x$

    $\therefore 25^{\log_5 4} = = 5^{2 \log_5 4} = 5^{\log_5 16} = 16$

17. **Answer: C**

    Solution

    We know that $\log_a b = \log b / \log a$.

    $\therefore \log_4 5.\log_{25} 64 = (\log 5/\log 4) \times (\log 64/\log 25)$

    $= (\log 5 / \log 4) \times (\log 4^3 / \log 5^2)$

    $= (\log 5 / \log 4) \times (3 \log 4 / 2 \log 5) = 3/2.$

18. **Answer: E**

    Solution

$\log_{25} 125 = m \Leftrightarrow 25^m = 125 \Leftrightarrow 5^{2m} = 5^3 \Leftrightarrow 2m = 3.$
$\Rightarrow m = 3/2$
And, $\log_{125} 25 = n \Leftrightarrow (125)^n = 25 \Leftrightarrow 5^{3n} = 5^2 \Leftrightarrow 3n = 2$
$\Rightarrow n = 2 / 3$
$\therefore \log_{25} 125 - \log_{125} 25 = 3/2 - 2/3 = (9-4)/6$
$\Rightarrow 5/6.$

19. **Answer: D**
Solution
$[\log_{10} (3.33 \log_{10} 1000)]^2 = [\log_{10} (10/3 \log_{10} 10^3)]^2$
$= [\log_{10} (10 \log_{10} 10)]^2$
$= [\log_{10} 10)]^2 = 1^2 = 1.$ ($\because \log_{10} 10 = 1$)

20. **Answer: C**
Solution
$1 / 2 \log_{10} 36 - 2\log_{10} 3 + \log_{10} 15$
$= \log_{10} 36^{1/2} - \log_{10} 3^2 + \log_{10} 15$
$= \log_{10} 6 - \log_{10} 9 + \log_{10} 15$
$= \log_{10} (6 \times 15) / 9 = \log_{10} (90 / 9) = \log_{10} 10 = 1$

21. **Answer: B**
Solution
Let $\log_{10} 0.00001 = x$
$\Rightarrow x = \log_{10} (10)^{-5}$
$\Rightarrow \log_{10} 1 - \log_{10} (10)^5 = 0 - 5 = -5.$

22. **Answer: B**
Solution
$\{\log_{14} (10)\} / \{\log_{196} (10)\} = \log_{14} (10) / \log_{14^2}(10)$
{($\therefore \log_a b$ C. = (1 / b) $\log_a$ C)
$\{(\log_{13} 10) / (1 / 2) \log_{14} 10\} = \{1 / (1 /2)\} = 2$

23. **Answer: A**
Solution
$\log_{100} 0.001 = \log_{100} (1 / 1000)$
$= \log_{100} 1 - \log_{10^2} 1000$
$= 0 - (1 / 2) \log_{10} 10^3 = -(3/2) \times \log_{10} 10 = -3/2$

24. **Answer: A**
Solution
$2 \log (6 / 16) + \log (512 / 216) + \log (6 / 2)$
$= \log (6 / 16)^2 + \log (512 / 216) + \log (6 / 2)$
$= \log (6^2 \times 512 \times 6) / (16^2 \times 216 \times 2)^2 = \log (6^2 \times 2^9 \times 6) / (2^4)^2 \times 5^3 \times 2)$
$= \log \{(2^9 \times 6^3) / (2^8 \times 6^3 \times 2)\}^2 = \log (2^9 \times 6^3) / (2^9 \times 6^3)$
$= \log 1 = 0.$

25. **Answer: B**
Solution
Take 9^{x-1} as the common term. The equation then reduces to
$9^{x-1} (9^2 - 1) = 90$
$9^{x-1} = 90$
Thus, x -1 = 0 or x = 1.

26. **Answer: A**
Solution
$\log_2 (2/3) + (\log_2 (9/4) / \log_2 4)$
$= \log_2 (2/3) + 1/2 \log_2 (3/2)^2$
$= \log_2 (2/3) + 1/2 (2 \log_2 3/2)$
$= \log_2 2/3 + \log_2 3/2 = \log_2 1 = 0$

27. **Answer: E**
Solution
$1/\log_3 2 + 2/\log_9 4 - 3/\log_{27} 8$
Converting to base 2.
we get, $\log_2 3 + 2\log_{2^2} 9 - 3\log_{2^3} 27 = \log_2 3 + (4 \log_2 3)/2 - (9 \log_2 3)/3 = 3\log_2 3 - 3\log_2 3 = 0$

28. **Answer: A**
Solution
$\log 56700 = \log(5.67 \times 10000)$
$= \log 5.67 + \log 10000 = \log 5.67 + 4$

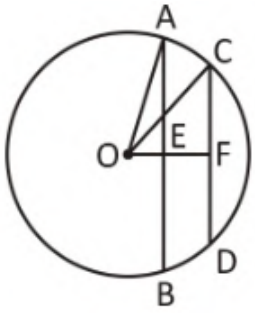
29. **Answer: B**
Solution
$\log 0.0487 = \log (4.87/100) = \log 4.87 - \log 100 = \log 4.87 - 2$

30. **Answer: B**
Solution
$\log_{10} 250 = \log_{10}(1000/4) = \log_{10} 1000 - 2\log_{10} 2$
$3 - 2 \times 0.301 = 2.398$

GEOMETRY SOLUTIONS

1. **Answer: C**

Solution



AB = 48 cm and CD = 14 cm

∴ AE = EB = 24 cm and CF = FD = 7 cm

Radii, OA = OC = 25 cm

In ΔAOE, By Pythagoras theorem

$OE = \sqrt{(OA^2 - AE^2)}$

$= \sqrt{(625 - 576)} = \sqrt{49} = 7cm$

Again, In ΔCOF, By Pythagoras theorem

$OF = \sqrt{(OC^2 - CF^2)}$

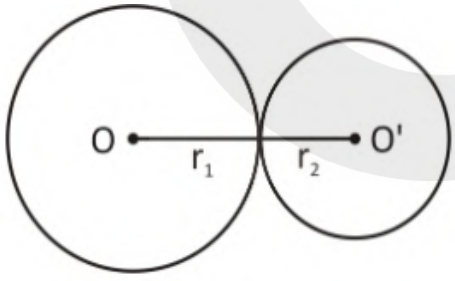
$= \sqrt{(625 - 49)} = \sqrt{576} = 24$ cm

Distance between chords, EF = OF – OE = 24 – 7 = 17 cm

Hence option C.

2. **Answer: D**

Solution



Radius of one circle = diameter/2 = 16/2 = r_1 = 8cm

OO' = 15 cm

$r_1 + r_2 = 15$

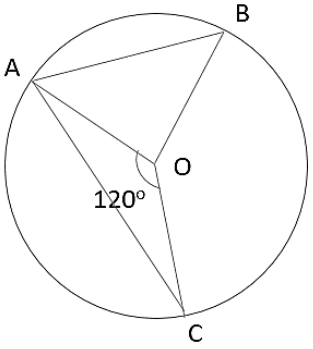
$8 + r_2 = 15$

$r_2 = 15 - 8 = 7$ cm

Hence option D.

3. **Answer: B**

Solution



We know that,

∠BOA + ∠AOC + ∠BOC = 360°

90° + 120° + ∠BOC = 360°

∠BOC = 360° – 210° = 150°

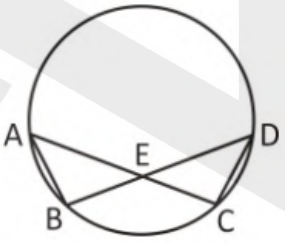
Note: The angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

∴ ∠BAC = 1 / 2 ∠BOC = 1 / 2 × 150= 75°

Hence option B.

4. **Answer: A**

Solution



We know that, Exterior angle is equal to the sum of two interior opposite angles.

∴ ∠BEC = ∠EDC + ∠ECD

140 = ∠EDC + 30°

∠EDC = 110°

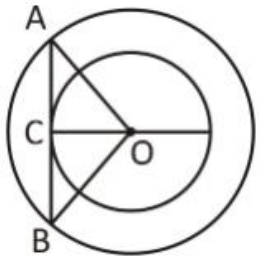
∴ ∠BAC = ∠EDC = 110°

[∵ Angles on the same arc]

Hence option A.

5. **Answer: A**

Solution



OC = 4 cm and OA = 6 cm

In ΔAOC, By Pythagoras theorem,

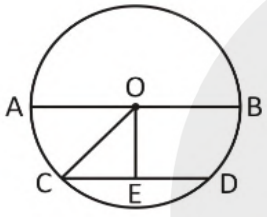
$AC = \sqrt{(OA^2 - OC^2)} = \sqrt{(6^2 - 4^2)} = \sqrt{20}$ cm $= 2\sqrt{5}$ cm

$\therefore AB = 2 \times AC = 2 \times 2\sqrt{5} = 4\sqrt{5}$ cm

Hence option A.

6. **Answer: C**

Solution



Diameter, AB = 30 cm

$\therefore$ Radius, AO = OC = 15 cm

Chord, CD = 24 cm

$\therefore$ CE = ED = 12 cm

In ΔCOE, By Pythagoras theorem

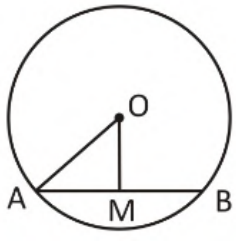
$OE = \sqrt{(OC^2 - CE^2)} = \sqrt{(15^2 - 12^2)} = 9$ cm

Hence option C.

7. **Answer: C**

Solution

Solution



AB = 30 cm

$\therefore$ AM = MB = 15 cm

OM = $\sqrt{64}$ cm = 8 cm

In ΔAOM, By Pythagoras theorem

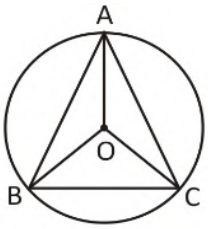
$\therefore$ Radius OA

$= \sqrt{(OM^2 + AM^2)} = \sqrt{(8^2 + 15^2)} = \sqrt{289} = 17$ cm

Hence option C.

8. **Answer: B**

Solution



OA = OB = OC = Circum-radius

In ΔABC, we know that

$\angle ABC + \angle BCA + \angle BAC = 180°$

$\angle BAC = 180° - 80° - 40° = 60°$

Note: The angle subtended by an arc of a circle at the center is double the angle subtended by it at any point on the remaining part of the circle.

$\therefore \angle BOC = 2 \times \angle BAC = 2 \times 60° = 120°$

Hence option B.

9. **Answer: B**

Solution

Let the number of sides of a polygon be n. Then,

Sum of interior angles = $(2n - 4) \times 90°$

Sum of exterior angles = 360°

$\therefore (2n - 4) \times 90° = 3 \times 360°$

$2n - 4 = 12$

$2n = 16$

$n = 8$

The number of sides of the polygon is 8.

Hence option B.

10. **Answer: D**

Solution

Let the number of sides of a regular polygon be n. Then,

$180º - (360º / n) = 140º$

$180° \times n - 360° = 140 \times n$

$40° \times n = 360° \rightarrow \therefore n = 9$

The number of sides of the polygon is 9.

Hence option D.

11. **Answer: A**

Solution

If the number of sides of a regular polygon be n. Then,

Sum of interior angles = 1260°

$(2n - 4) \times 90° = 1260°$

$2n - 4 = 14$

2n = 18

n = 9

The number of sides of the polygon is 9.

Hence option A.

12. **Answer: C**

Solution

Sum of exterior angles of a regular polygon = 360°

But, (360º / 100º) = 3.6 ≠ a whole number

Clearly, the angle 100° doesn't completely divide 360°which means we can't get a total of 360°by adding 100°to n times.

On the other hand, the rest of the angles satisfy the condition.

For instance:

40° + 40° + ... + 9 times = 360°

90° + 90° + ... + 4 times = 360°

72° + 72° + ... + 5 times = 360°

45° + 45° + ····· + 8 times = 360°

Therefore, it's clear that 100° can't be an angle of a regular polygon.

Hence, option C is correct.

13. **Answer: A**

Solution

Let the number of sides of a regular polygon be n.

Then,

180º - (360º / n) = 4 × (360º / n)

180°= 5 × (360^0/n)

180°× n = 1800°

n = 10

Hence, option A is correct.

14. **Answer: D**

Solution

Let number of sides of a regular polygon = n

∴ 180°– (360° / n) – (360° / n) = 144°

180°× n - 720° = 144° × n

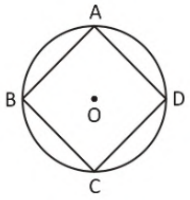
36°× n = 720°

n = 20

Hence, option D is correct.

15. **Answer: C**

Solution



The sum of opposite angles of a concyclic quadrilateral is 180°.

∴ ∠A + ∠C = 180°

6x + 3y = 180° ...(i)

∠B + ∠D = 180°

5x + 4y = 180° ...(ii)

As the RHS is equal on both sides we can write,

6x + 3y = 5x + 4y

x = y

∴ x:y = 1:1

Hence option C.

16. **Answer: A**

Solution

∠BAC = 65°

∠ACB = 90° [∵ Angle of semi-circle]

In ΔABC, we know that

∠ABC = 180 – 90– 65° = 25°

∴ ∠ABC + ∠ADC = 180°

∠ADC = 180 – ∠ABC = 180 – 25= 155°

Hence, option A is correct.

17. **Answer: C**

Solution

Let the number of sides of a polygon be n. Then,

180º – (360º / n) – (360º / n) = 160º

180°× n – 720°= 160°× n

20°× n = 720°

n = 36

Hence, option C is correct.

18. **Answer: B**

Solution

Sum of the interior angles of a regular polygon of n sides = (2n – 4) × 90°

∴ (2n – 4) × 90° = 1440°

2n – 4 = 16

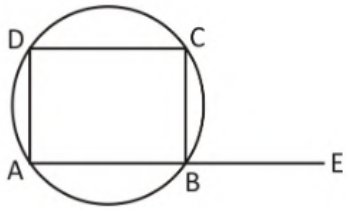
2n = 20

n = 10

Hence, option B is correct.

19. **Answer: A**

Solution



∠ABC + ∠ADC = 180°

∠CBE = 70°

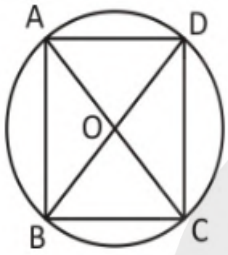
∴∠ABC = 180°– ∠CBE = 180°– 70°= 110°

∴∠ADC = 180°– ∠ABC = 180°– 110° = 70°

Hence, option A is correct.

20. **Answer: A**

Solution



The angle subtended at the centre by an arc is twice to that of angle subtended at the circumference.

∴ ∠CAD = (1 / 2) ∠COD = 60º

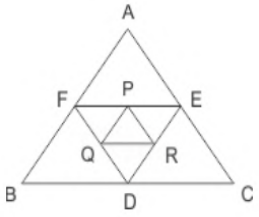
∴∠BAD = ∠BAC + ∠CAD = 50°+ 60° = 110°

∴∠BCD = 180°– ∠BAD = 180° – 110°= 70°

Hence, option A is correct.

21. **Answer: C**

Solution



Given that,

D, E and F are midpoints of BC, CA and AB and P, Q and R are midpoints of EF, FD and DE

we know that,

Area of ΔABC = 4 ΔDEF

But area of ABC = 128 sq. cm.

4*ΔDEF = 128

⇒ ΔDEF = 128 / 4

= 32 sq. units

And area ΔDEF = 4 ΔPQR

⇒ 4 ΔPQR = 32 = 32 / 4 = 8 sq. units.

Hence, option C is correct.

22. **Answer: C**

Solution

Let the number of sides of two regular polygons be x and 2x respectively. Then,

{180º – (360º / x)} : { 180º – (360º / 2x) = 4:5

{180º (x – 2) / x} × {x / 180º (x – 1)} = 4 / 5

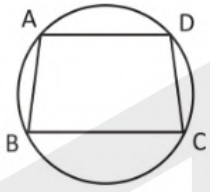
5x – 10 = 4x – 4 → ∴ x = 6

The number of sides of these polygons is (6,12) respectively.

Hence option C.

23. **Answer: C**

Solution



∠ABC + ∠CDA = 180°

∠CDA = 180°– ∠ABC = 180°– 80° = 100°

We know that,

∠BCD + ∠CDA = 180°

∴ ∠BCD = 180° – ∠CDA = 180° – 100° = 80°

Hence, option C is correct.

24. **Answer: B**

Solution

∴ Sum of the angles of quadrilateral = 5x + 4x + 3x + 6x = 360°

or, 18x = 360°

∴ x = 20

The largest angle of the quadrilateral = 6 × 20° = 120°

∴ Smallest angle of the triangle = 120°/4 = 30°

If the 2nd largest angle of the triangle be y°, then the largest angle of the triangle = (y + 40)°

⇒ y° + (y + 40)° + 30° = 180°

∴ 2y = 110°

∴ y = 55°

Hence, Option B is correct

25. **Answer: C**

Solution

CX = AX = 12 / 2 = 6 cm

And BX = (√3 / 2) × 12 = 6√3 cm

BY and AC are two chords of circle

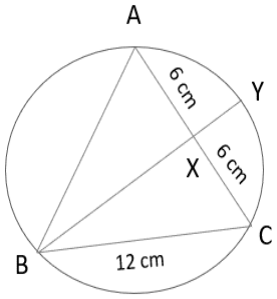
Therefore BX × XY = CX × AX

XY = (6 × 6) / 6√3 = (6 / √3) cm

BY = BX + XY = 6√3 + (6 / √3) = 24 / √3 cm

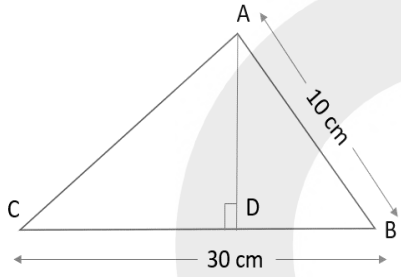
Therefore, 2 × BX × BY = 2 × 6√3 × (24 / √3) = 228 cm^2

Hence option C.



26. **Answer**: C

Solution



Let AD be altitude of ΔABC.

Area of ΔABC = 120 = (1 / 2) × BC × AD

⇒ 120 = (1 / 2) × 30 × AD

⇒ AD = 8 cm

As we know, AB = 10 cm and AD = 8 cm

Now, by the Pythagorean theorem, we get

∴ In ΔABC, BD = 6 cm

∴ CD = BC – BD = (30 – 6) = 24 cm

In ΔACD,

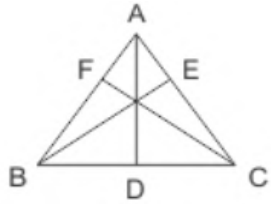
$AC^2 = CD^2 + AD^2 = 24^2 + 8^2 = 640$.

∴ AC = 8√10 cm

Hence, option C is correct.

27. **Answer: B**

Solution



In ΔABC, a = BC = 10 cm, b = AC = 8 cm and c = AB = 8 cm

Let AD, BE and CF are the medians of ΔABC.

In geometry, Apollonius' theorem is a theorem relating the length of a median of a triangle to the lengths of its side. It states that "the sum of the squares of any two sides of any triangle equals twice the square on half the third side, together with twice the square on the median bisecting the third side".

∴ $AB^2 + AC^2 = 2(AD^2 + BD^2)$

∴ $(8)^2 + (8)^2 = 2(AD^2 + 52)$

∴ $128 = 2(AD^2 + 25)$

∴ AD = √39 cm

Using $AB^2 + BC^2 = 2(BE^2 + AE^2)$, we get

BE = √66 cm

Similarly, using $AC^2 + BC^2 = 2(CF^2 + AF^2)$, we get

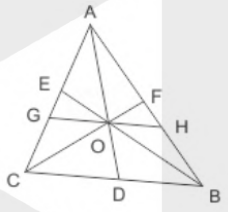
CF = √66 cm

Thus, AD + BE + CF = (√39 + 2√66) cm

Hence, option B is correct.

28. **Answer: B**

Solution



We use the property that medians of a triangle divide each other in the ratio 2 : 1.

∴ AO : OD = BO : OE = CO : OF = 2 : 1

Now, consider ΔAOG and ΔADC. We can see that they are similar by the AAA property.

∴ Their sides are in the same ratio.

Specifically, AO : AD = 2 : 3 = AG : AC

∴ AG/AC = 2/3

∴ AG = 2/3 × AC

∴ AG = 2/3 × (AG + GC) = 2/3 × (AG + 20)

∴ AG = 40 cm

∴ AC = AG + GC = 40+20 cm = 60 cm

Hence, option B is correct.

29. **Answer: C**

Solution

Let the sides of the triangle be a, b, c and circumradius be R.

Geometric mean of a, b and c = $abc^{1/3} = 4$

∴ $abc = 4^3 = 64$

Area of circumcircle = $\pi R^2 = 36\pi$

∴ Circumradius = R = 6

Now, area of triangle = (abc/4R) = 64 / (4 × 6) = 8/3 sq. units

Hence, option C is correct.

30. **Answer: A**

Solution

Let the other two sides be x and y.

Given, x + y = 71 cm

and, $61^2 = x^2 + y^2$ [By Pythagoras theorem]

$(x + y)^2 = x^2 + y^2 + 2xy$

$\Rightarrow 71^2 = 61^2 + 2xy$

$\Rightarrow 5041 = 3721 + 2xy$

$\Rightarrow 2xy = 5041 - 3721 = 1320$

$(x - y)^2 = x^2 + y^2 - 2xy$

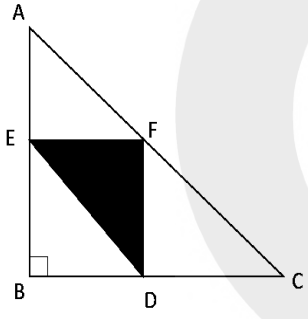
$\Rightarrow (x - y)^2 = 61^2 - 1320 = 3721 - 1320 = 2401$

$\Rightarrow x - y = 49$ cm

Hence option A.

31. **Answer: D**

Solution



Sides are 5, 12 and 13 cm.

Triangle ABC is a right angled triangle where ∠B = 90°.

Therefore, area of the triangle

= (1 / 2) × base × height

= (1 / 2) × 5 × 12 = 30 cm^2

Applying the midpoint theorem which says that if we join the mid points of the sides of a triangle, the 4 triangles thus made will be equal in areas.

∴ The area of the triangle DEF

= (1 / 4) × area of triangle ABC

So, Area of ΔDEF

= (1 / 4) × Area of Triangle ABC

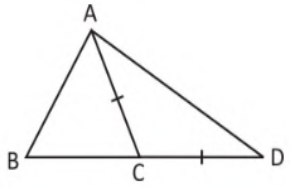
= (1 / 4) × 30 cm^2

= 7.5 cm^2

Hence, option D is correct.

32. **Answer: D**

Solution



We know that, Exterior angle is sum of opposite interior angles

∴ ∠CAD + ∠ADC = ∠ACB = 68°

2∠ADC = 68° [∵ AC = DC ⇒ ∠CAD = ∠ADC]

∠ADC = 34°

In ΔABD, we know that

∠ABC = 180° – ∠BAD – ∠ADB = 180° – 112° – 34° = 34°

Hence, option D is correct.

33. **Answer: B**

Solution

We know that, the sum of the angles of a triangle is 180°

∠A + ∠B + ∠C = 180°

∴ ∠B = 180°– 150° = 30° [∵ ∠A + ∠C = 120°]

It is also given,

∠A + 3∠B = 180°

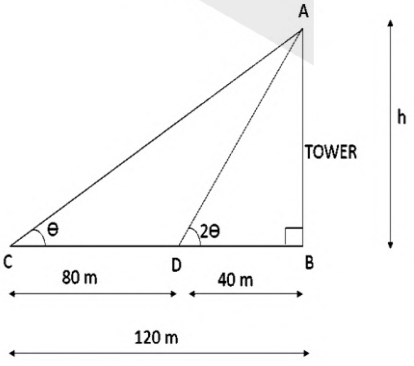
⇒ ∠A + 3 × 30° = 180°

⇒ ∠A = 180° – 90° = 90°

Hence, option B is correct.

34. **Answer: B**

Solution



Given, BC = 120 m and CD = 80 m

∴ BD = BC – CD = 120 – 80 = 40 m

Let, the height of the tower, AB = h metre

And, ∠ACB = θ and ∠ADB = 2θ

In ΔABC,

tan θ = AB / BC = h / 120 ⋯(i)

Now, in ΔABD,

$\tan 2\Theta = (AB / BD)$

$2\tan\Theta / (1 - \tan^2\Theta) = h / 40$ [$\because \tan 2\Theta = 2\tan\Theta / (1 - \tan^2\Theta)$]

$80 \tan \Theta = h(1 - \tan^2\Theta)$

$80 \times (h / 120) = h \{1 - (h / 120)^2\}$ ⋯ [From eqn.(i)]

$2 / 3 = 1 - (h / 120)^2$

$(h / 120)^2 = 1 - (2 / 3) = 1 / 3$

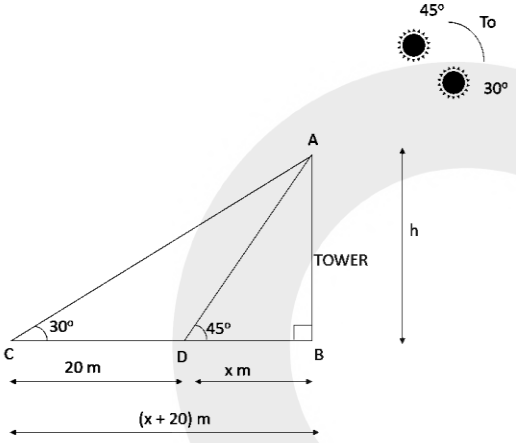
$h / 120 = \sqrt{} (1 / 3) = 1 / \sqrt{3}$

$h = 120 / \sqrt{3}$ m

Hence, option B is correct.

35. **Answer: C**

Solution



Let, the height of the pillar, AB = h metre.

When the sun's angle of elevation was 45°, then the length of shadow of the pillar is BD = x m.

And, when the sun's angle of elevation is 30°, then the length of shadow of the pillar is BC.

When the sun changes from 45° to 30°, then the length of shadow of the pillar increases CD = 20 (given)

BC = CD + BD = (20 + x) m

In ΔABD,

$\tan 45^\circ = AB / BD$

$1 = h / x$

$h = x$ ⋯(i)

Now, in ΔABC,

$\tan 30^\circ = AB / BC$

$1 / \sqrt{3} = h / (x + 20)$

$h\sqrt{3} - x = 20$

$h\sqrt{3} - h = 20$ [From eq. (i)]

$h (\sqrt{3} - 1) = 20$

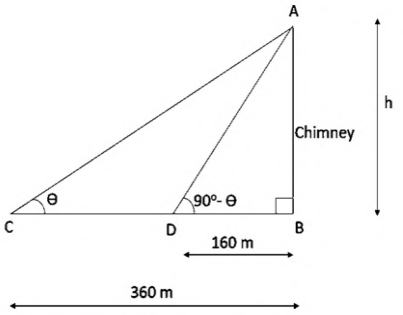
$h = 20 / (\sqrt{3} - 1) \times \{(\sqrt{3} + 1) / (\sqrt{3} + 1)\}$

$= 10 (\sqrt{3} + 1)$ m

Hence, option C is correct.

36. **Answer: B**

Solution



Given, distance BD = 160 m and BC = 360 m

Let, the height of the chimney = h metre

∠ACB and ∠ADB are complementary.

∴ ∠ACB = Θand ∠ADB = (90 – Θ)

In ΔABC,

$\tan \Theta = AB / BC = h / 360$

Now, in ΔABD,

$\tan (90^\circ - \Theta) = AB / BD$

$\cot \Theta = h / 160$ .....(ii) [$\because \tan (90^\circ - \Theta) = \cot \Theta$]

By multiplying both equations (i) and (ii),

$\tan \Theta \cot \Theta = h^2 / 57600$

$1 = h^2 / 57600$ [$\because \tan \Theta \cot \Theta = 1$]

$h = \sqrt{57600} = 240$ m

Hence, option B is correct.

37. **Answer: B**

Solution

From above given figure,

We can prove that AXCY is a parallelogram.

Similarly, BXDY is a parallelogram.

Now, AXCY is a parallelogram.

⇒ AY || CX

[∴ Opposite sides of a parallelogram are parallel]

⇒ PY || QX ⋯ (1)

Also, BXDY is a parallelogram

⇒ DX || BY [∴ Opposite sides of a parallelogram are parallel]

⇒ PX || QY ⋯(2)

Thus, in a quadrilateral PXQY, from (i) and (ii), we have, PY || QX and PX || QY

⇒ PXQY is a parallelogram.

Hence, option B is correct.

38. **Answer: C**

Solution

Given that, The ratio of number of sides of two regular polygons = 2 : 3

and the ratio of interior angles of two regular polygons = 6 : 7

Each interior angle of a regular polygon of n sides = $\frac{2n-4}{n}$ right angles

Let the number of sides be 2y and 3y respectively.

According to question,

$\therefore \frac{\frac{2\times 2y-4}{2y}}{\frac{2\times 3y-4}{3y}} = \frac{6}{7}$

$\Rightarrow \frac{4y-4}{12y-8} = \frac{2}{7}$

$\Rightarrow \frac{y-1}{3y-2} = \frac{2}{7}$

$\Rightarrow 7y - 7 = 6y - 4$

$\Rightarrow y = 7 - 4 = 3$

$\therefore$ Number of sides = $2y = 2 \times 3 = 6$ and $3y = 3 \times 3 = 9$.

Hence, option C is correct.

39. **Answer: C**

Solution

From given figure, we can see that

$CC_1 = 1$, $OC_1 = 1 + r$

$OC = AC - AO = CD - AO = 2 - r$ [AC and CD are the radii of the bigger circle]

$\therefore (OC_1)^2 = (CC_1)^2 + (OC)^2$

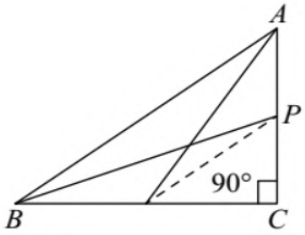
$\Rightarrow (1 + r)^2 = 1^2 + (2 - r)^2$

$\Rightarrow 1 + r^2 + 2r = 1 + 4 + r^2 - 4r \Rightarrow 2r + 4r = 4 \Rightarrow 6r = 4$

$\Rightarrow r = 2/3$Hence, option C is correct.

40. **Answer: A**

Solution



From given figure, we have

$AQ^2 = AC^2 + QC^2$

$BP^2 = BC^2 + CP^2$

$AQ^2 + BP^2 = (AC^2 + BC^2) + (QC^2 + CP^2) = AB^2 + PQ^2$.

$\therefore AQ^2 + BP^2 = AB^2 + (\frac{1}{2}AB.^2$ $\quad \because PQ = \frac{1}{2}AB$

$AQ^2 + BP^2 = \frac{5}{4} AB^2 \Rightarrow 4(AQ^2 + BP^2) = 5AB^2$

Hence, option A is correct.

MENSURATION SOLUTIONS

1. **Answer: B**

Solution

If a is side of the new cube, then

$A^3 = (3^3+4^3+5^3)$

$=27+64+125$

$= 216 = 6^3$

Therefore, a = 6 cm

Surface area of the new cube

$=6a^2=6\times6^2=216cm^2$.

2. **Answer: C**

Solution

Let the edge = a cm

So increase by 100% = a + a = 2a

Total surface Area of original cube = 6a2

TSA of new cube = $6(2a)^2 = 24a^2$

Increase in area = $24a^2–6a^2 = 18a^2$

Increase % = $18a^2/6a^2\times100 = 300\%$

3. **Answer: B**

Solution

Total volume of water displaced =(6 × 40) m^3 = 240 m^3

Rise in water level = (240/50×20) = 0.24m = 24cm.

4. **Answer: A**

Solution

Clearly, we have r = 6 cm and h = 8 cm.

Volume = $(\frac{1}{3}) \times\pi r^2h = \pi\times6^2\times8/3 = 96\pi cm^3$

5. **Answer: E**

Solution

Volume of the block = (6 × 9 × 12) cm^3 = 648 cm^3

Side of the largest cube = H.C.F. of 6 cm, 9 cm, 12 cm = 3 cm.

Volume of this cube = (3 × 3 × 3) cu.cm = 27 cu.cm

Number of cubes = 648/27 = 24.

6. **Answer: D**

Solution

Area of the wet surface = [2(lb + bh + lh) – lb]

= 2(bh + lh) + lb

= [2 (4 × 1.25 + 8 × 1.25) + 8 × 4]m^2 = 62 m^2

7. **Answer: D**

Solution

2(10×6) = 2(10+6)×h

h = 60/16 = 15/4 m

This, Volume = $(10\times6\times15/4)m^3 = 225m^3$.

8. **Answer: E**

Solution

No. of bricks = volume of the wall/volume of 1 brick

= (800 × 400 × 205) / (25 × 10.25 × 8) = 32000.

9. **Answer: C**

Solution

Number of cubes=(20 × 20 × 20) / (4 × 4 × 4) = 125.

10. **Answer: B**

Solution

Let the edge of the cube be a.

Then, $6a^2 = 1536$

→ a = 16 cm.

Volume = $a^3 = 16^3$ = 4096 cu.cm

11. **Answer: A**

Solution

$l^2+b^2 = (diagonal)^2 = 97$

Also, lb=12

$(l+b)^2 = l^2+b^2+2lb = 97 + 24 = 121$

(l + b) = 11.

Perimeter = 2(l + b) = 22 cm.

12. **Answer: E**

Solution

Given that,

Volume of cube = $343x^3$

$\Rightarrow (side)^3 = (7x)^3$

$\Rightarrow$ side = 7x

$\therefore$ Diameter of sphere = Side of cube = 7x

and, radius of sphere (r) = 7x/2

Now, the surface area of sphere = $4\pi r^2 = 4\pi \times (7x/2)^2$ = $49\pi x^2$.

13. **Answer: E**

Solution

Given that, diameter of big sphere = 16 cm

$\therefore$ radius of big sphere (R) = 8 cm

Let radius of each small sphere = r

We know that,

Volume of each small sphere = Volume of big sphere/Number of small sphere

$\Rightarrow 4/3 \times \pi r^3 = 4\pi R^3/3 \div 64$

$\Rightarrow r^3 = (8)^3/64 = 2$

$\Rightarrow$ r = 2 cm

Now, the surface area of each small sphere = $4\pi r^2$ = $4\pi(2)^2 = 16\pi$ cm^2.

14. **Answer: A**

Solution

Given that, side of cuboid (a) = 3 cm

And, length of cuboid (l) = 1 cm, breadth of cuboid (b) = 2 cm and height of cuboid (h) = 4 cm

Now, Total surface area of both cube and cuboid = surface area of cube + surface area of cuboid

$= 6a^2 + 2(lb + bh + hl)$

$= 6 \times (3)^2 + 2(1\times2 + 2\times4 + 4\times1)$

$= 54 + 28 = 82\ cm^2$

Hence, both cube and cuboid can be painted.

15. **Answer: C**

Solution

Sphere fits exactly in the cube.

∴ Side of cube = diameter of sphere

∴ Radius of sphere = 6

∴ Radius of the sphere whose volume is required = $6\sqrt{2}$

Volume = $4/3 \times \pi \times 2\sqrt{2} \times 216$

$= 8\pi\times \sqrt{2} \times 72 = 576\ \pi\sqrt{2}$

16. **Answer: A**

Solution

Let the breadth of the wall be x meters.

Then, Height = 10x meters and Length = 20x meters.

$x \times 10x \times 20x = 25$

$\Rightarrow x^3 = 25/200 = 1/8$

$\Rightarrow x = 1/2$ m

$\Rightarrow x = (1/2) \times 100 = 50$ cm.

17. **Answer: D**

Solution

Volume of the wall = $(3600\times 800 \times 60)$ cu.cm

Volume of bricks = 90% of the volume of the wall = $[(90/100) \times 3600 \times 800 \times 60]$ cu.cm

Volume of 1 brick = $(24\times 16 \times 10)$ cu.cm

Number of bricks = $[(90/100) \times (3600 \times 800 \times 60)\ ]/(24 \times 16 \times 10) = 40500$.

18. **Answer: A**

Solution

Volume of the metal used in the box = External Volume – Internal Volume = $[(50 \times 40 \times 23) - (44 \times 34 \times 20)]$ cu.cm = 16080 cu.cm

Weight of the metal = $[(16080 \times 1.5)/1000]$ kg = 24.12 kg.

19. **Answer: E**

Solution

Volume of the large cube = $(1^3+6^3+8^3) = 729$ cu.cm.

Let the edge of the large cube be a.

So, $a^3 = 729 \rightarrow a = 9$ cm.

Required ratio = $6(1^2+6^2+8^2)/6(9^2) = 101/81$

20. **Answer: E**

Solution

External radius $R = 5$cm (since diameter = 10 cm)

Internal radius $r = 4$cm (since thickness = 1 cm)

Length $h = 18$cm

Volume of iron:

$$V = \pi h(R^2 - r^2) = \frac{22}{7} \times 18 \times (5^2 - 4^2)$$
$$= \frac{22}{7} \times 18 \times (25 - 16)$$
$$= \frac{22}{7} \times 18 \times 9 = \frac{22}{7} \times 162$$
$$= \frac{3564}{7} = 509.142857 \text{ cu.cm}$$

Weight = Volume × Density $= 509.14 \times 6 = 3054.86$g $= 3.05486$kg

This is not among the options.

21. **Answer: D**

Solution

Height = $\sqrt[3]{2744} = 14$ cm

h = 14 cm, r = 7 cm.

So, l = $\sqrt{[7^2 + 14^2]} = \sqrt{245} = 7\sqrt{5}$cm

Total surface area = $\pi rl+\pi r^2$

$= 22/7 \times 7 \times 7\sqrt{5} + 22/7 \times 7 \times 7$

$= 154(\sqrt{5}+1)$

$= (3.236 \times 154)$ sq.cm = 498.35 sq.cm

22. **Answer: A**

Solution

Given length of iron rod = 14 mts and diameter of rod = 2 cms

$\Rightarrow$ r = 1/100 mts

Therefore, Volume of one rod = $\pi r^2 h = (22/7 \times [1/100]^2 \times 14) = 11/2500\ m^3$

Given volume of iron = 1.87 cu. m

Therefore, Number of rods = $(1.87 \times 2500/11) = 425$.

23. **Answer: B**

Solution

1 hectare = 10,000 sq. m

So, Area = (1.8×10000) sq. m

Depth = 4/100 m = 1/25 m

Therefore, Volume = Area × Depth = $18000 \times (1/25) = 720$ cu. m

24. **Answer: E**

Solution

Let the radius of the sphere and cylinder be 'r'.

So, volume of sphere = Volume of cylinder Volume of sphere = $4/3\ \pi r^3$

$4/3\ \pi r^3 = \pi r^3 h$

$h = 4/3\ r$

TSA of sphere = $4\pi r^2$

TSA of cylinder = $2\pi r\ (r + h)$

$= 2\pi r\ (r + 4/3\ r)$

$= 14/3\ \pi r^2$

Required ratio = $(14/3\ \pi r^2)/4\pi r^2 = 7:6$.

25. Answer: B

Solution

Let side of square = x cm

Side of another square = (x + 5) cm

$\therefore x^2 + (x + 5)^2 = 325$

$x^2 + x^2 + 25 + 10x = 325$

$2x^2 + 10x - 300 = 0$

$x^2 + 5x - 150 = 0$

$x^2 + 15x - 10x - 150 = 0$

$x(x + 15) - 10\ (x + 15) = 0$

$(x - 10)(x + 15) = 0$

$\therefore x = 10$ (-15 not possible)

$\therefore$ side of one square = 10 cm

side of another square = 15 cm.

26. Answer: A

Solution

Let length and breadth of a rectangle be 'l cm' and 'b cm' respectively.

ATQ,

$l \times b = 480$ ---(i)

And, $2(l + b) = 92$

$l + b = 46$

$l = 46 - b$ ---(ii)

On solving (i) & (ii), we get:

b = 16 [$\because$ length is always greater than breadth]

& l = 30

Now, circumscribing circle of maximum possible area can only be drawn when diameter of the circle is equal to the diagonal of the rectangle.

So, radius of the circle = $\frac{1}{2} \times (\sqrt{l^2 + b^2})$

$= \frac{1}{2} \times \sqrt{1156}$

= 17 cm

So, circumference of the circle = 2 × 22/7 × 17 = 748/7 cm = 106 6/7 cm.

27. Answer: C

Solution

Volume of the given solid sphere = $4/3 \times \pi r^3$

$= 4/3 \times \pi \times 6 \times 6 \times 6$

$= 288\ \pi\ cm^3$

Now, Volume of new sphere = $4/3\ \pi r^3$

$= 4/3 \times \pi \times 3 \times 3 \times 3$

$= 36\ \pi\ cm^3$

$\therefore$ Remaining volume = $288\ \pi - 36\ \pi = 252\pi$

Now, volume of the cylinder = $\pi r^2 H$

$\Rightarrow \pi \times 3 \times 3 \times H = 252\ \pi$

$\therefore H = 252/9 = 28$ cm

28. Answer: E

Solution

Let the length of the base be 2x and the width of the base be 3x. Then, the surface area of the prism can be expressed as:

Surface area = $2(2x)(6) + 2(3x)(6) + 2(2x)(3x) = 24x + 36x + 12x^2 = 12x^2 + 60x$

Given, surface area = 432 cm^2

$12x^2 + 60x = 432$

$x^2 + 5x - 36 = 0$

$(x + 9)(x - 4) = 0$

Since x cannot be negative, x = 4.

Therefore, the length of the base is 2(4) = 8 cm and the width of the base is 3(4) = 12 cm.

The dimensions of the rectangular prism are: length = 8 cm, width = 12 cm, and height = 6 cm.

Ratio = 4:6:3.

29. Answer: D

Solution

Volume required in the tank = (200 × 150 × 2) cu.m = 60000 cu.m

Length of water column flown in1 min =(20 × 1000)/60 m =1000/3 m

Volume flown per minute = 1.5 × 1.25 × (1000/3) cu.m= 625 cu.m.

Required time = (60000/625)min = 96min

30. Answer: D

Solution

Let radius & height of conical tent be 'r cm' & 'h cm' respectively.

So, radius of hemispherical bowl = r cm.

ATQ,

$\frac{1}{3}\ \pi r^2 h / 3\pi r^2 = 7/1$

⇒h/3×3 = 7/1

h = 63 cm

Now,

Radius of conical tent = $\sqrt{65^2 - 63^2}$

= $\sqrt{256}$ = 16 cm

Required area = 4 × $(16)^2$

= 1024 cm^2.

31. Answer: E

Solution

Let radius of cylinder = r cm

And, let height of cylinder = h cm

ATQ –

2r = ⅔ (r + h)

3r = r + h

h = 2r

Curved surface area of cylinder = 2πrh

And, volume of cylinder = $\pi r^2 h$

$2\pi rh/\pi r^2 h$ = 1/8

r = 16 cm

h = 32 cm

So, l = $\sqrt{r^2 + h^2}$

l = $\sqrt{256 + 1024}$

l = $\sqrt{1280}$

l = $\sqrt{4 \times 4 \times 4 \times 4 \times 5}$

l = $16\sqrt{5}$ cm.

32. Answer: B

Solution

ATQ, radius of sphere is R cm and of hemisphere is r cm

$4\pi R^2/3\pi r^2$ = 3/1

R^2/r^2 = 9/4

⟹ R/r = 3/2

Given,

$R^2 + r^2$ = 42

If R = 3x cm and r = 2x cm

Then $9x^2 + 4x^2$ = 42

So, X=2

R = 6 cm and r = 4 cm

Required volume (of sphere) = 4/3 πR^3 = 4/3 $\pi(6)^3$ = 288π cm^3

33. Answer: B

Solution

Let radius be r cm

176 = 2 × 22/7r

⟹ r = 28 cm ⇒ l = 56 cm

Let length, breadth of rectangle be l, b cm respectively

Square is attached along breadth of rectangle, edge of square = b cm

Increase in area = area of square

b^2 = 169 ⇒ b = 13 cm

Area of rectangle = lb = 56 × 13 = 728 cm^2.

34. Answer: E

Solution

Let radius of circular vessel = r cm

ATQ –

2πr – 2r = 90

15r/7 = 45

r = 21 cm

Total volume of circular vessel = πr^2 = 22/7 × 21 × 21 = 1386 cm^2

Total useful material = 1386 – 161 = 1225 cm^2

Volume of one square box = 1225/25 = 49 cm^2

So, side of one square box = a^2 = 49

a = 7 cm

35. Answer: B

Solution

Let radius & height of the cone be '9r' & '10r' respectively.

So, radius of hemispherical base = 9r

ATQ,

10r + 9r = 38

19r = 38

r = 2 cm

Required volume = ⅓ $\pi(9r)^2$ × (10r) + ⅔ $\pi(9r)^3$

= ⅓ $\pi(9r)^2$ [10r + 18r]

= ⅓ $\pi 81r^2$ × 28r

= 1/3 × 22/7 × 81 × 2 × 2 × 28 × 2

= 19008 cm^3

TRIGONOMETRY SOLUTIONS

1. **Answer: B**

   Solution

   When 0° < q < 90°, both sine (sin q) and cosine (cos q) functions are positive

   Since both sin q and cos q are positive, their sum, sin q + cos q, will be greater than 1

   Therefore, the correct option is "greater than 1"

2. **Answer: D**

   Solution

   cot 30° = cot (90° – 60°) = tan 60°

   cot 75° = cot (90° – 15°) = tan 15°

   $= \frac{\tan 60° + \tan 15°}{\tan 15° - \tan 60°} = -1$

3. **Answer: D**

   Solution

   Cos q + sec q = 2

   Cos q +1/cos q = 2

   $\cos^2 q - 2\cos q + 1 = 0$

   (cos q – 1)2 = 0

   Cos q = 1

   Sec q = 1

   $\cos^6 q + \sec^6 q = 1 + 1 = 2$

4. **Answer: C**

   Solution

   tan 7q . tan 2q = 1

   tan 7q = $\frac{1}{\tan 2q} = \cot 2q$

   $\tan 7q = \tan(90° - 2q)$

   $7q = 90° - 2q$

   $9q = 90°$

   $q = 10°$

5. **Answer: A**

   Solution

   $\frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} = \frac{5}{3}$

   $= 5\sec\theta - 5\tan\theta = 3\sec\theta + 3\tan\theta$

   $= 2\sec\theta = 8\tan\theta$

   $= \frac{\tan\theta}{\sec\theta} = \frac{2}{8} = \frac{1}{4}$

   $= \sin\theta = \frac{1}{4}$

6. **Answer: A**

   Solution

   $\tan q + \cot q = 2$

   On squaring both sides,

   $\tan^2 q + \cot^2 q + 2\tan q \cot q = 4$

   $\tan^2 q + \cot^2 q = 4 - 2 = 2$

7. **Answer: B**

   Solution

   $\frac{1}{1+\tan^2\theta} + \frac{1}{1+\cot^2\theta}$

   $\frac{1}{\sec^2 q} + \frac{1}{cosec^2 q}$

   $= \sin^2 q + \cos^2 q = 1$

8. **Answer: D**

   Solution

   $r\sin q = 1$

   $r\cos q = \sqrt{3}$

   $= \frac{\sin q}{\cos q} = \frac{1}{\sqrt{3}}$

   $= \sqrt{3}\tan q + 1$

   $= \sqrt{3} \times \frac{1}{\sqrt{3}} + 1 = 2$

9. **Answer: A**

   Solution

   $\tan q + \cot q = 2$

   $\tan q + \frac{1}{\tan q} = 2$

   $= \frac{\tan^2 q + 1}{\tan q} = 2$

   $= \tan^2 q + 1 = 2\tan q$

   $= \tan^2 q - 2\tan q + 1 = 0$

   $= (\tan q - 1)^2 = 0$

   $= \tan q - 1 = 0$

   $\tan q = 1$

   Hence, $q = 45°$

10. **Answer: B**

    Solution

    $\frac{AB}{BC} = \frac{2}{1}$

    $= AB = 2k, BC = k$

    $AC = \sqrt{(2k)^2 + k^2} = \sqrt{5k^2}$

    $= \sqrt{5}k$

    $\sin A + \cot C = \frac{BC}{AC} + \frac{BC}{AB}$

    $= \frac{k}{\sqrt{5}k} + \frac{k}{2k}$

    $= \frac{1}{\sqrt{5}} + \frac{1}{2} = \frac{2+\sqrt{5}}{2\sqrt{5}}$

11. **Answer: D**

    Solution

    When q = 60°

$cosq = \frac{1}{2}, \cos^2 q = \frac{1}{4}$

$cosq > \cos^2 q$

12. **Answer: B**

Solution

$tanx = sin45°.cos45° + \sin 30°$

$\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}} + \frac{1}{2} = \frac{1}{2} + \frac{1}{2} = 1$

$= \tan x = \tan 45°$

$= x = 45°$

13. **Answer: B**

Solution

$DE = 36 - 24 = 12\,metre$

From $\Delta\, ADE$,

$sin\,60° = \frac{DE}{AD}$

$\frac{\sqrt{3}}{2} = \frac{12}{AD}$

$AD = 12 \times \frac{2}{\sqrt{3}} = 8\sqrt{3}$ metre

14. **Answer: A**

Solution

The value of tan(45°) is 1.

In trigonometry, the tangent function (tan) is defined as the ratio of the length of

the opposite side to the length of the adjacent side in a right triangle.

In a 45-45-90 degree triangle, the two legs (sides adjacent to the 45° angles) are

congruent, and the ratio of the length of the opposite side to the length of the

adjacent side is 1. Therefore, the value of tan(45°) is 1.

15. **Answer: E**

Solution

To find the value of $\cos^2 60° + \cos^2 45°$, we can directly substitute the values and

simplify:

$\cos^2 60° = (1/2)^2 = 1/4$

$\cos^2 45° = (\sqrt{2}/2)^2 = 2/4 = 1/2$

$\cos^2 60° + \cos^2 45° = 1/4 + 1/2 = 3/4$

Therefore, $\cos^2 60° + \cos^2 45° = 3/4$.

16. **Answer: D**

Solution

We know that tan(60°) = √3 and cot(30°) = √3.

∴ tan(60°) + cot(30°) = √3 + √3 = 2√3.

17. **Answer: E**

Solution

In the first quadrant, sin(x) is positive.

Using the Pythagorean identity, sin^2(x) + cos^2(x) = 1, we can find sin(x):

sin^2(x) + (1/2)^2 = 1

sin^2(x) + 1/4 = 1

sin^2(x) = 3/4

sin(x) = √(3/4) = √3/2

∴ sin(x) = √3/2.

18. **Answer: D**

Solution

We know that sec(45°) = √2 and

sec(60°) = 2.

∴ sec(45°) + sec(60°) = √2 + 2.

19. **Answer: C**

Solution

In the second quadrant, cos(x) is negative.

Using the Pythagorean identity,

$\sin^2(x) + \cos^2(x) = 1$, we can find cos(x):

$(3/5)^2 + \cos^2(x) = 1$

$9/25 + \cos^2(x) = 1$

$\cos^2(x) = 16/25$

cos(x) = -4/5 (taking the negative value)

∴ cos(x) = -4/5.

20. **Answer: E**

Solution

We know that tan(π/3) = √3 and sin(π/6) = 1/2.

Therefore, tan(π/3) + sin(π/6) = √3 + 1/2.

21. **Answer: E**

Solution

In the third quadrant, sin(x) is negative.

Using the Pythagorean identity, sin^2(x) + cos^2(x) = 1, we can find sin(x):

sin^2(x) + (3/5)^2 = 1

sin^2(x) + 9/25 = 1

sin^2(x) = 16/25

sin(x) = -4/5 (taking the negative value)

Therefore, sin(x) = -4/5.

22. **Answer: D**

Solution

$\sin^2 q + \cos^2 q + sec^2 q + cosec^2 q + \tan^2 q + \cot^2 q$

$= 1 + \sec^2 q - \tan^2 q + cosec^2 q - \cot^2 q + 2(\tan^2 q + \cot^2 q)$

$= 3 + 2((tanq - cotq)^2 + 2) > 7$

$[(tanq - cotq)^2 > 0]$

23. **Answer: A**

Solution

$\frac{4}{1+\tan^2 \alpha} + \frac{3}{1+\cot^2 \alpha} + 3\sin^2 \alpha$

$= \frac{4}{\sec^2 \alpha} + \frac{1}{cosec^2 \alpha} + 3\sin^2 \alpha$

$= 4\cos^2 \alpha + \sin^2 \alpha + 3\sin^2 \alpha$

$= 4(\cos^2 \alpha + \sin^2 \alpha) = 4$

24. **Answer:** A

Solution

$7\sin^2 q + 3\cos^2 q = 4$

On dividing both sides by $\cos^2 q$

$7\tan^2 q + 3 = 4\sec^2 q$

$7\tan^2 q + 3 = 4(1 + \tan^2 q)$

$7\tan^2 q + 3 = 4 + 4\tan^2 q$

$7\tan^2 q - 4\tan^2 q = 4 - 3$

$3\tan^2 q = 1$

$\tan^2 q = \frac{1}{3}$

$tanq = \frac{1}{\sqrt{3}}$

25. **Answer: A**

Solution

$sin A + \sin^2 A = 1$

$sin A = 1 - \sin^2 A = \cos^2 A$

$\cos^2 A + \cos^4 A$

$= \cos^2 A + (\cos^2 A)^2$

$= \cos^2 A + \sin^2 A = 1$

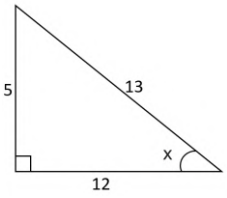
26. **Answer: E**

Solution

In the second quadrant, sec(x) is negative.

Since tan(x) = opposite/adjacent = 5/12

∴ By Pythagorean triplet, the hypotenuse will be 13.



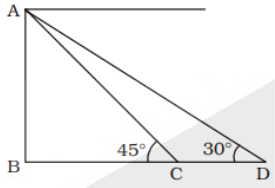
We know, sec(x) = hypotenuse / adjacent = 13/12

But as sec x is negative in the 2nd quadrant, sec(x) = -13/12.

∴ sec(x) = -13/12.

27. **Answer: A**

Solution



AB = Height of light-post = h metre

CD = 200 metre;

C and D are positions of ships angle ACB = 45°; angle ADB = 30°

In $\Delta ABC, tan\ 45° = \frac{AB}{BC}$

$1 = \frac{AB}{BC}$

AB = BC = h metre

In triangle ABD, $tan\ 30° = \frac{AB}{BD}$

$\frac{1}{\sqrt{3}} = \frac{h}{h+200}$

$\sqrt{3}h - h = 200$

$h(\sqrt{3} - 1) = 200$

$h = \frac{200}{\sqrt{3}-1} = \frac{200(\sqrt{3}+1)}{2}$

= 100 (1.73 + 1) metre = 273 metre

28. **Answer: A**

Solution

$\cos q + \sin q = m$

$\sec q + cosec\ q = n$

$= \frac{1}{\cos q} + \frac{1}{\sin q} = n$

$\frac{\sin q + \cos q}{\sin q . \cos q} = n$

$n(m^2 - 1) = \frac{\sin q + \cos q}{\sin q . \cos q}[(\sin q + \cos q)^2 - 1]$

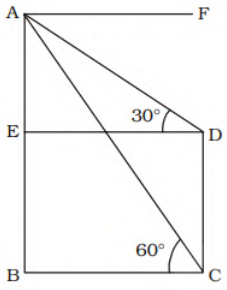
$= \frac{\sin q + \cos q}{\sin q . \cos q} \times 2\sin q . \cos q$

$= 2(\sin q + \cos q)$

$= 2m$

29. **Answer: A**

Solution



AB = Height of building = 60 metre

CD = Height of tower = h metre

Angle FAD = angle ADE = 30°

Angle FAC = Angle ACB = 60°

From Triangle ABC,

$tan60° = \frac{AB}{BC}$

$\sqrt{3} = \frac{60}{BC}$

$BC = \frac{60}{\sqrt{3}} = 20\sqrt{3}\, metre = DE$

From Triangle ADE,

$tan\ 30° = \frac{AE}{ED}$

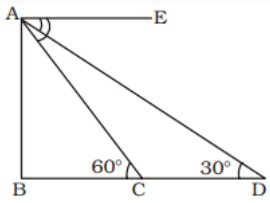
$\frac{1}{\sqrt{3}} = \frac{60-h}{20\sqrt{3}}$

$60 - h = \frac{20\sqrt{3}}{\sqrt{3}} = 20$

h = 60 – 20 = 40 metre

30. **Answer: C**

Solution



AB = Height of observation tower = h metre

C and D = Positions of boat

BC = 50 metre

Let, CD = x metre

Angle ACB = 60° = Angle EAC

Angle ADB = 30° = Angle EAD

In Triangle ABC,

$tan\ 60° = \frac{AB}{BC}$

$\sqrt{3} = \frac{h}{50}$

$h = 50\sqrt{3}\, metre$

In Triangle ABD,

$tan\ 30° = \frac{AB}{BD}$

$1/\sqrt{3} = \frac{50\sqrt{3}}{50+x}$

$50 + x = 50\sqrt{3} \times \sqrt{3} = 150$

x = 150 – 50 = 100 metre

Speed of boat = $\frac{Distance}{Time}$ =

$\left(\frac{100}{8}\right) m/sec = \frac{100}{8} \times \frac{18}{5} kmph$ = 45 km/hr

CO-ORDINATE GEOMETRY SOLUTIONS

1. **Answer: D**

   Solution

   slope = $\frac{y_2-y_1}{x_2-x_1} = \frac{3+5}{-5-x} = \frac{8}{-5-x} = \frac{-4}{3}$

   → 24 = 20 + 4x

   → x = 1

2. **Answer: D**

   Solution

   Slope of line 2x – 3 y = 11 is –2/–3 = 2/3

   Slope of line 3x + ky = 4 is –3/k

   Also, product of slopes of two perpendicular lines is -1

   → 2/3 × –3/k = –1

   → –2k = –1

   → k = –2 × –1= 2

3. **Answer: B**

   Solution

   Slope of the line passing through the points (-5, 1) and (-2, 0) = $\frac{0-1}{-2+5}$ =–1/3

   Let slope of line perpendicular to it = m

   Also, product of slope of two perpendicular lines = -1

   → m × –1/3 = –1

   → m = (–1) × (–3) = 3

4. **Answer: B**

   Solution

   The line 3x + y = –6 will intercept the x-axis at y = 0

   Thus, substituting value of y in above equation

   → 3x + 0 = –6

   → x = –6/3 = –2

   Thus, the line will intercept x axis at (-2, 0)

5. **Answer: C**

   Solution

   Q (a, b) after reflection in the y-axis = (-a, b)

   Reflection of point (-a, b) in the x-axis is (-a, -b)

   According to ques,

   → (–a, –b) = (–6, 2)

   → –a = –6 and –b = 2

   ∴ Coordinates of Point Q = (6, –2)

6. **Answer: B**

   Solution

   (x, y) = $(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n})$

   (x, y) = $(\frac{4(7/2)+1(6)}{4+1}, \frac{4(6)+1(1)}{4+1})$

   (x, y) = (4, 5)

7. **Answer: B**

   Solution

   Slope, m = –2/5 and y-intercept, c = 6

   Equation of line = y = mx + c

   → y = 2x/5 + 6

   → y = (–2x+30)/5

   → 5y = 2x+30

   → 2x+5y = 30

8. **Answer: A**

   Solution

   Reflection of point (x, y) in line x = a is (-x+2a, y)

   Now, Reflection of point (-3, 1) in line x = 2

   = [3+2(-2), 1] = (-1, 1)

9. **Answer: C**

   Solution

   Reflection of point (x, y) in line y=a is (x,y+2a)

   Now, Reflection of point (5, –1) in line y=2

   = [5, 1+2(2)] = (5,5)

10. **Answer: C**

    Solution

    slope m = $\frac{y_2-y_1}{x_2-x_1} = \frac{-4+1}{4-5} = 3$

    slope of the parallel line = m = 3.

11. **Answer: A**

    Solution

    Mid-point = $(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}) \Rightarrow \frac{-5+x}{2} = -2 \Rightarrow x =$ –4+5 = 1

12. **Answer: D**

    Solution

    A line which intercept x-axis at (–3, 0) and y-axis at (0, 6)

    → Slope, m = $\frac{6-0}{0+3}$ = 6/3 = 2

    ∴ Equation of line passing through point (x_1, y_1) and having slope m is $(y-y_1) = m(x-x_1)$

    → Equation of line = (y–0) = 2(x+3)

→ y = 2x+6

13. **Answer: A**

Solution

Slope of line ax + by is = –a/b

→ Slope of 2x – 5y = 12 is = $\frac{-2}{-5}$

= $\frac{2}{5}$

14. **Answer: A**

Solution

P (a, b) after reflection at the origin = (–a, –b)

Reflection of point (–a, –b) in the y-axis is (a, –b)

According to question,

→ (a, –b) = (4,3)

→ a = 4 and b = –3

∴ Coordinates of Point P = (4, –3)

15. **Answer: A**

Solution

Reflection of point (x, y) in the line x=a is = (–x+2a, y)

and in line y = b is = (x, –y+2b)

Thus, reflection of the point (5, 2) in the line x = -3 = [((–5) + 2 × (–3)), 2] = (–11, 2)

16. **Answer: A**

Solution

centroid = $(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}) = (\frac{1+4-2}{3}, \frac{-5+0+2}{3})$ = (1, –1)

17. **Answer: C**

Solution

Slope of line: ax–4y = –6 = a/4

According to question,

→ a/4 = –3/2 * 4 = –6

→ a=–3/2 × 4 = –6

18. **Answer: A**

Solution

Slope of line passing through points (2, 1) and (6, 3)

= $\frac{3-1}{6-2} = \frac{2}{4} = \frac{1}{2}$

Slope of two parallel lines is always equal.

→ Slope of the line parallel to the line having slope 1/2 = 1/2

19. **Answer: A**

Solution

The equation of the line is: x + 1 = 0

The formulae for finding the image of a point (x_1, y_1) to the line ax + by + c = 0 is given by

$$\frac{(x-x_1)}{a} = \frac{y-y_1}{b} = \frac{-2(ax_1+by_1+c)}{(a^2+b^2)}$$

Here, a= 1, b = 0 and c = 1

So, we have

(x–5)/1 = –2(5+1)/1 and (y+2) = 0

→ y = –2 and x = –7

20. **Answer: A**

Solution

y = mx + c

y = -x/3 + 6

3y = -x + 18

x + 3y = 18

x + 3y = 18

21. **Answer: A**

Solution

slope = m = -a/b = -5/3, c = -3

eqn of perpendicular line is

x = (-1/m)x + c

y = (3/5)x – 3

5y = 3x – 15

3x – 5y = 15

22. **Answer: D**

Solution

The equation of the line is: y + 2 = 0

The formulae for finding the image of a point (x_1, y_1) to the line ax + by + c = 0 is given by:

$$\frac{(x-x_1)}{a} = \frac{y-y_1}{b} = \frac{-2(ax_1+by_1+c)}{(a^2+b^2)}$$

Here, a = 0, b = 1 and c = 2

So, we have

(y+3)/1 = 2(–3+2)/1 and (x–4) = 0

→ y = –1 and x = 4

23. **Answer: D**

Solution

Slope of line passing through points (4, -2) and (-3, 5)

$= \frac{5+2}{-3-4} = \frac{7}{-7} = -1$

Slope of two parallel lines is always equal.

→ Slope of the line parallel to the line having slope -1 = –1

24. Answer: B

Solution

Reflection of point (x, y) in line x=a is (-x+2a, y)

Now, Reflection of point (-2, 5) in line x= -1

= [2+2(-1), 5] = (0, 5)

25. **Answer: B**

Solution

The distance from x-axis is equal to its ordinate i.e., 3

26. Answer: C

Solution

Using section formula, the coordinates of point that divides line joining A = (x_1, y_1) and B = (x_2, y_2) in the ratio a : b

= $(\frac{ax_2+bx_1}{a+b}, \frac{ay_2+by_1}{a+b})$

Let the ratio in which the segment joining (-1, -12) and (3, 4) divided by the x-axis = k : 1

Since, the line segment is divided by x-axis, thus y coordinate of the point will be zero, let the point of intersection = (x, 0)

Now, point P (x, 0) divides (-1, -12) and (3, 4) in ratio = k : 1

→ $0 = \frac{(4\times k)+(-12\times 1)}{k+10}$

→ 4k–12 =0

→ k = 12/4 = 3

∴ Required ratio = 3 : 1

27. **Answer: B**

Solution

Slope of line making an angle of 45° with the positive x- axis = tan (45°)

→ Slope, m = 1

y-intercept, c = ¾

Equation of line having slope mmm and y intercept c is: y = mx + c

→ y = x + 3/4

→ 4y = 4x + 3

→ 4x–4y = –3

28. **Answer: B**

Solution

Slope of line 3x - 2y = 6 = $-\frac{3}{-2} = \frac{3}{2}$

Product of slopes of two perpendicular lines = -1

Let slope of line passing through origin = m

→ m × 3/2 = –1

→ m = –2/3

Equation of line passing through origin and having slope m is y = mx (Since y intercept is zero)

→ y = –2x/3

→ 3y = –2x → 2x + 3y = 0

Solving the above equations, we get the intersection point M = (18/13, –12/13)

29. **Answer: C**

Solution

Coordinates of mid-point of line joining points (x_1, y_1) and (x_2, y_2) = $(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2})$

Let coordinates of Point A = (x, y)

P = (3, 1) and B = (5, –4)

→ 3 = (x+5)/2

→ x = 6–5 = 1

Similarly, 1= (y–4)/2

→ y = 2+4 = 6

Thus, coordinates of Point A = (1, 6)

30. **Answer: A**

Solution

When a line intercepts y-axis at a point, then x-coordinate of that point is 0.

Let the line intercepts y-axis at (0, *y*)

Equation of line = 4x–3 = –6

Putting x=0 in above equation, we get:

→ (4×0) – 3y = –6

→ 3y = 6

→ y = 6/3 = 2

∴ The line 4x - 3y = -6 will intercept the y-axis at = (0, 2)

PERMUTATION & COMBINATION SOLUTIONS

1. **Answer: A**

Solution

The word 'DEALING' has 7 different letters.

When the vowels EAI are always together, they can be supposed to form one letter.

Then, we have to arrange the letters DNLG (EAI).

Now, 5 (4 + 1 = 5) letters can be arranged in 5! = 120 ways.

The vowels (EAI) can be arranged among themselves in 3! = 6 ways.

Required number of ways = (120 × 6) = 720.

2. **Answer: C**

Solution

We may have (3 Boys and 2 girls) or (4 Boys and 1 girl) or (5 boys only).

Required number of ways = $({}^7C_3 \times {}^6C_2) + ({}^7C_4 \times {}^6C_1) + ({}^7C_5)$

= 525 + 210 + 21 = 756

3. **Answer: D**

Solution

Since each desired number is divisible by 5, so we must have 5 at the unit place. So, there is 1 way of doing it.

The tens place can now be filled by any of the remaining 5 digits (1, 2, 3, 6, 7). So, there are 5 ways of filling the tens place.

The hundreds place can now be filled by any of the remaining 4 digits. So, there are 4 ways of filling it.

Required number of numbers = (1 × 5 × 4) = 20

4. **Answer: D**

Solution

We may have(1 black and 2 non-black) or (2 black and 1 non-black) or (3 black).

Required number of ways = $({}^3C_1 \times {}^6C_2) + ({}^3C_2 \times {}^6C_1) + ({}^3C_3)$ = 45 + 18 + 1 = 64

5. **Answer: E**

Solution

There are 6 letters in the given word, out of which there are 3 vowels and 3 consonants.

Let us mark these positions as under:

(1) (2) (3) (4) (5) (6)

Now, 3 vowels can be placed at any of the three places out 4, marked 1, 3, 5.

Number of ways of arranging the vowels = ${}^3P_3 = 3! = 6$.

Also, the 3 consonants can be arranged at the remaining 3 positions.

Number of ways of these arrangements = ${}^3P_3 = 3! = 6$.

Total number of ways = (6 × 6) = 36.

6. **Answer: B**

Solution

Required number of ways = $({}^7C_5 \times {}^3C_2) = ({}^7C_2 \times {}^3C_1)$ = 21 × 3 = 63.

7. **Answer: C**

Solution

The word 'TROPICAL' contains 8 different letters.

When the vowels OIA are always together, they can be supposed to form one letter.

Then, we have to arrange the letters TRPCL (OIA).

Now, 6 Letters can be arranged in 6! = 720 ways.

The vowels (OIA) can be arranged among themselves in 3! = 6 ways.

Required number of ways = (720 × 6) = 4320

8. **Answer: E**

Solution

'LOGARITHMS' contains 10 different letters.

Required number of words = Number of arrangements of 10 letters, taking 4 at a time.

= ${}^{10}P_4 = (10 \times 9 \times 8 \times 7 \times 6) = 30240$

9. **Answer: D**

Solution

Numbers between 99 and 1000 are all three-digit numbers.

Total number of 3 digit numbers having at least one of their digits as 6 = (Total numbers of three-digit numbers) – (Total number of 3 digit numbers in which 6 does not appear at all)

= (9 × 10 × 10) – (8 × 9 × 9)

= 900 – 648 = 252

10. **Answer: B**

Solution

The required number of ways

(a) 1 white and 2 others = ${}^4C_1.{}^6C_2 = 4 \times 15 = 60$

(b) 2 white and 1 other = ${}^4C_2.{}^6C_1 = 6 \times 6 = 36$

(c) All the 3 white = ${}^4C_3 = 4$

Total =60 + 36 + 4 = 100

11. **Answer: D**

Solution

In this case, since nothing is mentioned about whether the prizes are distinct or identical we can take the prizes to be distinct. Thus, each prize can be given in 8 ways – a total of 8^6 ways.

12. **Answer: C**

Solution

We need to assume that the 5 Indians are one person, so also for the 6 Dutch and the 7 Pakistanis. These three groups of people can be arranged amongst themselves in 3! ways.

Also, within themselves the 5 Indians, the 6 Dutch and the 7 Pakistanis can be arranged in 7!, 6! and 5! ways respectively. Thus, the answer is 3!7!6!5!

13. **Answer: E**

Solution

For a 4 digit number the thousand place cannot be a 0 leaving us with three options 3, 4 and 5.

Therefore, ways to make a 4 digit number using the digits 0, 3, 4 and 5 with repetition being Allowed = 3 × 4 × 4 × 4 = 192

14. **Answer: B**

Solution

We need to count words starting with P. These words would be represented by

P _ _ _ _ .

The letters ATNA can be arranged in 4! / (2!) ways in the four places, i.e. a total of 12 ways

15. **Answer: A**

Solution

The first digit can only be 2 (one way), the second digit can be filled in seven ways, the third in six ways and the fourth in five ways. A total of 1 × 7 × 6 × 5 = 210 ways.

16. **Answer: D**

Solution

For each digit, there would be 5 options (0, 2, 4, 6, 8). But it cannot start with 0.

Hence, the total number of numbers would be 4 × 5 × 5 × 5 = 500.

17. **Answer: C**

Solution

No. Of ways to choose a Envelope = $^6C_1 = 6$

No. Of ways to choose a stamp = $^5C_1 = 5$

Total ways = 6 × 5 = 30

18. **Answer: D**

Solution

From the first suite we have 13 options leaving us with a single Option for the other three.

Thus, 13 × 1 × 1 × 1.

19. **Answer: A**

Solution

First the six plates can be arranged in 6! Ways. This will leave us with seven places for four cups. Thus, total ways = $6! \times {}^7C_4 \times 4! = 6! \times {}^7P_4$.

20. **Answer: C**

Solution

If the number of players is n, $^nC_2$ should be equal to 55. Trial and error gives us the value as 11

21. **Answer: C**

Solution

For each selection there are three ways of doing it. Thus, there are a total of 3 × 3 × 3 × 3 × 3 = 243.

22. **Answer: B**

Solution

The guys can be seated alternately in 8! Ways and similarly the girls can bee seated in 8! Ways. If before the guys were and seated on odd numbered chair while the girls on even they can also be seated the other way round.

Thus, total number of ways = 2 × 8! × 8! (Note: We do not need to use circular arrangements here because the seats are numbered.)

23. **Answer: B**

Solution

In order to form triangles from the given points, we can either select two points from the first line and one point from the second OR select one point from the first line and two from the second.

This can be done in:

$^9C_1 \times {}^{10}C_2 + {}^9C_2 \times {}^{10}C_1 = 405 + 360 = 765$ ways.

24. **Answer: D**

Solution

First step – arrange six boys around the table according to the circular permutations rule, i.e. in 5! Ways. [Number of ways of arranging n people on a circular arrangement = (n-1)!]

Second step – now, we have seven places and have to arrange six girls on these places. This can be done in $^6P_6$ = 6! ways. Hence, the total number of ways = 5! × 6!

25. **Answer: A**

Solution

A team of 6 members has to be selected from the 10 players. This can be done in $^9C_6$ Or 84 ways.
Now, the captain can be selected from these 6 players in 6 ways.
Therefore, total ways the selection can be made is 84 × 6 = 504

26. **Answer: B**

Solution

The total number of stations = 20
From 20 stations we have to choose any two stations and the direction of travel (i.e., Mumbai to Delhi is different from Delhi to Mumbai) in $^{20}P_2$ ways.
$^{20}P_2$ = 20 × 19 = 380

27. **Answer: E**

Solution

Divide the numbers into three-digit numbers and four-digit numbers.
Number of three-digit numbers = 2 × 3 × 2 = 12.
Number of four-digit numbers starting with 10 = 2 × 1 = 2.
Total = 12+2 = 14.

28. **Answer: B**

Solution

3-digit numbers = 2 × 4 × 4 – 1 = 31 (-1 is because the number 200 cannot be counted);
4-digit numbers starting with 10 = 4 × 4 = 16, Number of 4 digit numbers starting with 11 = 4 × 4 = 16
Total numbers = 31 + 16 + 16 = 63

29. **Answer: E**

Solution

A_ _ _ _ _W/W_ _ _ _ _A. The letters to go into the spaces are C, R, L, I, N, G. Since all of them are distinct the number of ways of arranging them would be 6!.
Total ways = 6! × 2.

30. **Answer: D**

Solution

The digits are 3, 4, 5, 6, 5, 4, 3. In this 7 digit number there are 4 odd places and 3 even places – OEOEOEO. The four odd digits 3, 5, 5, 3 can be arranged in 4 odd places in 4!/2!2! = 6 ways [as 3 and 5 are both occurring twice].
Similarly, the even digits 4, 6, 4 can be arranged in three even places in 3!/2! = 3 ways.
Total no. of ways = 6×3 = 18

31. **Answer: B**

Solution

Since it says not more than 5, the sets possible are of 0, 1, 2, 3, 4 or 5 numbers.
Therefore total no. of ways = $^{15}C_0 + {}^{15}C_1 + {}^{15}C_2 + {}^{15}C_3 + {}^{15}C_4 + {}^{15}C_5$ ($^nC_0 = 1$)
= 1 + 15 + 105 + 455 + 1365 + 3003 = 4944

32. **Answer: C**

Solution

Since there are eleven symmetric letters, the number of passwords that can be formed would be = 11 × 10 × 9 × 8 = 7920.

33. **Answer: E**

Solution

This would be given by the number of passwords having:
1 symmetric and 2 asymmetric letters + 2 symmetric and 1 asymmetric letter + 3 symmetric and 0 asymmetric letters.
$^{11}C_1 \times {}^{15}C_2 \times 3! + {}^{11}C_2 \times {}^{15}C_1 \times 3! + {}^{11}C_3 \times 3!$ = 11×105×6 + 55 × 15 × 6 + 11 × 10 × 9 = 6930 + 4950 + 990 = 12870

34. **Answer: E**

Solution

The 5 letter word can be rearranged in 5!=120 Ways without any of the letters repeating.
The first 24 of these words will start with A.
Then the 25th word will start will CA _ _ _.
The remaining 3 letters can be rearranged in 3!=6 Ways. i.e. 6 words exist that start with CA.
The next word starts with CE _ _ _. In 6 ways as well.
The next word starts with CH and then A, i.e., CHA _ _.
The first of the words will be CHAES. The next word will be CHASE.
Therefore, the rank of CHASE will be 24 + 6 + 6 + 2 = 38.

35. **Answer: C**

Solution

The caps are different; The hooks are identical. If none of the hooks is to remain empty, then we can hang the caps in one of the following ways:

Case a. Number of ways of achieving the first option 2 – 2 – 1

Two caps out of the 5 can be selected in $^5C_2$ ways.

Another 2 out of the remaining 3 can be selected in $^3C_2$ ways and the last cap can be selected in $^1C_1$ way.

However, as the hooks are identical, the two different ways of selecting which hook holds the first two caps and which one holds the second set of two caps will look the same. Hence, we need to divide the result by 2. Therefore, total number of ways of achieving the 2 – 2 – 1 option is ways $^5C_2 \times {}^3C_2 = 15$ ways

Case b. Number of ways of achieving the second option 3 – 1 – 1

Three caps out of the 5 can be selected in $^5C_3$ ways.

As the hooks are identical, the remaining two caps can go into the two identical looking hooks in only one way.

Therefore, total number of ways of getting the 3 – 1 – 1 option is $^5C_3 = 10 = 10$ ways.

Total ways in which the 5 caps can be packed in 3 identical hooks = number of ways of achieving Case a + number of ways of achieving Case b= 15 + 10 = 25 ways.

36. **Answer: A**

Solution

Let the number of sides be n.

The number of diagonals is given by $^nC_2 - n$

Therefore, $^nC_2 - n = 35$, $n>0$

$^nC_2 - n = 35$

$n^2 - 3n - 70 = 0$

$n^2 - 10n + 7n - 70 = 0$

$n(n - 10) + 7(n - 10) = 0$

$n = -7$ or $n = 10$. As $n>0$, n will not be –7. Therefore, $n = 10$

37. **Answer: C**

Solution

The first letter from the left can be chosen in 26 ways because there are 26 alphabets.

Having chosen this, the second letter can be chosen in 26 ways.

Having chosen the first two letters, the third letter can be chosen in 26 ways.

All the three letters can be chosen in 26 x 26 x 26 =17576 ways.

It implies that the maximum possible number of five letter palindromes is 17576 because the fourth letter is the same as the second letter and the fifth letter is the same as the first letter.

38. **Answer: E**

Solution

We are having digits 1, 2, 3, 4 & 5 and numbers greater than 3000 are to be formed, no digit is repeated.

The number can be 4 digits but greater than 3000 or 5 digits.

Number of 4-digit numbers greater than 3000 are 3 or 4 or 5 can occupy a thousand place = 3 x $^4P_3$ = 3 x 24 = 72.

5-digit numbers = $^5P_5 = 5! = 120$

So, the total numbers greater than 3000 = 72 + 120 = 192.

39. **Answer: B**

Solution

The number of arrangements of 4 different digits taken 4 at a time is given by $^4P_4 = 4! = 24$.

All the four digits will occur an equal number of times at each of the positions, namely ones, tens, hundreds, thousands.

Thus, each digit will occur 24/4 = 6 times in each of the position. The sum of digits in one's position will be 6 x (2+4+6+8) = 120. Similar is the case in ten's, hundred's and thousand's places.

Therefore, the sum will be $120 + 120 \times 10 + 120 \times 100 + 120 \times 1000 = 1333320$

40. **Answer: D**

Solution

We first count the number of committee in which

(i). Mr. B is a member

(ii). the ones in which he is not

Case (i): As Mr. B agrees to be in committee only where Mrs. C is a member.

Now we are left with (7-1) men and (4-2) ladies (Mrs. A is not willing to join) = 8 people

We can choose 1 more in $^8C_1 = 8$ ways.

Case (ii): If Mr. B is not a member then we left with (7 + 4 – 1) people.
we can select 3 from 10 in $^{10}C_3$ = 120 ways.
Thus, the total number of ways is 120 + 8 = 128 ways

41. **Answer: A**

Solution

We will consider x = 7 to x = 1
For a = 7, b + c = 5. Number of solutions = 4 [values of b and c being (1,4), (4,1), (2,3), (3,2)]
For a = 6, b + c = 6. Number of solutions = 5 [values of b and c being (2,4), (4,2), (1,5), (5,1), (3,3)]
For a = 5, b + c = 7. Number of solutions = 6 [values of b and c being (3,4), (4,3), (2,5), (5,2), (6,1), (1,6)]
For a = 4, b + c = 8. Number of solutions = 7 [values of b and c being (4,4), (7,1), (1,7), (3,5), (5,3) (2,6), (6,2)]
For a = 3, b + c = 9. Number of solutions = 6 [values of b and c being (5,4), (4,5), (6,3), (3,6), (7,2), (2,7)]
For a =2, b + c = 10. Number of solutions = 5 [values of b and c being (5,5), (6,4), (7,3), (3,7)]
For a = 1, b + c = 11 Number of solutions = 4 [values of b and c being (6,5), (5,6), (4,7), (7,4)]
Hence, number of solutions 37.

42. **Answer: E**

Solution

A chess board consists of 9 parallel lines × 9 parallel lines. For a rectangle, we need to select 2 parallel lines and two other parallel lines that are perpendicular to the first set. (As all squares are rectangle). Hence, $^{9}C_2 \times {}^{9}C_2 = 1296$.

43. **Answer: B**

Solution

10 lakh distinct 3 digit initials are needed.
Let the number of required symbols be 'n'.
Therefore, using 'n' symbols we can form n * n * n = n^3 distinct 3 digit initials.
Note: distinct initials are different from initials where the digits are different.
For instance, AAA and BBB are acceptable combinations in the case of distinct initials while they are not permitted when the digits of the initials need to be different.
This n^3 different initials = 10 lakh
i.e., $n^3=10^6$ (10 lakh = 10^6)
→ n = 10^2 = 100
Hence, there needs to be a minimum of 100 symbols to achieve the objective

44. **Answer: A**

Solution

Let the number of Red T-shirts be 'a'.
Let the number of Orange T-shirts be 'b'.
Let the number of Yellow T-shirts be 'c'.
20a + 5b + c =1000; a + b + c = 100
Solving the above two equations by eliminating c,
19a + 4b = 900
b = (900 – 19a)/4
b = 225 – 19a/4 ⋯(1)
b being the number of T-shirts, is a positive integer, and is less than 99, as each of the other two types have at least one T-shirt in the combination i.e. : 0 < b < 99 ⋯(2)
Substituting (1) in (2),
0 < 225 – 19a/4 < 99
-225 < -19a/4 < (99 -225)
→ 4 × 225 > 19a > 126 × 4 → 900/19 > a > 504
a is the integer between 47 and 27 ⋯(3)
From (1), it is clear, a should be multiple of 4.
Hence possible values of a are (28, 32, 36, 40, 44)
For a=28 and 32, a + b >100
For all other values of a, we get the desired solution:
a=36, b=54, c=10
a=40, b=35, c=25
a=44, b=16, c=40
Three solutions are possible

45. **Answer: C**

Solution

The possible outcomes that satisfy the condition of "at least one flat gets the wrong order" are:
One flat gets the wrong order or two flat get the wrong order or all three flats get the wrong order.
We can calculate each of these cases and then add them together or approach this problem from a different angle.
The only case which is left out of the condition is the case where no wrong orders are delivered.
If we determine the total number of ways the four orders can be delivered and then subtract the one case from it, the remainder will be the three cases above.

There is only one way for no wrong orders delivered to occur. This is the same as everyone gets the right order.
1 × 1 × 1 = 1
Determine the total number of ways the three orders can be delivered.
3 × 2 × 1 = 6
The number of ways at least one house gets the wrong order is:
6 – 1 = 5
Therefore, there are 5 ways for at least one house to get the wrong order.

46. **Answer: C**
Solution
Let Mukesh ride horse A and Suresh B.
Two horses A and B, in a race of 6 horses... A has to finish before B.
if A finishes 1... B could be in any of the other 5 positions in 5 ways and other horses finish in 4! Ways, so total ways 5*4!
if A finishes 2... B could be in any of the last 4 positions in 4 ways. But the other positions could be filled in 4! ways, so the total ways 4*4!
if A finishes 3rd... B could be in any of the last 3 positions in 3 ways, but the other positions could be filled in 4! ways, so total ways 3*4!
if A finishes 4th... B could be in any of the last 2 positions in 2 ways, but the other positions could be filled in 4! ways, so total ways... 2 * 4!
if A finishes 5th... B has to be 6th and the top 4 positions could be filled in 4! ways..
A cannot finish 6th, since he has to be ahead of B
Therefore, total number of ways = 5 × 4! + 4 × 4! + 3 × 4! + 2 × 4! + 4! = 120 + 96 + 72 + 48 + 24 = 360

47. **Answer: E**
Solution
If the word started with the letter K then the remaining 5 positions can be filled in 5! Ways.
If it started with H then the remaining 5 positions can be filled in 5! Ways.
Similarly, if it started with I the remaining 5 positions can be filled in 5! Ways.
If it starts with K then the second letter would be C, then the 4 positions can be filled in 4! Ways.
Similarly, then the second letter would be H, I, N and 4 positions can be filled in 4! ways for each.
Lastly the second letter will be O, with CHIN already in order will be the 1st word With KO as the beginning.
The required word KOCHIN can be obtained after the 3 × 5! + 4 × 4! = 456 Ways i.e., KOCHIN is the 457th word.

48. **Answer: C**
Solution
All the numbers will have 1 at the thousands place.
1 _ _ _.
Now we are left with 7 digits.
Numbers that can be formed = 1×7×6×5 = 210.

49. **Answer: E**
Solution
The smallest number in the series is 1000, a 4-digit number.
The largest number in the series is 3000, the only 4-digit number to start with 3.
The leftmost digit (thousands place) of each of the 4-digit numbers other than 3000 can take one of the 2 values 1 or 2.
The next 3 digits (hundreds, tens and units place) can take any of the 5 values 0 or 1 or 2 or 3 or 4.
Hence, there are 2 x 5 x 5 x 5 or 250 numbers from 1000 to 3000.
Including 3000, there will be 251 such numbers.

50. **Answer: A**
Solution
Take all the socks as 1 and all the skirts as 1.
Total elements = 8
For total elements = 8!
For socks = 2!, For skirts = 2!
Total ways = 8! × 2! × 2! = 161280.

PROBABILITY SOLUTIONS

1. **Answer: C**
Solution
Here, S = {1, 2, 3, 4, ..., 19, 20}.
Let E = event of getting a multiple of 4 or 5 = {4, 5, 8, 10, 12, 15, 16, 20}.
P(E) = n(E)/n(S) = 8/20.
Hence C.

2. **Answer: A**
Solution
Let S be the sample space
Then n(S) = no of ways of drawing 2 balls out of (5+6) = $^{11}C_2$ = 55
Let E = event of getting both balls of same colour
Then, n(E) = no of ways (2 balls out of 5) or (2 balls out of 6) = $^5C_2$+$^6C_2$ = 10+15 = 25
Therefore, P(E) = n(E)/n(S) = 25/55 = 5/11.
Hence A.

3. **Answer: A**
Solution
Let S be the sample space.
Then, n(S) = number of ways of drawing 3 balls out of 15 = $^{15}C_3$ = 455.
Let E = event of getting all the 3 red balls.
n(E) = $^5C_3$ = 10 => P(E) = n(E)/n(S) = 10/455 = 2/91.
Hence A.

4. **Answer: B**
Solution
Total number of outcomes possible,
n(S) = 5 + 15 = 20
Total number of prizes,
n(E) = 5
P(E) = n(E)/n(S) = 5/20 = 1/4
Hence B.

5. **Answer: D**
Solution
Let S be the sample space.
Then, n(S) = $^{52}C_2$ = 1326.
Let E = event of getting 2 queens out of 4.
n(E) = $^4C_2$ = 6.
P(E) = n(E)/n(S) = 6/1326 = 1/221.
Hence D.

6. **Answer: E**
Solution
Total number of balls = (8 + 7 + 6) = 21.
Let E = Event that the ball drawn is neither red nor green = Event that the ball drawn is blue.
n(E) = 8.
P(E) = n(E)/n(S) = 8/20 = 2/5.
Hence E.

7. **Answer: A**
Solution
Clearly, there are 52 cards, out of which there are 12 face cards. (Jack, Queen and King only)
P (getting a face card) = 12/52 = 3/13.
Hence A.

8. **Answer: B**
Solution
Total number of balls = (6 + 8) = 14.
Number of blue balls = 8.
P (drawing a white ball) = 8/14 = 4/7.
Hence B.

9. **Answer: D**
Solution
Let S be the sample space.
Then, n(S) = $^{52}C_2$ =(52 × 51)/(2 × 1) = 1326.
Let E = event of getting 1 club and 1 diamond.
n(E)= number of ways of choosing 1 club out of 13 and 1 diamond out of 13 = $^{13}C_1 \times {}^{13}C_1$ = 169.
P(E) = n(E)/n(S) = 169/1326 = 13/102.
Hence D.

10. **Answer: C**
Solution
Here S = {TTT, TTH, THT, HTT, THH, HTH, HHT, HHH}
Let E = event of getting at most two tails.
Then E = {HHH, TTH, THT, HTT, THH, HTH, HHT}.
P(E) = n(E)/n(S) = 7/8.
Hence C.

11. **Answer: E**
Solution
In two throws of a die, n(S) = (6 × 6) = 36.
Let E = event of getting a sum = {(2, 6), (6, 2), (4, 4), (5, 3), (3, 5)}.
P(E) = n(E)/n(S) = 5/36.
Hence E.

12. **Answer: B**
Solution
Here S= {TTT, TTH, THT, HTT, THH, HTH, HHT, HHH}.

Let E = event of getting at least two tails = {TTT, TTH, HTT, THT}.
P(E) = n(E)/n(S) = 4/8 = 1/2
Hence B.

13. **Answer: E**
Solution
Let, S = sample space
E – event of selecting 1 boy and 2 girls.
Then, n(S) = Number ways of selecting 3 students out of 25 = $^{26}C_3$ = 2600.
n(E) = $^{10}C_2 \times {}^{16}C_1$ = 720.
P(E) = n(E)/n(s) = 720/2600 = 18/65.
Hence E.

14. **Answer: A**
Solution
P(A) = 2/7, P(B) = 1/9.
Let C be the event that both are selected.
P(C) = P(A) × P(B) as A and B are independent events:
= (1/7) × (2/9) = 2/63.
Hence A.

15. **Answer: D**
Solution
P(A') = 5/7, P(B') = 8/9.
Let C be the event that none are selected.
P(C) = P(A) × P(B) as A and B are independent events:
= (5/7) × (8/9) = 40/63.
Hence D.

16. **Answer: B**
Solution
P(A') = 5/7, P(B') = 8/9.
Let C be the event that both are selected or A is selected but not B or B is but not A.
P(C) = 1 – none of them are selected = 1 – 40/63 = 23/63.
Hence B.

17. **Answer: C**
Solution
Required probability:
$^{4}C_1 \times {}^{4}C_1 \times {}^{4}C_1 / {}^{52}C_3$ = 4×4×4/22100 = 16/5525
Hence C.

18. **Answer: A**
Solution
Total cases of checking in the hotels = 5 × 5 × 5 = 125 ways.
Cases when 3 men are checking in different hotels = 5×4×3 = 60 ways.
Required probability = 60/125 = 12/25.
Hence A.

19. **Answer: C**
Solution
Here, n(S) = 52.
Let E = event of getting a queen of the club or a king of heart.
Then, n(E) = 2.
P(E) = n(E)/n(S) = 2/52 = 1/26.
Hence C.

20. **Answer: B**
Solution
Here n(S) = (6 × 6) = 36
Let E = event of getting a total more than 8 = {(3,6), (4,5), (4,6), (5,4), (5,5), (5,6), (6,3), (6,4), (6,5), (6,6)}
Therefore, P(E) = n(E)/n(S) = 10/36 = 5/18.
Hence B.

21. **Answer: D**
Solution
Total number of balls = (3 + 4 + 2) = 9.
Let S be the sample space.
Then, n(S) = Number of ways of drawing 2 balls out of 9 = $^{9}C_2$ = 36.
Let E = Event of drawing 2 balls, none of which is blue.
n(E) = Number of ways of drawing 2 balls out of (3 + 4) balls = $^{7}C_2$ = 21
Therefore, P(E) = n(E)/n(S) = 21/36 = 7/12.
Hence D.

22. **Answer: D**
Solution
P(first letter is not vowel) = 3/8
P(second letter is not vowel) = 2/7
So, the probability that none of the letters would be vowels is = 3/8×2/7 = 3/28.
Hence D.

23. **Answer: C**
Solution
Total balls = 5+6+4 = 15
n(S) = Number of ways of drawing 3 balls = $^{15}C_3$ = 455
n(E) = Ways to get all white balls = $^{5}C_3$ = 10.
P(E) = n(E)/n(S) = 10/455 = 2/91.
Hence C.

24. **Answer: E**

Solution

In a simultaneous throw of two dice, we have n(S) = (6 × 6) = 36. Then,

E= {(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5)}

n(E) = 9.

P(E) = n(E)/n(S) = 9/36 = 1/4.

Hence E.

25. **Answer: E**

Solution

One person can select one car out of 3= $^3C_1$ ways = 3.

Hence, three persons can select one car out of 3 in 3 × 3 × 3 = 27.

Therefore, the probability that all three apply for the same car is 3/27. So, the correct option 1/9

Hence E.

26. **Answer: A**

Solution

Here S = {1,2,3,4,5,6}

Let E be the event of getting the multiple of 2

Then, E = {2, 4, 6}

P(E) = n(E)/n(S) = 3/6 = 1/2.

Hence A.

27. **Answer: B**

Solution

The two events mentioned are independent.

The first roll of the die is independent of the second roll.

Therefore, the probabilities can be directly multiplied.

P(getting first 5) = 1/6P(no second 3) = 5/6

Therefore P(getting first 5 and no second 3) = 1/6 × 5/6 = 5/36.

Hence B.

28. **Answer: A**

Solution

The possible outcomes are as follows:

(5H, 0T),(0H, 5T), (H, 4T), (T, 4H), (2H, 3T) (3H, 2T), i.e. 6 outcomes in all.

Therefore, the probability that the tail appears an odd number of times = 3/6 = 1/2.

Hence A.

29. **Answer: C**

Solution

We can get sum 15 if

Case 1: 6, 6, 3

Total outcome = 3!/2! = 3

Favourable outcome (getting 5 at first throw) = 0

Case 2: 6, 5, 4

Total outcomes = 3! = 6

Favourable outcome (getting 5 at first throw) = 2 (5, 6, 4 or 5, 4, 6)

Case 3: 5, 5, 5

Total outcome = 1

Favourable outcome (getting 4 at first throw) = 1

Probability = 3/10

Hence C.

30. **Answer: B**

Solution

Probability of the three selling all their tickets are 1/6, 1/4 and 3/9

Hence, probability that atleast one of them sell all the tickets is

= 1 – (none of them sell all his tickets)

= 1 – (5/6 × 3/4 × 6/9)

= 1 – 5/12 = 7/12

Hence B.

31. **Answer: D**

Solution

We know that Total probability = 1

Given probability of black stones = ¼

Probability of blue and white stones = 1 – 1/4 = ¾

But, given blue + white stones = 8+10 = 18

Hence,

3/4 → 18

1 → ?

∴ 18 × 4/3 = 24.

Hence, the total number of stones in the box = 24.

Hence D.

32. **Answer: E**

Solution

Given total students in the class = 50

Students who are taking Economics = 14 and

Students who are taking Calculus = 29

Students who are taking both subjects = 50–(14 + 29) = 50 – 43 = 7

Students who are taking calculus only = 29 – 7 = 22

Probability that a randomly chosen student from this group is taking only the Calculus class = 22/50 = 11/25.

Hence E.

33. **Answer: B**

Solution

If a number ends with 5 or 0 then the number will be divisible by 5

So, the last digit will be 5.

$n(E) = {}^4P_3$ (rest 4 numbers selection) × 1,

$n(S) = {}^5P_4$

Required probability = n(E)/n(S) = 4!/5! = 1/5.

Hence B.

34. **Answer: D**

Solution

Total number of 5-digit numbers

= 5! = 120

Now to be multiple of 4, the last 2 digits must be divisible by 4. i.e. they

must be 12, 24, 32, 52. They can be filled in 4 ways.

The other remaining 3 places can be filled with 3! = 6 ways.

Required probability = (6×4)/120 = 1/5.

Hence D.

35. **Answer: D**

Solution

The probability of death by each bullet is 0.5.

This states that the probability of being alive is = 1-0.5 = 0.5.

Also, criminal fires 4 bullets at policeman

So, to stay alive the police must miss all the 4 bullets.

Hence, the probability of police still alive is $(0.5)^4$ = 0.0625

Hence D.

36. **Answer: A**

Solution

Since the two bags are identical, the probability of choosing either bag is 1/2.

Once we have chosen a bag, the probability of drawing a red marble from that bag is 5/10, or 1/2, since there are 5 red marbles and 5 blue marbles in each bag.

Therefore, the overall probability of choosing one of the bags and then drawing a red marble is:

P(red) = P(choose bag 1) × P(draw red from bag 1) + P(choose bag 2) × P(draw red from bag 2)

P(red) = (1/2) × (1/2) + (1/2) × (1/2)

P(red) = 1/2

So, the probability of drawing a red marble is 1/2, regardless of which bag you choose.

Hence A.

37. **Answer: D**

Solution

The event definition would be marble transferred is red and red marble drawn

Or

Marble transferred is yellow and red marble is drawn.

The answer will be given by:

(5/9) × (8/17) + (4/9) × (7/17) = 68/153 = 4/9

Hence D.

38. **Answer: C**

Solution

Required probability = 1 - probability that no pair is selected = $1 - {}^{10}C_4/{}^{20}C_4 = 1 - 224/323 = 99/323$.

Hence C.

39. **Answer: E**

Solution

The total number of ways in which the six identical balls can be distributed amongst the three boxes such that each box can get 0, 1, 2, 3, 4, 5 or 6 balls is given by the formula:

${}^{n+r-1}C_r$, where n is the number of identical balls and r is the number of boxes.

This will give the sample space as ${}^8C_2 = 56$.

So, the probability of getting equal number of balls in all the boxes is = 1 (2 balls in each box)

Hence E.

40. **Answer: A**

Solution

Total number of ways of drawing two notes from the pocket containing 12 notes is ${}^{12}C_2$ ways. The number of ways in which two-hundred-rupee notes can be drawn = ${}^5C_2$

The probability of choosing two-hundred-rupee notes = ${}^5C_2/{}^{12}C_2 = 5/33$

∴ Odds in favor of the event = favorable ways : unfavorable ways = 5 : 28.

Hence A.

41. **Answer: B**

Solution

Two cases:

Case 1: 0 at last place

So, 4 choices for 1st digit, 3 for 2nd, 2 for 3rd and 1 for 4th.
So, numbers = 4 × 3 × 2 × 1 = 24
Case 2: 8 at last place
For 5-digit number 0 cannot be placed at 1st place or cannot be 1st digit.
So, 3 choices (1, 3, 7) for 1st digit, 3 for 2nd, 2 for 3rd and 1 for 4th.
So, numbers = 3 × 3 × 2 × 1 = 18
So, total choices = 24+18 = 42
Also, Number total 5-digit numbers that can be formed from 0, 1, 3, 7, and 8.
0 not allowed at 1st place, so 4 choices for 1st place, 4 for 2nd, 3 for 3rd, 2 for 4th and 1 for 5th. So, total = 4 × 4 × 3 × 2 × 1 = 96
So, required probability = 42/96 = 7/16.
Hence B.

42. **Answer: C**
Solution
4x and x = 5x Red marbles
4+8 = 12 blue marbles
So 5x/(5x+12) = 5/11
On solving we get, x = 2
5 × 2 = 10 Red marbles.
Hence C.

43. **Answer: D**
Solution
As the win leads to multiplying the amount by 1.5 and loss leads to multiplying the amount by 0.5, we will multiply the initial amount by 1.5 thrice and by 0.5 thrice (in any order).
The overall result will remain the same.
So, final amount with the person will be (in all cases):
= 256(1.5)(1.5)(1.5)(0.5)(0.5)(0.5) = 81
Hence, the final result is: 256 – 81 = 175
A loss of Rs.175
Hence D.

44. **Answer: D**
Solution
The radius of the coin is 2 cm.
Since the coin should not cross the edge, the centre of the coin should at least be 2 cm away from the edge of the tile.
If we left 4 cm from all the edges, then the centre can lie in a square of 6 × 6.
So, Probability = Area of square / Area of tile = 36/100 = 0.36
Hence D.

45. **Answer: A**
Solution
Probability of hitting dart board = 1 – 0.25 = 0.75
If the dart hits, then probability of hitting within the circle of radius 2 m = Area of the circle/Area of the board = 4π /16 = π/4
Hence, the resultant probability = 0.75×π/4 = 3π/16.
Hence A.

46. **Answer: B**
Solution
Let's denote the fair die as D1 and the loaded die as D2.
The probability of choosing D1 is 0.5, and the probability of choosing D2 is also 0.5.
If we roll D1, the probability of rolling a 6 is 1/6, since it is a fair die.
If we roll D2, the probability of rolling a 6 is 2/7, since the probability of rolling any other number is 1 – 2/7 = 5/7, and this probability is evenly distributed among the other 5 numbers.
To find the overall probability of rolling a 6, we can use the law of total probability:
P(rolling a 6) = P(rolling a 6 | D1) × P(D1) + P(rolling a 6 | D2) × P(D2)
P(rolling a 6) = (1/6) × 0.5 + (2/7) × 0.5
P(rolling a 6) ≈ 0.226.
Hence B.

47. **Answer: A**
Solution
For divisibility by 5, we need the units' digit to be either 0 or 5.
The units digit in the powers of 7 follow the pattern – 7, 9, 3, 1, 7, 9, 3, 1, 7, 9...
Hence, divide 1 to 100 into four groups of 25 elements each as follows.
A = 1, 5, 9,... →25 elements
B = 2, 6, 10,... →25 elements
C = 3, 7, 11,... →25 elements
D = 4, 8, 12,... →25 elements
So, the combination can be of elements AC or BD.

Therefore, for the first number selected from any of these groups, we have 25 available options for the second element.
Hence, P(E) = 25/100 = 1/4
Hence A.

48. **Answer: C**

Solution

Now, X and Y contradict each other = [X lies and Y true] or [X true and Y lies]
= P(X) × P(Y–lie) + P(X–lie) × P(Y)
= (4/5 × 1/4) + (1/5 × 3/4) = 7/20
= (7/20 × 100) = 35%
Hence C.

49. **Answer: A**

Solution

We know that the sum of two sides in a triangle is greater than the third side.
Therefore, in following cases, the triangle is not formed.
(1,2,3), (1,3,4), (1,4,5), (2,3,5).
Total number of triangles formed using 5 lines = $^5C_3$
Therefore, required probability = $4/^5C_3$ = 4/10 = 2/5
Hence A.

50. **Answer: B**

Solution

In a circle of n different persons, the total number of arrangements possible = (n – 1)!
Total number of arrangements = n(S) = (10 – 1)! = 9!
Taking three persons as a unit, total persons = 8
Therefore no. of ways for these 13 persons to around the circular table = (8 – 1)! = 7!
In any given unit, 3 particular person can sit in 3!.
Hence total number of ways that any three person can sit = n(E) = 7! × 3!
Therefore P (E) = probability of three persons sitting together = n(E) / n(S) = 7! × 3! / 9! = 1/12
Hence B.

Let, X = Event that X speaks the truth
Y = Event that Y speaks the truth
Then P(X) = 80/100 = 4/5
P(Y) = 75/100 = 3/4
P(X–lie) = 1–4/5 = 1/5
P(Y–lie) = 1–3/4 = 1/4

CLOCKS & CALENDARS SOLUTIONS

1. **Answer: C**
   Solution
   Angle traced by hour hand in 13/3 hrs. = ((360/12) × (13/3))° = 130°
   Angle traced by min. hand in 20 min = ((360/60) × 20)° = 120°
   Required angle = (130 – 120)° = 10°.
2. **Answer: C**
   Solution
   Count the number of odd days from the year 2007 onwards to get the sum equal to 0 odd day.

Year	Odd Day
2007	1
2008	2
2009	1
2010	1
2011	1
2012	2
2013	1
2014	1
2015	1
2016	2
2017	1

   Sum = 14 odd days 0 odd days.
   Calendar for the year 2018 will be the same as for the year 2007.
3. **Answer: C**
   Solution
   The century divisible by 400 is a leap year.
   The year 1700 is not a leap year.
4. **Answer: D**
   Solution
   The year 2006 is an ordinary year. So, it has 1 odd day.
   So, the day on 22nd Nov 2007 will be 1 day beyond the day on 22nd Nov 2006.
   But 22nd Nov 2007 is Thursday.
   22 – 8 = 14 days.
   Therefore, 2 weeks ago it is same day.
   Thus, 8th Nov 2006 is Wednesday.
5. **Answer: B**
   Solution
   The year 2007 is an ordinary year. So, it has 1 odd day.
   1st day of the year 2007 was Monday.
   1st day of the year 2008 will be 1 day beyond Monday.
   Hence, it will be Tuesday.
6. **Answer: B**
   Solution
   Since at 5:25 the minute hand will be at 5 and the angle between them will be same as the distance covered in degree by the hour hand in 25 minutes
   Required angle = Distance of hour hand = Speed × Time = 1/2 × 25 = 12.5°
7. **Answer: C**
   Solution
   100 years consists of 24 leap years and 76 non-leap years.
   (100th year is not a leap year if it cannot be divided with 400).
8. **Answer: C**
   Solution
   On 31st December 2005 it was Saturday.
   Number of odd days from the year 2006 to the year 2009 = (1 + 1 + 2 + 1) = 5 days.
   ∴ On 31st December 2009, it was Thursday.
   Thus, on 1st Jan 2010 it is Friday.
9. **Answer: B**
   Solution
   Each day of the week is repeated after 7 days.
   So, after 63 days, it will be Monday.
   ∴ After 61 days, it will be Saturday.
10. **Answer: A**
    Solution
    The year 2004 is a leap year. So, it has 2 odd days.
    But Feb 2004 not included because we are calculating from March 2004 to March 2005. So, it has 1 odd day only.
    ∴ The day on 6th March 2005 will be 1 day beyond the day on 6th March 2004.
    Given that, 6th March, 2005 is Monday.
    ∴ 6th March 2004 is Sunday (1 day before to 6th March 2005).
11. **Answer: C**
    Solution
    The year 2004 is a leap year. It has 2 odd days.

∴ The day on 8th Feb 2004 is 2 days before the day on 8th Feb 2005.
Hence, this day is Sunday.

12. **Answer: C**
Solution
Since the clock gains 1 minute per hour, in 8 hours it will gain 1 minute/hour × 8 hours = 8 minutes.
The correct time after 8 hours would be 10:00 PM + 8 hours = 6:00 AM.
However, due to the clock gaining 8 minutes, it will show 6:00 AM + 8 minutes = 6:08 AM.

13. **Answer: B**
Solution
Since the clock loses 2 minutes per hour, in 6 hours it will lose 2 minutes/hour × 6 hours = 12 minutes.
The correct time after 6 hours would be 12:00 PM + 6 hours = 6:00 PM.
However, due to the clock losing 12 minutes, it will show 6:00 PM – 12 minutes = 5:48 PM

14. **Answer: B**
Solution
Since the clock gains 5 seconds per minute, in 3 hours and 45 minutes it will gain =
5 × (3 h × 60 + 45 minutes) = 1125 seconds = 18 minutes and 45 seconds.
The correct time after 3 hours and 45 minutes would be 8:00 AM + 3 hours and 45 minutes = 11:45 AM.
However, due to the clock gaining 18 minutes and 45 seconds, it will show 11:45 AM + 18 minutes and 45 seconds = 12:04 PM.

15. **Answer: C**
Solution
There are 7 days in a week, so after 59 days, it will be 59 days / 7 = 3 days later.
If today is Monday, after 3 days, it will be Thursday.

16. **Answer: A**
Solution
Since the clock gains 1 minute per hour, in 12 hours and 30 minutes it will gain 1 × 12.5 h = 12 minutes and 30 seconds.
The correct time after 12 hours and 30 minutes would be 6:00 PM + 12 hours and 30 minutes = 6:30 AM.
However, due to the clock gaining 12 mins 30 sec it will show 6:42 AM.

17. **Answer: C**
Solution
Out of the 12 months in a calendar year, the months of January, March, May, July, August, October, and December have exactly 31 days, which is a total of 7 months.
Therefore, the probability of a randomly selected month having exactly 31 days is 7/12.

18. **Answer: D**
Solution
In a non-leap year, there are 365 days, which is equivalent to 52 weeks and 1 day. Since January 1st falls on a Wednesday, the last day of the year will be 365 days later, which is also a Wednesday.

19. **Answer: C**
Solution
If the day before yesterday was Monday, then yesterday was Tuesday and today is Wednesday. The day after tomorrow would be 2 days later, which is Friday.

20. **Answer: C**
Solution
The rule for leap years is that a year is a leap year if it is divisible by 4, except for years that are divisible by 100 but not divisible by 400. This rule ensures that most years divisible by 4 are leap years, but not all years divisible by 100 are leap years, unless they are also divisible by 400.
The year 1800 is divisible by 4, but not divisible by 100, so it is not a leap year. The year 1900 is divisible by 4 and divisible by 100, but not divisible by 400, so it is not a leap year. However, the year 2000 is divisible by 4, divisible by 100, and divisible by 400, so it is a leap year.

21. **Answer: D**
Solution
If the 15th day of the month is a Wednesday, then the 25th day of the same month will be 10 days after the 15th day.
Since there are 7 days in a week, every 7th day falls on the same day of the week. So, 15 + 7 = 22nd of the month falls on Wednesday. ∴ 25th of the same month will fall on Saturday.

22. **Answer: D**
Solution

q = | $\frac{11}{2}$ m – 30h | = | $\frac{11}{2}$ × 30 – 30 × 6 | = | 165 – 180 | = 15°

23. **Answer: D**

Solution

On straight line means 180 degree angle.

180 = $\frac{11}{2}$ m– 30h

180 = $\frac{11}{2}$ m – 30 × 3

180 = $\frac{11}{2}$ m–90

(180+90) × 2 = 11m

m = 540/11 = 49 $\frac{1}{11}$

24. **Answer: B**

Solution

The hour hand moves 30° in one hour, while minute hand moves 360° in one hour.

So, if the hour hand has moved 28°, the minute hand will move $\frac{360 \times 28}{30} = 28 \times 12 = 336°$

25. **Answer: B**

Solution

Coincide means 0° angle.

0 = $\frac{11}{2}$m – 30 × 3

11m = 90 × 2 = 180

M = $\frac{180}{11}$ = 16 $\frac{4}{11}$

So, time = 3: 16 $\frac{4}{11}$

26. **Answer: A**

Solution

15th August 2010 = (2009 years + Period 1.1.2010 to 15.8.2010)

Odd days in 1600 years = 0

Odd days in 400 years = 0

9 years = (2 leap years + 7 ordinary years) = (2 × 2 + 7 × 1) = 11 odd days = 11/7 = 4 odd days.

Jan + Feb + March + April + May + June + July + Aug = (31 + 28 + 31 + 30 + 31 + 30 + 31 + 15) = 227 days

∴ 227 days = (32 weeks + 3 days) = 3 odd days.

Total number of odd days = (0 + 0 + 4 + 3) = 7 0 odd days. Given day is Sunday.

27. **Answer: A**

Solution

Since the clock loses 3 minutes every hour, in 4 hours and 30 minutes it will lose 3 minutes/hour × 4.5 hours = 13.5 minutes. The correct time after 4 hours and 30 minutes would be 9:00 AM + 4 hours and 30 minutes = 1:30 PM. However, due to the clock losing 13.5 minutes, it will show 1:30 PM – 13.5 minutes = 1:16 PM.

28. **Answer: B**

Solution

A calendar month has 30 days, and the month starts from a Monday. ∴ there are 4 weekends i.e., 4 Saturdays and 4 Sundays = 8 days / 30 days. ∴ the probability is 8/30 = 4/15. Therefore, the probability of a randomly selected day falling on a weekend is 4/15.

29. **Answer: B**

Solution

In a non-leap year, there are 365 days, which is equivalent to 52 weeks and 1 day. So, the day of the week on the same date will be shifted by 1 day forward in the following year.

Since March 1st, 2022, falls on a Tuesday, February 28th, 2023, will be 1 day before March 1st, 2022. So, it will fall on a Tuesday.

30. **Answer: B**

Solution

As we know, 1st Jan, 2001 was a Monday.

From 2001 till 2022 there will be 27 odd days (22 odd days for each year + 5 odd days of leap years)

∴ January 1st, 2023 falls on a Sunday.

∴ from 2023 to 2031 there are 10 odd days.

∴ January 1st, 2031 falls on a Wednesday.

∴ December 31st, 2030 falls on a Tuesday.

31. **Answer: A**

Solution

The clock loses 2 minutes every hour. After 5 hours and 30 minutes (5.5 hours), it would lose = 2 × 5.5 hours = 11 minutes

The actual time elapsed would be 5 hours and 30 minutes – 11 minutes = 5 hours 19 minutes

Therefore, the time shown on the clock would be 12:00 PM + 5 hours 19 minutes = 5:19 PM.

32. **Answer: E**

Solution

The clock gains 5 minutes every hour.

The clock shows 5:00 PM after 1.5 hours from 3:30 PM.

∴ in 1.5 hour, the clock gain = 1.5 × 5 = 7.5

So, the actual time when the clock shows 5:00 PM will be 5:00 PM + 7.5 minutes ≈ 5:08 PM.

33. **Answer: A**

Solution

The clock gains 6 minutes every hour. After 5 hours 30 mins (5.5 hours), it would gain 6 × 5.5 = 33 mins, Therefore, the time shown on the clock would be 7:45 AM + 5 hours 30 minutes + 33 minutes = 1:48 PM.

34. **Answer: B**

Solution

1st January, 0001 is a Monday. So 1st January, 0401 will also be a Monday, and so on as the cycle continues every 400 years.

∴ So 1st January, 1601 will also be a Monday.

Now, 1600 years has 0 odd days.

100 years till 1700 has 5 odd days

75 years contain 18 leap years and 57 ordinary years and therefore (36+57) or 93 or 2 odd days.

∴ 1775 years given 0+5+2=7 and so 0 odd days.

Also number of days from 1st Jan.1776 to 16th July,1776

Jan + Feb + March + April + May + June + July

= 31 + 29 + 31 + 30 + 31 + 30 + 16

= 198 days = 28 weeks + 2 days = 2 odd days

∴ Total number of odd days=0+2 = 2

Hence, the day on 16th July, 1776 was a 'Wednesday'.

35. **Answer: B**

Solution

Month	Odd Day
March	3
April	2
May	3
June	2
July	3
August	3
September	2
October	3

Total 21 odd days. 21/7 = 0.

∴ November starts with the same day.

36. **Answer: A**

Solution

We can say that,

During the interval we have two leap years as 1992 and 1996 and it contains February of both these years.

∴ The interval has (5 + 2) = 7 days = 0 odd day.

Hence, January 7, 1997, was also Tuesday.

37. **Answer: C**

Solution

According to question, we can say that.

At 9 O'clock, the minute hand is 9 × 5 = 45 minutes space behind the hour hand.

Hence, the minute hand will have to gain 45 – 30 = 15 minutes.

Therefore, 60 minutes is equal to the gain of 55 minutes spaces.

Hence, gain of 15 minutes spaces equals to

$$= \frac{60}{55} \times 15$$

Gain of 15 minutes spaces equals

$$= \frac{180}{11} = 16\frac{4}{11}$$

∴ Hour hand and minutes hands of a clock point in opposite direction after

9 o' clock at $\frac{4}{11}$ minutes past 9.

38. **Answer: C**

Solution

Since minute hand gains 5 minutes in every 60 minutes

⇒ second hand gains 5 seconds in every 60 seconds

∴ In every 60 seconds true time, it moves 65 seconds.

⇒ 65 × 6° = 390°.

Hence, required answer will be 390°.

39. **Answer: D**

Solution

X weeks + x days

7 days = 1 week

So, 7x days + x days = 8x days.

Hence, 8x is the answer.

40. **Answer: C**

Solution

Total number of odd days = 1600 years have 0 odd day + 300 years have 1 odd day + 49 years (12 leap years + 37 ordinary years) have 5 odd days + 26 days of January have 5 odd days = 0 + 1 + 5 + 5 = 4 odd days.

So, the day was Thursday.

41. **Answer: D**

Solution

We shall find the day on 1st April 2001.

1st April 2001 = (2000 years + Period from 1.1.2001 to 1.4.2001)

Odd days in 1600 years = 0

Odd days in 400 years = 0

Jan + Feb + March + April

= (31 + 28 + 31 + 1) = 91 days ≡ 0 odd days.

Total number of odd days = (0 + 0 + 0) = 0

On 1st April 2001 it was Sunday.

In April 2001 Wednesday falls on 4th, 11th, 18th and 25th.

42. **Answer: E**

Solution

Since the clock gains 1.5 minutes per hour, in 3 hours and 45 minutes (3.75 hours) it will gain = 1.5 × 3.75 = 5.625 minutes.

The correct time is 6:00 PM + 3 hours and 45 minutes + 5.625 minutes = 9:51 PM.

43. **Answer: A**

Solution

The clock is 1/3rd times slower.

So, every hour it loses 20 mins.

∴ in the first 2 hours it loses 40 mins.

In the next 15 mins, it loses another 5 mins i.e., 1/3rd of 15 mins.

Overall losing 45 mins.

∴ The clock after 2 hours 15 mins will show the time as 1:30 pm.

44. **Answer: E**

Solution

A palindrome means the digits are the same when read from left to right and right to left.

∴ there are 16 possible outcomes i.e.,

12:21 am, 01:10 am, 02:20 am, 03:30 am, 04:40 am, 05:50 am, 10:01 am, 11:11 am, 12:21 pm, 01:10 pm, 02:20 pm, 03:30 pm, 04:40 pm, 05:50 pm, 10:01 pm, 11:11 pm

45. **Answer: C**

Solution

ATQ,

The clock gains 30 sec/hour in the first 12 hours. ∴ it will gain 12 × 0.5 mins = 6 mins.

Similarly, it loses 15 sec/hour in the next 12 hours = 12 × 0.25 mins = 3 mins.

∴ in the entire day it gains 6 – 3 mins = 3 mins.

Thereby, it gains 15 mins in the next 5 days.

In the remaining 10 hours 30 mins (10.5 hours) it will gain = 0.5 × 10.5 = 5.25 mins.

∴ Total gain = 15 + 5.25 mins = 20.25 mins

∴ the time = 12:00 + 10:30 + 00:20.25 = 10:50 am

46. **Answer: A**

Solution

The clock loses 5 minutes per hour.

In a day it would lose, 5 × 24 = 120 mins = 2 hours.

So, in 4 days, it would lose 8 hours.

In the next 10 hours 30 minutes, it would lose 5 × 10.5 = 52.5 mins.

Overall loosing 8 hours 52.5 mins.

Subtracting 8 hours and 52.5 minutes from 10:00 AM on Friday, we get 1:08 AM on Friday.

47. **Answer: B**

Solution

In 400 consecutive years there are 97 leap years.

Hence in 400 consecutive years, February has the 29th day 97 times, and the remaining 11 months have the 29th day 400 × 11 = 4400 times. Therefore, 29th day of the month occurs (4400 + 97) = 4497 times.

48. **Answer: E**

Solution

Since, we do not know the year, we cannot find the exact day of the week.

49. **Answer: E**

Solution

Watch A gain 1 min/hour.

Watch B loses 2 min/hour.

So, at 6 pm the alarm is set for 6 am the next day i.e., after 12 hours.

As watch A gains 1 min/hour, the alarm will ring at 5:48 am (i.e., 12 mins prior to 6 am).

Till 5 o' clock, watch B has already lost 22 mins (i.e., 2 × 11 min/hour)

Watch B will lose 2 seconds/min.

So, in the next 48 mins, it will lose another 96 second (i.e., 1 min 36 seconds)

So, the alarm on watch B will ring 23 mins 36 seconds prior to the actual time, i.e., 5:24:24 am.

50. **Answer: B**

Solution

According to question, we know that,

Total time in hours from Sunday at 4 pm to the following Sunday at 8 am. = [(6 × 24) + 16] = 160 hours

Thus, the watch gains 6 + $10\frac{2}{3}$ = $16\frac{2}{3}$ minutes in 160 hours.

Now, $\frac{50}{3}$ minutes are gained in 160 hours.

∴ 1 minutes are gained in 160 × $\frac{3}{50}$

⇒ 6 minutes are gained in 160 × $\frac{3}{50}$ × 6

Required answer = $\frac{288}{5}$ hours = $57\frac{3}{5}$ hours.

Hence, the watch was correct on Wednesday at 1:36 am

DATA SUFFICIENCY SOLUTIONS

1. **Answer: D**

Solution

In none of the statement or in question, it is given any information about number therefore by using only percentage, we could not conclude our answer.

Hence, option D is correct.

2. **Answer: C**

Solution

In the room two window and one door are there.

From the statement III, we can conclude that the area of one window was 17sq. cm but we cannot find the area of the other window so we would not get our answer even by using all the statements.

Hence, option C is correct.

3. **Answer: D**

Solution

Let the number is a, b, c, d, and e where a is the first number, b is the second number and so on

According to question

$\frac{a+b+c+d}{4} = 6e$

a + b + c + d = 24e ⋯(i)

Statement I : Average of the first two numbers is equal to the average of the next two numbers

It means a + b = c + d ⋯(ii)

Statement II : Average of the first two numbers is four times of the fifth number.

$\frac{a+b}{2}$ = 4e, a + b = 8e ⋯(iii)

Statement III: The average of the all the numbers is 5

So sum of all the numbers = 5 × 5 = 25

a + b + c + d + e = 25

From the question statement given,

a + b + c + d = 24e

Or, 24e + e = 25

Therefore, e = 1

Clearly, only statement (iii) is sufficient to answer the question.

Hence, option D is correct.

4. **Answer: B**

Solution

From the Statement I, speed is less than 66 km/hr and from the Statement II, distance is more than 235 km so it is clear that he could not drive from Gujrat to Lucknow in less than 3 hours. So Statement I and Statement II are needed to get our answer.

In Statement III he can drive up to the speed of 180 km/hr but he drove at the speed of less than 66 km/hr so this statement is insufficient to get our answer.

Hence, option B is correct.

5. **Answer: C**

Solution

From the Statement I, the desired amount of mixture is 540 litres.

From the Statement II and Statement III, we can conclude that he had 540 litres of green paint and 360 litres of violet paint but we cannot conclude that how much quantity of yellow paint he has because the given ratio is the ratio to make the mixture. It is not the ratio of quantity he has.

Hence, option C is correct.

6. **Answer: D**

Solution

From Statement I:

Using the information given in above statement.

Mohish + 19 = Anurag

Here, we have no information about t and the actual age of Mohish .

So, we cannot find the answer.

Hence, statement I alone is not sufficient.

From Statement II:

Using the information given in above statement, we can say that

Mohish + Anurag = t

Here, we have no information about the present age of Mohish

So, we cannot find the answer.

Both Statement I and II:

From statement I:

Mohish + 19 =Anurag

From Statement II:

Mohish + Anurag = t

2 Mohish + 19 = t

Here, cannot find the actual age of Mohish and the real value of t.

So, we cannot find the answer.

Hence, Both statement I and II together are not sufficient.

Hence, option D is correct.

7. **Answer: C**

Solution

From the question, P : Q = 1 : 5

Statement I: after 6 years, the ratio will become 5 : 17

Statement II: 3 years ago, the ratio was 1 : 7

Therefore, statement I or Statement II alone is sufficient to get our answer, and their ages will be 9 years and 45 years respectively

Hence, option C is correct.

8. **Answer: A**

Solution

From the Statement I

Let the CP of each of two cheapest articles = x and the CP of costliest article = x + 1

Then, x + x + x + 1 = 70,

x = 23 → ∴ the CP of costliest article = 23 + 1 = 24

From the Statement II, we can say that the cost price of two articles is same and from Statement III, we can say that the cost price of costliest article is 4.34% more than the cost price of cheapest article therefore by combining both the statement we can also get our answer.

Hence, option A is correct.

9. **Answer: E**

Solution

From Statement I we can conclude the speed of the train and by combining Statement II and Statement III, we can conclude the distance between Delhi and Patna. But we cannot conclude how long it has stopped at each stoppage because the speed we concluded from the statement I is the speed of the train not the average speed of the entire journey.

Hence, option E is correct

10. **Answer: C**

Solution

By combining Statement I and Statement II we can conclude the age of Devansh and the age of Soham. So, we can find the sum as well.

Statement II and Statement III indirectly mean the same.

So by combining Statement I and statement III we can get our answer as well.

So, either Statement I and II together or Statement I and III together are sufficient.

Hence, option C is correct.

11. **Answer: E**

Solution

By combining statements, I and III we can conclude our answer as 500 meters

In statement II, only ratio of speed is given therefore it is not possible to get our answer only with the help of statement II.

By combining statements I and II also we can find the answer as the ratio between two people is given and in statement I the distance between the winner and loser is given so we can find the required distance as well.

So, Statement I and III together or Statement I and II together are sufficient.

Hence, option E is correct.

12. **Answer: E**

Solution

From the statement I,

We can conclude the sum of money Ratnesh had invested but nowhere it is mentioned the rate of interest Ratnesh had invested her money. Therefore, we cannot reach the answer by the statement alone.

From the statement II,

Difference between CI and SI is Rs.15 at the end of 2 years. But as the principal is not given here, we can't deduce the rate of interest using statement II alone.

But, from the statement I, we concluded the sum of money and to this statement we can conclude the rate of interest.

Therefore, if we combine statement I and statement II then we can conclude that the rate of interest was 10% per annum and the sum of money was Rs. 1500 now we can calculate the amount Ratnesh will receive al the end of 2 years.

Therefore, the data in both the statements I and II together is necessary to answer the question

Hence, option E is correct.

13. **Answer: E**

Solution

In the question, the distance is given.

In statement I, the ratio of speed is given.

From the statement I we can conclude that P's speed : Q's speed = 4 : 5 but we cannot find out speed of individual trains.

In statement II the average speed of both the trains is given so we cannot find out speed of individual trains.
If we combine statement I with statement II, then we can get as P's speed = 40 km per hr Q's speed = 50 km per hr
Therefore, the data in both the statements I and II is necessary to answer the question.
Hence, option E is correct.

14. **Answer: C**

Solution

Let the population of the city 4 years ago i.e. in the year 2016 = x
Then, from the statement I, 1.4641 × x = 292820
x = 200000
From the statement II,
If it was increased by 10% each year then,
$x \times \frac{110}{100} \times \frac{110}{100} \times \frac{110}{100} \times \frac{110}{100}$ = 292820
From here, we can get the value of x = 200000
∴ Either Statement I or Statement II alone is sufficient to answer the question.
Hence, option C is correct.

15. **Answer: E**

Solution

Let the two-digit number = ab = 10a + b
From the statement, I, we can conclude that,
a + b = 15
From the statement II, we can conclude that, 10b + a = 10a + b + 27
9(b – a) = 27
If we solve the both the equation then we can get our answer as 69
Therefore, the data in both the statements I and II together is necessary to answer the question.
Hence, option E is correct.

16. **Answer: D**

Solution

From the statement I, 30x + 4y = 216 ⋯(i)
From the statement II, y = 54 – 7.5x
4y + 30x = 216 ⋯(ii)
Here, both the equations are same it means we have two variable and one equation therefore we could not conclude the value of two variable from one equation.
Therefore, the data in both the statements I and II is not sufficient to answer the question.
Hence, option D is correct

17. **Answer: C**

Solution

From the statement I, we can conclude the number of days P alone will take to complete the work.
From the statement II, efficiency is given so it is easy to find the number of days, P alone or Q alone will take to complete the work.
Therefore, either Statement I or Statement II alone is sufficient to answer the question.
Hence, option C is correct.

18. **Answer: B**

Solution

The time taken by me from 9 am to 2 pm = 5 hours
From the statement I, speed of my brother = 28 km per hour
From the statement II, the time taken by my brother from 10 am to 1:30 pm = 3:30 hours = 3.5 hours
From the question and statement II, we can conclude the ratio of time = 5 : 3.5 = 50 : 35 = 10 : 7
Now we know that, speed = inversely proportional to time
So, the ratio of speed = 7 : 10
Let the distance between by house and the school is 70 km
Then my speed = 70/7 = 10 km per hour
And my brother speed = 70/10 = 7 km per hour
Now, we can conclude the time at which I will meet my brother but from the statement II, we could not conclude distance or any ratio.
Therefore, the data in statements II alone is sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question. Hence, option B is correct.

19. **Answer: C**

Solution

From statement I:
In 1 day, Tejas finishes $1/40^{th}$ of the work.
In 1 day, Tanmay finishes $1/50^{th}$ of the work.
The work was completed in 20 days.
Part of work completed by Tejas and Tanmay in 20 days
$= 20\left(\frac{1}{40} + \frac{1}{50}\right) = \frac{9}{10}$
Part of work completed by Sachin

= 1 – $\frac{9}{10} = \frac{1}{10}$

Amount received as payment will be proportional to the amount of work done.

Amount paid to Sachin = $\frac{1600 \times 1}{10}$ = Rs160

Similarly, from statement II we also can find the share of Sachin.

∴ Either of the statements alone is sufficient to answer this question.

Hence, option C is correct.

20. **Answer: A**

Solution

From statement I:

Share of profit of A and B will be in the ratio of 4 : 7

After two years,

Let the share of profit of B be Rs. Y

∴ $\frac{4}{7} = \frac{190000}{y}$

⇒ y = $\frac{190000 \times 7}{y}$

⇒ y = Rs. 332500

∴ Statement I alone is sufficient to answer.

From statement II: Only A's investment is given so we can't find the profit of B.

Hence, option A is correct.

21. **Answer: D**

Solution

From Statements I and II:

45% of girls participated in the event, which is equals to 135

∴ 100% ⇒ $\frac{135}{45} \times 100 = 300$

∴ Total number of girls in the college = 300

The number of boys who participated in the event.

⇒ 300 × $\frac{100}{120}$ = 250

Total number of boys participated is 250, which is equals to 50%

∴ 100% ⇒ $\frac{250}{50} \times 100 = 500$

∴ Total number of boys in the college = 500

∴ Statement I and II together are necessary to answer the questions.

Hence, option D is correct.

22. **Answer: C**

Solution

From statement I:

Let the total population of Karachi be X.

⇒ 35X – 27X = 32000

⇒ 8X = 32000

⇒ X = 4000

∴ The total population 35X + 27X = 62X

62X = 62 × 4000 = 248000

From statement II:

Let the total population of Karachi be X.

So, population of Patna = $\frac{100X}{60} = \frac{5X}{3}$

∴ $\frac{5X}{3} - X$ = 45000

⇒ 2X = 135000

⇒ X = 67500

∴ Statement I or II alone is sufficient to answer the question.

Hence, option C is correct.

23. **Answer: D**

Solution

From the statement I :

We can conclude, the radius of the circle = 28 cm therefore the diameter of the circle = 56cm.

From the statement II :

We can conclude that the largest side of the triangle = 56 cm and it is circumscribed by a circle therefore it will be a right – angled triangle because diameter makes 90° at any point on the circumference.

From the statement III :

We can conclude that it is an isosceles triangle.

If we combine all the statements then we can conclude that we need to find the perimeter of an isosceles right – angle triangle the hypotenuse of which is 56 cm (the diameter of the circle).

By the Pythagorean theorem, we can get our unique answer.

Hence, option D is correct

24. **Answer: A**

Solution

From the question, the length of train U = 300 meters and let the speed = x m/sec

And the length of train B = 350 meters and the speed = y m per sec

From the statement I, distance = speed × time

650 = (x + y) × 10

x + y = 130 ⋯(i)

From the statement II,

x – y = 10 ⋯(ii)

If we combine the statement I , and II and solve the equation then we can get our answer.

From the statement III, x + y = 130 ⋯(iii)

If we combine statement II, and III then we can get our answer.

Hence, option A is correct.

25. **Answer: A**

Solution

From the statement I, we can conclude that the CP = 25

From the statement II, when he sells 100 units that time scale weighs only 80 units

Let the cp of one unit = Rs. 1 then the CP of 80 units = Rs. 80 and the SP of 80 units = Rs. 100

From here we can conclude the profit percentage as 25%. Here we need to calculate percentage value so we don't have need to exact cost price.

From the statement III, let he sells total 100 units.

And the CP of one unit = Rs. 1

Then the SP of 100 units = 120% of 100 = 120 and the net profit percentage = 50% therefore, CP = 80 it means he uses 20% less weight

If he had sold the article at CP then his profit would have been 25%

Hence, option A is correct.

26. **Answer: C**

Solution

From Statement I:

Let Arun = 4x, Varun = 5x

Vedant = 4x + 4

From Statement II:

Arun = 20 years, Vedant = x, Varun = x + 1

Statement III:

Let age of Mohan = m years

Vedant = m – 5 years

From Statement I + Statement II:

Arun = 20 years, Varun = $\frac{20}{4x}$ × 5x= 25 years

Vedant = 25 – 1 = 24 years

Statement I and II both are necessary to answer the question.

Hence, option C is correct.

27. **Answer: A**

Solution

From Statement I:

Let radius = r, height = h

Curved surface area = 2πrh

396 = 2 x $\frac{22}{7}$ x r x h

r × h = 63

Total Surface area = $2\pi rh + 2\pi r^2$

1628 = $\frac{22}{7}(rh + r^2)$

$rh + r^2 = 259$

$63 + r^2 = 259$

$r^2 = 259 - 63 = 196$

r = 14

r × h = 63

14 × h = 63

h = 4.5 cm

Statement I is alone sufficient to answer the question.

From Statement II:

r = 3h + 0.5

Statement II is not alone sufficient to answer the question.

Hence, option A is correct.

28. **Answer: A**

Solution

Let the invested amount be Rs. x

From statement I:

⇒ SI = $\frac{P \times r \times n}{100}$

Increase in rate of interest = $\frac{116}{6} - \frac{88}{6} = \frac{14}{3}$ %

Increase in SI because of increased rate of interest:

⇒ x × $\frac{14}{3} \times \frac{1}{100}$

This increase = Increase in income = 5600

⇒ x × $\frac{14}{3} \times \frac{1}{100}$ = 5600

⇒ x = $\frac{560000 \times 3}{14}$ = 1,20,000

From statement II:

⇒ SI = x

⇒ x = $\frac{x \times 20 \times 5}{100}$

From here x cannot be calculated

∴ Statement I alone is sufficient while statement II is not.

Hence, option A is correct.

29. **Answer: A**

Solution

From statement I, Isha and Esha take 10 days to complete the project implies one of them (Isha or Esha) takes less than 20 days and the other takes more than 20 days. Esha and Rhea take 20 days to complete the project implies one of them (Esha or Rhea) takes less than 40 days and the other takes

more than 40 days. Esha cannot take less than 20 days because if Esha takes less than 20 days than Esha and Rhea should take less than 20 days to complete the project.(but given Esha and Rhea take 20 days to complete the project).
Isha takes less than 20 days and she is the fastest.
From statement II, we do not know anything about Esha, so who is fastest cannot be determined.
Hence, option A is correct.

30. **Answer: E**

Solution

From I, Let speed of Trishan = 3x
speed of Henil = 2x
From II, we cannot calculate distance between city A and B.
From III, difference between speed of Trishan and Henil is given, so we cannot calculate the answers.
from statement, I, II, III together,
Difference between their speeds = 3x – 2x = 50
x = 50
speed of Trishan = 3 × 50 = 150
speed of Henil = 2 × 50 = 100
Let distance = D
D/100 – D/150 = 4
3D – 2D/300 = 4
D = 1200
Hence, option E is correct.

31. **Answer: B**

Solution

Using the statement I we cannot answer the question because no speed is given.
But using II we can.
Since the cycles are going to meet 30 minutes , which means , the relative speed =60 + 80 = 140 km/hr.
So the gap = speed x time .So we can find the distance .
Hence option B is correct.

32. **Answer: C**

Solution

Clearly N is greater than 60. Because average only increases if the value is greater than 60.
Also M denotes the number of children , so it will be the positive value.
So by first statement we can get (1, 64), (2, 63), (3, 62), (4 , 61) since M > 60.
But by statement II N is a prime number. So (4, 61) is correct.
So 4 children with average weight 61 are added in the group.
Hence C is correct.

33. **Answer: C**

Solution

As given in statements I and II, we have
P = Q = K,
Q + K + A = 64
and A = Q + k.
take Q + K = A in second,
we get 2A = 64 or A = 32.
Thus, Q + K = 32 and Q = K.
So, Q = K = 16. Thus, P = 16.
Hence, option C is correct.

34. **Answer: E**

Solution

From statement I,
Let the amount invests by D = x
4800 = x × (1 + R/100)^3
So, Statement I alone is not sufficient to answer the question.
From statement II,
R = 10%
So, Statement II alone is not sufficient to answer the question.
From statement III,
Let the amount invests by C = x
Let amount invests by D = y
SI = y × 2 × R/100 = yR/50
CI = x × (1 + R/100)2 – x
CI – SI = 1200
or
SI – CI = 1200
So, Statement III alone is not sufficient to answer the question.
Hence, option E is correct.

35. **Answer: A**

Solution

From statement I,
p1/p3 = 4/5
p1/p4 = 1/3
So, Statement I alone is not sufficient to answer the question.

From Statement II,
p2 + p3 + p4 = 125
(p1 – 5)/(p4 – 5) = 3/11
So, Statement II alone is not sufficient to answer the question.
From statement III,
p4 = 300/100 × p1
p4: p1 = 3: 1
2x = 40
x = 20
p1= 20 years
p4 = 3 × 20 = 60 years
So, Statement III alone is not sufficient to answer the question.
From statement I and II,
Let p1 and p4's present age be x and 3x respectively.
(p1 – 5)/(p4 – 5) = 3/11
(x – 5)/(3x – 5) = 3/11
→ 11x – 55 = 9x – 15
∴ 2x = 40
∴ x = 20
The present age of p1 and p4 is 20 and 60 years respectively.
p3's present age = 20/4 × 5 = 25 years
p2's present age = 125 – (25 + 60) = 125 – 85 = 40 years
Sum of the ages of p1 and p2 = (20 + 40) = 60 years
Hence, statement I and II alone is sufficient to answer the given question.
Hence, option A is correct.

36. **Answer: E**

Solution

Height of cone/height of the cylinder = 2/1
So, Statement I alone is not sufficient to answer the question
From statement II,
Area of the square = 9
Side of the square = 3
Perimeter of the square = 3 × 4 = 12 cm
Height of the cylinder = 12 cm
So, Statement II alone is not sufficient to answer the question.
From statement III,
Radius of the cone = length of the rectangle
Perimeter of the rectangle = 2 × (l + b) = 20
So, Statement III alone is not sufficient to answer the question.
Hence, option E is correct.

37. **Answer: A**

Solution

From statement I,
Ethanol and petrol in X = 3: 2
Ethanol and petrol in Y = 4: 3
So, Statement I alone is not sufficient to answer the question.
From statement II,
Vessel Y mixture = 28
Ratio of the Ethanol and petrol in X = 17: 12
So, Statement II alone is not sufficient to answer the question.
From statement III,
Mixture of Z = 10
Ratio of Ethanol and petrol in Z = 5: 4
So, Statement III alone is not sufficient to answer the question,
From I and II
Ethanol in 28 liters of Y = 4/7 × 28 = 16 liters
Petrol in 28 liters of Y = 3/7 × 28 = 12 liters
3x + 16/2x + 12 = 17/12
34x + 204 = 36x + 192
2x = 12
x = 6
Ethanol in vessel X = 3 × 6 = 18 liters
So, Statement I and II are necessary to answer the question.
Hence, option A is correct.

38. **Answer: C**

Solution

From statement I,
Number of females from B in 2020
= 4/7 × 280=160
Number of males from B in 2020
= 3/7 × 280=120
So, statement I alone is not sufficient to answer the question.
From statement II,
60% of the total number of employees from B in 1997 to 2000 is males.
So, statement II alone is not sufficient to answer the question.

From I and II,
Total number of employees from B in 1997 to 2000 = 300 + 100 + 350 + 280 = 1030
Number of males from B in 1997 to 2000 = 1030 × 60/100 = 618
Number of males from B in 1997 to 1999 = 618 – 120 = 498
Number of females from B in 1997 to 1999 = (300 + 100 + 350) – 498 = 252
Required percentage = 252/750 × 100 = 33.6%
Both the statements are necessary to answer the question.
Hence, option C is correct.

39. **Answer: A**
Solution
From I: Let the total work lcm(24,36) = 72 units Work done by A,C and D in one day = 72/24 = 3 units Work done by C and D in one day = 72/36 = 2 units
Work done by A in one day = 3 – 2 = 1 unit
Therefore, number of days taken by A to complete the work = 72/1 = 72 days
The time taken by B to complete the work = 72/2 = 36 days
This statement is alone sufficient.
From II: Let the efficiency of C and D be x and 2x resp.
1/72 + x + 2x = 1/24
=>x = 1/108
This statement is not sufficient to answer.
Hence, option A is correct.

40. **Answer: C**
Solution
From I: Let the required number be (10x+y).
Now, 10x + y = 10y + x + 27
=> (x – y) = 3
So, the possible values of x and y are 96,85,74,63,52 and 41.
We cannot find a unique solution.
So, this statement is not alone to solve the question.
From II: Two-digit numbers which are multiple of 9 = 18,27,36,45,54,63,72,81,90,99
We cannot find unique solution.
So, this statement alone is not sufficient.
Combining both the equations. Let the required number be (10x+y).
10x + y = 10y + x + 27
=> (x – y) = 3
So, the possible values of x and y are 96,85,74,63,52 and 41.
Two-digit numbers which are multiple of 9 = 18,27,36,45,54,63,72,81,90,99.
So, 63 is the required two-digit number.
So, the factors of 63 are 1,3,7,9 21,63.
Both are necessary to answer.
Hence, option C is correct.

41. **Answer: E**
Solution
From Statement I:
Time = 50 seconds
Relative speed = $\frac{length\ of\ trains}{time}$
= $\frac{length\ of\ trains}{50}$
From Statement II:
Length of train B = 640m
Difference of speed = 28 km/h
From Statement III:
Let the speeds = 19x, 26x
Statement I + Statement II + Statement III
26x – 19x = 28
7x = 28
x = 4
Speed = 104 km/h, 76 km/h
$(104 + 76) \times \frac{5}{18} = \frac{(a + 640)}{50}$
50 × 50 = a + 640
2500 – 640 = a
a = 1860 m
Therefore, all the statements are necessary to answer the question.
Hence, option E is correct.

42. **Answer: C**
Solution
Let the cost of the Pen be Rs. x, the cost of the Sharpener be Rs. y, the cost of the pencil be Rs. p and the cost of the Eraser be Rs. q.
From Statement I:
0.3x + 0.2y > 0.4p + 0.6q
⇒ 0.3x + 0.2y > 0.4p + 0.6q
⇒ $x + \frac{2}{3}y > \frac{4}{3}p + 2q$
So the sum of cost of the Pen and 2/3 of cost of the Sharpener is more than the sum of the cost of 4/3 of

the pencil (4/3 implies more than 1 here) and twice of that eraser (twice implies 2 erasers). Evidently, the sum cost of the Pen and that of the Sharpener must be more than the sum of the cost of the pencil and that of the eraser. So, statement I alone is sufficient.

From Statement II:

$2x + 3y < p + 1.5q$

Clearly, the sum of the cost of the pencil and 1 and a half of the Eraser is more than the sum of the cost of the two Pen s and that of the three Sharpener s

So, statement II alone is also sufficient.

Hence, either of the statements is sufficient to reach the answer.

Hence, option C is correct.

43. **Answer: B**

Solution

The maximum triangles would be in case all these 6 points are non-collinear. In such a case the number of triangles is ${}^6C_3$

$= \frac{6 \times 5 \times 4}{1 \times 2 \times 3} = 20$

Hence, statement I is incorrect.

In other case if surveyor take the position that A and B coincide on the first straight line, C and D coincide on the second straight line, E and F coincide on the third straight line and all these coincidences happen at 3 points which are on the same straight line in such a case there would be 0 triangles formed,

Hence, statement II is correct.

Therefore, option B is correct.

44. **Answer: C**

Solution

Statement I:

Let the speed of man in still water be 5x and speed of stream be 2x

Upstream speed = 36 km/hr

So, $5x - 2x = 36$

$3x = 36$

$x = 12$ km/hr

Downstream speed = 5x + 2x = 7x = 7 x 12 = 84 km/hr

Let the distance covered upstream be x km

So, the distance covered downstream be x Km

So, $\frac{x}{36} + \frac{x}{84} = 3\frac{8}{14}$

x = 90 km

Time taken to row a boat downstream to cover the same distance covered upstream

$= \frac{90}{84}$ = 1.07 hours

Therefore, statement I alone is sufficient to answer the question

Statement II :

Let the speed of man in still water be 5x and the speed of stream be 2x

Upstream speed = 36 km/hr.

So, $5x - 2x = 36$

$3x = 36$; $x = 12$ km/hr.

Downstream speed = 5x + 2x = 7x = 7 × 12 = 84 km/hr.

Distance covered upstream = 36 × 2.5 = 90 km

Time taken to row a boat downstream to cover the same distance covered upstream

$= \frac{90}{84}$ =1.07 hours

Therefore, statement II alone is sufficient to answer the question

So, the data either in Statement I or in Statement II alone are sufficient to answer the question.

Hence, option C is correct.

45. **Answer: D**

Solution

Statement I:

Let, the amount of water and the amount of milk initially in the container = 5x litres and 13x litres, respectively

Amount of water sold by Suraj = $\frac{5}{18}$ x 72= 20 litres

Amount of milk sold by Suraj = $\frac{13}{18}$ x 72 =52 litres

Since the amount of water and the amount of milk in 44 litres of the mixture which has been mixed later is not given

Therefore, statement I alone is not sufficient to answer the question.

Statement II:

Let, the amount of water and the amount of milk initially in the container = 5x litres and 13x litres, respectively.

Amount of water sold by Suraj = $\frac{5}{18}$ x 72= 20 litres

Amount of milk sold by Suraj = $\frac{13}{18}$ x 72 =52 litres

Amount of water in 44 litres of mixture

= $\frac{5}{11}$ x 44 =20 litres

Amount of milk in 44 litres of mixture

= $\frac{6}{11}$ x 44 =24 litres

But the amount of mixture after mixing 44 litres of another mixture is not given.

Therefore, statement II alone is not sufficient to answer the question.

Combining statement I and statement II:

Let, the amount of water and the amount of milk initially in the container = 5x litres and 13x litres, respectively.

And the amount of water and the amount of milk after mixing 44 litres of mixture = 4y litres and 9y litres, respectively.

Amount of water sold by Suraj = $\frac{5}{18}$ x 72 = 20 litres

Amount of milk sold by Suraj = $\frac{13}{18}$ x 72 = 52 litres

Amount of water in 44 litres of mixture = $\frac{5}{11}$ x 44 = 20 litres

Amount of milk in 44 litres of mixture = $\frac{6}{11}$ x 44 = 24 litres

So, 5x – 20 + 20 = 4y

5x = 4y; y = $\frac{5x}{4}$

And 13x – 52 + 24 = 9y

13x – 28 = 9 × $\frac{5x}{4}$

52x – 112 = 45y

7x = 112; x = 16

The amount of water mixed by Suraj = 5x = 80 litres

So, the data in both statements I and II together are necessary to answer the question.

Hence, option D is correct.

46. **Answer: D**

Solution

Neither of the statements is independently sufficient as the average age and the strength of different classes are mentioned in two different statements.

Combining both the statements, even if the class XIII forms 25% of the total strength, the average age will be less than 20 years.

This is shown below:

Let the strength of the college be 10k and that of the class XIII students be 2.5k. In that case the strength of the remaining classes is 7.5k. Hence, the average age of the college.

$\Rightarrow \frac{2.5k \times 21 + 7.5k \times 19}{10k}$ = 19.5 years

Hence, the average age will be less than 20 years. If the strength of the XIII class is less than 25%, the average age would be less than 19.5 years.

Hence, option D is correct.

47. **Answer: A**

Solution

From I alone,

Joined on 2010 or before = 22

Joined after 2010 = 8

Retired on 2020 or before = 11

Retired after 2020 = 19

Assuming, that the six persons who retired in or before 2020 are from the 22 persons who joined before 2010, the least number of people who had at least 10 years of service is (22 – 11) = 11

Hence, I alone is sufficient.

From II alone,

Joined on 2010 or before = 11

Joined after 2010 = 19

Retired before 2020 = 11

Retired 2020 or after = 19

The least number of people who have at least 10 years of service could be (30 – 0) = 30 or (11 – 11) = 0.

Hence, II alone is not sufficient.

Hence, option A is correct.

48. **Answer: A**

Solution

From statement I,

The time taken by P to cross Q, when they move in opposite directions is half the time taken by P to cross Q, when they move in the same direction.

i.e. $\frac{(l_p + l_q)}{(S_p + S_q)} = \frac{(l_p + l_q)}{2(S_p - S_q)}$

using this $S_p : S_q$ can be found as 3 : 1

As the length of train P is less and it moves at a faster rate, it takes less time to cross the platform, irrespective of the platform length.

Statement II alone is not sufficient, as it does not mention the ratio of the speeds of trains P and Q.

Hence, option A is correct.

49. **Answer: B**

Solution

Let the ten's place and unit's place digits of the number be x and y respectively.

And, the number be 10x+y = z

A → 2y = 7x

B → (10y+x)-(10x + y) = 45 → y – x = 5

C → z =27

Hence, either C alone or A and B together are sufficient to answer.

Hence, option B is correct.

50. **Answer: D**

Solution

From statement I,

m1= x

m2 = 2x

m3= (x + 1000)

m4 = 3x

So, Statement I alone is not sufficient to answer the question

From Statement II,

m3's share = Rs.4000

So, Statement I alone is not sufficient to answer the question

From statement III,

Profit ratio of m3/m4 = 2/3

So, Statement III alone is not sufficient to answer the question

From I, II and III

Profit ratio of m1, m2, m3 and m4 = x × 12: 2x × 12: (x + 1000) × 6: 3x × 6

=12x: 24x: (6x + 6000): 18x

(6x + 6000)/18x = 2/3

6x + 6000 = 12x

x = 1000

Profit ratio = 12000: 24000: 12000: 18000

= 2: 4: 2: 3

m2's profit share = 4/2 × 4000 = 8000

All the statements are necessary to answer the question.

Hence, option D is correct.

DATA COMPARISON SOLUTIONS

1. **Answer:** A

Solution

Quantity I: $\left(x-\frac{2}{3}x\right)+4400=\frac{x}{3}+4400$

$\frac{3}{5}\left(\frac{x}{3}+4400\right)+7500=39000$

$\frac{3}{5}\left(\frac{x}{3}+4400\right)=31500$ → ∴ x = 1,44,300

Quantity II: $\frac{7}{8}\times\frac{8}{11}\times x=91{,}000$ ∴ x = 1, 43,000

∴ Quantity I > Quantity II

2. **Answer:** A

Solution

Quantity I: 5a + 7b = 45 – eqn (1)

125a = 92b ; 125a–92b=0 – eqn (2)

Multiply eqn 2 by 25

125a + 175b = 1125

Subtract eqn 2 from 1

	125a	+ 175b	=	1125
–	125a	– 92b	=	0
		267b	=	1125

b = 1125 / 267 = 375 / 89

∴ a = 276/89

b – a = 99/89 ≈ 1.11

Quantity II: c + $\frac{1}{d}$ = 1, $d+\frac{1}{a}=1$

cd + 1 = d –(1)

da + 1 = a –(2)

Substitute value of d in (2)

(cd+1)a+1 = a

acd + a +1 = a

acd = -1

∴ Quantity I > Quantity II

3. **Answer:** C

Solution

Quantity I:

884 / 2 = 442

442 + 6 = 448

448 / 2 = 224

224 + 6 = **230**

230 /2 = 115

Quantity II: 230

Quantity I = Quantity II

4. **Answer:** A

Solution

Quantity I: 325 + (34 × 1)

359 + (34 × 2)

427 + (34 × 3)

529 + (34 × 4)

665 + (34 × 5)

Quantity II: 664

Quantity I > Quantity II

5. **Answer:** B

Solution

Quantity I: 5

5×3+3=18

18×3+3=57

57×3+3=174

174×3+3=**525**

Quantity II: 5

5×4–4=16

16×4–4 = 60

60×4–4 = 236

236 × 4 – 4 = **940**

∴ Quantity II > Quantity I

6. **Answer:** B

Solution

Quantity I:

12^2–4

13^2–3

14^2–2

15^2–1

16^2–0

17^2+1

18^2+2

19^2+3 = **364**

Quantity II: 365

Quantity II > Quantity I

7. **Answer:** A

Solution

Quantity I: 304 + 123 × 23 + 10 ÷ 30 = 3133.33

Quantity II: 523 + 250 × 6 ÷ 15 – 12 = 611

Quantity I > Quantity II

8. **Answer:** B

Solution

Quantity I: Total Savings = (5500 × 3) + (5250 × 4) + (5330 × 5) = 53490

Total Expense = 12550

Average Monthly Income = (Total Expense + Total Savings) / 12 = (12550 +53490)/12 = 66040 / 12 = 5503.33 ≈ 5503

Quantity II: 7! = 7 × 6 × 5 × 4 × 3 × 2 × 1
= 7 × 2 × 3 × 5 × 2 × 2 × 3 × 2 × 1
= $7^1 \times 5^1 \times 3^2 \times 2^4$
Thus, the smallest perfect square number which is divisible by 7! is
($7^1 \times 5^1 \times 3^2 \times 2^4$) × (5 × 7) = 5040 × 35 = 176400
Quantity II > Quantity I

9. **Answer:** E
Solution
Quantity I: Let the seven consecutive numbers be x, (x + 1), (x + 2), (x + 3), (x + 4), (x + 5) and (x + 6).
Then, x + (x + 1) + (x + 2) + (x + 3) + (x + 4) + (x + 5) + (x + 6) = 196 ⇒ 7x + 21 = 196 ⇒ 7x = 175 ⇒ x = 25
Thus, the numbers are 25, 26, 27, 28, 29, 30 and 31.
Of these numbers, only two numbers i.e. 29 and 31, are prime.
Quantity II: $111x^2-222x-333=0$
Product = c/a = −333 × 111 = −36963
Sum = −b/a = 222
(333, −111)
∴ 333/111=3 and −111/3 = −1
∴ roots are −3, 1
Difference = −3−1 = −4 or 1−(−3) = 1+3 = 4

10. **Answer:** B
Solution
Quantity I: Σ total marks = 460
Σ marks obtained = 391
Total % = $\frac{\sum marks\ obtained}{\sum total\ marks} \times 100 = \frac{391}{460} \times 100 = 85\%$
Quantity II: SP = 17x
CP = 9x
%Profit = $\frac{SP-CP}{CP} \times 100 = \frac{8x}{9x} \times 100 = 88.89\%$
∴ Quantity II > Quantity I

11. **Answer:** A
Solution
Quantity I:
$\frac{500-x}{x} \times 100 = 5\left(\frac{420-x}{x} \times 100\right)$
∴ 500−x = 5(420−x)
∴ 4x = 1600
∴ x= 400
Quantity II:
25
25 × 1 + 1
26 × 2 + 2
54 × 3 + 3
165 × 4 + 4
664
∴ Quantity II = 165
∴ Quantity I > Quantity II

12. **Answer:** D
Solution
Quantity I:
$x^2 + 16x = 161$
∴ $x^2 + 16x-161 = 0$
∴ $x^2+23x-7x-161 = 0$
∴ (x+23)(x−7)=0
∴ x=−23 or x = 7
Quantity II:
$2y^2+8y - 42 = 0$
∴ $2(y^2+4y-21)=0$ i.e. $y^2+ 4y - 21 = 0$
∴ $y^2-3y+ 7y - 21 = 0$
∴ (y−3)(y+7)=0
∴ y = 3 or y = −7
Relation cannot be established.

13. **Answer:** B
Solution
Quantity I:
$43x^2 + 11x+57 = 13x^2 + 56$
$30x^2+11x+1=0$
$30x^2 + 6x + 5x + 1 = 0$
(5x+1)(6x + 1) = 0
x= −1/5 or x = −1/6
Quantity II:
$y^2 - 16y+39 = 0$
$y^2-13y-3y+39 = 0$
(y−13)(y−3) = 0
y = 13 or y = 3 → ∴ y > x
∴ Quantity II > Quantity I

14. **Answer:** B
Solution
Quantity I:
$12x^2 + 65x + 97 = 4x^2 + 17x+ 33$
$8x^2+48x+64 = 0$

$8(x^2+6x+8) = 0$ i.e., $x^2+6x+8=0$

$x^2+4x+2x+8 = 0$

$(x+4)(x+2) = 0$

$x = -4$ or $x = -2$

Quantity II:

$y^2 - 22y+117 = 0$

$y^2-13y-9y+117 = 0$

$(y-13)(y-9) = 0$

$y = 13$ or $y = 9$ → ∴ $y>x$

∴ Quantity II > Quantity I

15. **Answer:** B

Solution

Quantity I: $\frac{60}{100} \times 500 + \frac{75}{100} \times 320 = 540$

Quantity II: Let the total number of students be x.

Let A and B represent the sets of students who passed in Java and Python respectively.

Then, number of students passed in one or both the subjects

= n (A ∪ B) = n (A) + n (B) – n (A ∩ B) = 80% of x + 85% of x – 75% of x

$= \frac{80}{100}x + \frac{85}{100}x - \frac{75}{100}x = \frac{90}{100}x = \frac{9}{10}x$

∴ Students who failed in both the subjects = $x - \frac{9}{10}x = \frac{x}{10}$

$\therefore \frac{x}{10} = 60$ → $\therefore x = 600$

Quantity II > Quantity I

16. **Answer:** B

Solution

Quantity I: Weight of tin = 6 kg.

Total weight of alloy = 42 kg.

Required % = $\frac{6}{42} \times 100 = 14.28\%$

Quantity II: $\frac{x}{100} \times 92 = 15$

$\therefore x = 16.30\%$

Quantity II > Quantity I

17. **Answer:** C

Solution

Quantity I:

132 = 11 × 12

156 = 12 × 13

182 = 13 × 14

210 = 14 × 15

Quantity I = Quantity II

Quantity II:

x : x(x + 1)

7 : 7(8) = 7:56

14 : 14(15) = 210

18. **Answer:** B

Solution

Quantity I: Each number is the preceding number multiplied by – 2. So, the required number is –128.

Quantity II: Numbers are squares of consecutive numbers. So, the next number is = 64.

Quantity I < Quantity II

19. **Answer:** A

Solution

Quantity I:

Let the age of Sujay = 7x, Akash = 8x

8x – 7x = 4 → ∴ x = 4

Age of Sujay = 7 × 4 = 28 years

Quantity II:

Total age of Sujay and Dhiraj = 25 × 2 = 50 years

Let the age of Sujay = x, Dhiraj = x + 2

x + x + 2 = 50

2x = 50 – 2 → 2x = 48 → ∴ x = 24

Age of Sujay = 24 years

Quantity I > Quantity II

Hence option A.

20. **Answer:** B

Solution

Quantity I:

Let height = x cm, radius = x + 2 cm, The curved surface area of the cylinder = 220 cm^2

$2 \pi r h = 220$

$2 \times 22/7 \times (x+2)x = 220$

$x^2 + 2x = 35$

$x^2 + 7x - 5x - 35 = 0$

$x (x + 7) - 5 (x + 7) = 0$

$(x + 7) (x - 5) = 0$ → ∴ $x = 5, -7$

Radius = 7 cm, Height = 5 cm

Volume = $\pi r^2 h$

$= 22 / 7 \times 7 \times 7 \times 5 = 770 cm^3$

Quantity II:

$= 780\ cm^3$

Quantity I < Quantity II

Hence option B.

21. **Answer:** A

Solution

Quantity I:

Because percentage of girls = 65%,

So, the percentage of boys = 100 – 65% = 35%

Quantity II:

Last year,

Girls was 300 × 60% = 180, boys = 300 × 40% = 120

This year, because girls are increased by 10% and total students remain same.

girls = 180 × 110% = 198, boys = 300 – 198 = 102

% of boys = (102/300) × 100 = 34%

Quantity I > Quantity II

Hence option A.

22. **Answer:** B

Solution

Quantity I: Find the volume of a cylinder with radius 15cm and height 21 cm.

Volume of cylinder = $\pi r^2 h$ = 22/7 × 15 × 15 × 21 = 14850

Quantity II: Find the volume of a sphere with radius 21 cm

Volume of sphere = (4/3) πr^3 = 4/3 × 22/7 × 21 × 21 × 21 = 38808

Hence, Quantity I < Quantity II

23. **Answer:** A

Solution

Quantity I:

Distance travelled by the bus = 240 km

Time taken = 4 hours

∴ Speed of the bus = 240/4 = 60 km/hr

Speed after 20% increment = 60 × (120/100) = 72 km/hr

Now distance need to travel = 240 × 3 = 720 km

∴Time take = 720/72 = 10 hours

Quantity II:

Distance between point A and B = 150 km

half of distance = 75 km

Time taken to cover 75 km at a speed of 25 km/hr = $\frac{75}{25}$ = 3 hours

Time taken to cover 75 km at a speed of 30 km/hr = $\frac{75}{30}$ = 2.5 hours

And we also know that bus took 45/60 = 3/4= 0.75

∴ total time = 3 hr + 2.5 hr + 0.75 hr = 6.25

Here, Quantity I > Quantity II

24. **Answer:** A

Solution

Quantity I:

Let speed of boat = x kmph, speed of stream = y kmph

upstream speed = x – y, downstream speed = x + y

x + y = $\frac{72}{9}$ = 8 ⋯ (1)

x – y = $\frac{40}{10}$ = 4 ⋯(2)

equation 1 + equation 2

x = 6

Speed of the boat = 6 kmph

Quantity II:

Let speed of boat = x kmph , speed of steam = 1 kmph

Downstream speed = x + 1

$\frac{36}{9}$ = x + 1 → ∴ 4 – 1 = x → ∴ x = 3

Speed of boat = 3 kmph

Quantity I > Quantity II

Hence, option A

25. **Answer:** B

Solution

Quantity I:

Area of the square = 26 × 26 = 676 cm^2

Area of the square = Area of the rectangle

Area of the rectangle = 676 cm^2

Quantity II:

Length of the rectangle = 6x, Breadth = 5x

Perimeter = 132

2 (l + b) = 132

6x + 5x = 66

11x = 66

x = 6

Length = 36cm, breadth = 30cm

Area of the rectangle = 36 × 30

= 1080 cm^2

Quantity I < Quantity II

Hence, option B.

26. **Answer:** B

Solution

Quantity I:

Total number of way in which two balls can be drawn

$n(S) = {}^{25}C_2 = \frac{25 \times 24}{2} = 300$

Total number of ways in which both the balls drawn are either Black or Red

$n(E) = {}^{4}C_2 + {}^{5}C_2 = 6 + 10$

$n(E) = 16$

Probability that both the balls are either Black or Red

$n(P) = \frac{n(E)}{n(S)} = \frac{16}{300}$

Quantity II:

Total number of ways in which four ball can be drawn

$n(S) = {}^{25}C_4 = \frac{25 \times 24 \times 23 \times 22}{4 \times 3 \times 2 \times 1}$

$n(S) = 12,650$

Total number of ways in which all the balls are of different colour.

$n(E) = {}^{7}C_1 \times {}^{4}C_1 \times {}^{5}C_1 \times {}^{9}C_1 = 7 \times 4 \times 5 \times 9$

$n(E) = 1260$

Probability that both the balls are of different colour

$n(P) = \frac{n(E)}{n(S)} = \frac{1260}{12650}$

$n(P) = \frac{126}{1265}$

Quantity I < Quantity II

Hence, option B.

27. **Answer:** A

Solution

Solution:

Relative speed = 100+80 = 180 kmph

= 180 x 5/18 = 50 m/s

Let the length of the other train = x metres

(250 + x) / 9 = 50

(250 + x) = 450

X = 200

Quantity I: The length of other train is 200 m.

Quantity II: The length of the first train shrinks by 3/4th of that of the other train

Therefore, 3/4 of 200 = 150 m

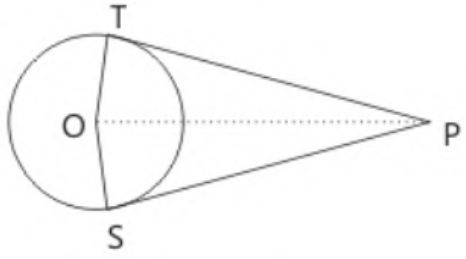
Now, length of the first train = 250 – 150 = 100 m

Hence, Quantity I > Quantity II

Hence option A.

28. **Answer:** C

Solution



From the above figure

∠OTP = 90°, (∵ Radius makes an angle of 90° with the tangent at the point of tangency).

Quantity I:

In triangle OTP

$PO^2 = PT^2 + OT^2 = 30^2 + 16^2$

$PO = \sqrt{(30^2 + 16^2)} = 34$ cm

Quantity II:

Length of the hypotenuse2 = $15^2 + 8^2$

Therefore, hypotenuse = 17 cm

Therefore, double of it = 34 cm

Clearly, quantity I = quantity II

Hence option C.

29. **Answer:** C

Solution

Quantity I: Perimeter of the square = 2 × 2 (27 + 15) = 2 × 84 = 168 cm

let the side of a square be a.

Then, 4a = 168 cm → ∴ a = 42 cm

Diameter of the circle = 42 cm

∴ Radius = 42/2 = 21 cm

∴ Perimeter of the semicircle = πr + 2r

= 22/7 × 21 + 2 × 21 = 108

Quantity II: Perimeter of another semicircle with a radius of 21 cm

= 22/7 × 21 + 2 × 21 = 108

Hence, Quantity I = Quantity II

30. **Answer:** D

Solution

Since no absolute value related to salary is given, we can't find either of the quantities.

Hence, no relation can be established.

It is impossible to determine a relationship.

Hence option D.

31. **Answer:** B

Solution

Quantity I: LCM (856, 124) =

2	856	124
2	428	62
	214	31

= 856 × 124 = 26536

∴ 106625 + 26536 – 56132 = 77029

Quantity II: 98467 + 652 × 26 = 115419

Quantity II > Quantity I

32. **Answer:** A

Solution

Quantity I: 13p + 5 = 29q + 26

13p – 29q = 21

p = (21 + 29q)/13

The number is completely divisible by 13 only when q = 6.

∴ p = 15 when q = 6

∴ the no. is = 13p+5 = 13(15)+5=200

Quantity II:

```
          0 0 0 7 4
    2 6 4|1 9 6 8 3
        - 0
          1 9
        -   0
          1 9 6
        -     0
          1 9 6 8
        - 1 8 4 8
            1 2 0 3
          - 1 0 5 6
              1 4 7
```

Quantity I > Quantity II

33. **Answer:** E

Solution

Quantity I: $6x^2 - 101x + 33 = 0$

∴$6x^2-2x-99x + 33 = 0$

∴ (3x-1)(2x-33) = 0

∴ x = 1/3 (i.e. 0.33) or x = 33/2 (i.e. 16.5)

Quantity II: $y^2 - 34y = -93$

∴ $y^2 - 34y + 93 = 0$

∴ $y^2-3y-31y+ 93 = 0$

∴ (y-3)(y-31) = 0

∴ y = 3 or y = 31

∴ Relation cannot be established.

34. **Answer:** E

Solution

Quantity I: $-110x + 27x^2 = 19x^2 - 78$

$8x^2 - 110x+78 = 0$

$2(4x^2 - 55x +39) = 0$ i.e., $4x^2 - 55x + 39 = 0$

$4x^2 - 52x-3x + 39 = 0$

(x-13)(4x-3)=0

x = 13 or x = 3/4 (i.e., 0.75)

Quantity II: $18y^2-41y+21=0$

$18y^2-27y-14y+ 21 = 0$

(2y-3)(9y7) = 0

y = 3/2 (i.e., 1.5) or y = 7/9 (i.e., 0.77)

Relation cannot be established.

35. **Answer:** E

Solution

Quantity I: $5x^2 - 42x = -85$

$5x^2-42x+85 = 0$

$5x^2-25x-17x+85 = 0$

(x - 5)(5x – 17) = 0

x = 5 or x = 17/5 (i.e. 3.4)

Quantity II: $y^2+8y - 9 = 0$

$y^2+9y-y-9=0$

(y+9)(y-1) = 0

y = 9 or 1

Relation cannot be established.

36. **Answer:** E

Solution

Quantity I: $6x^2 - 11x+4=0$

$6x^2-3x-8x + 4 = 0$

(2x-1)(3x-4) = 0

x = 1/2 (i.e. 0.5) or x = 4/3 (i.e. 1.33)

Quantity II: $32y^2-124y+16= 6y^2 - 16y$

$26y^2-108y+16=0$

$2(13y^2 - 54y+8)=0$ i.e. $13y^2 - 54y+8=0$

$13y^2-52y-2y+8=0$

(y-4)(13y2) = 0

y = 4 or y 2/13 (i.e. 0.15)

Relation cannot be established.

37. **Answer:** B

Solution Quantity I:

$\therefore \frac{80}{100} \times (P + Q) = \frac{10}{100} \times (P - Q)$

$\therefore \frac{8}{10} \times (P + Q) = \frac{1}{10} \times (P - Q)$

$\therefore \frac{8}{10} \times (P + Q) = \frac{1}{10} \times (P - Q)$

$\therefore 8 \times (P + Q) = 1 \times (P - Q)$

$\therefore 8P + 8Q = P - Q$

$\therefore 7P = -8Q$

$\frac{P}{Q} = \frac{-8}{7}$

Quantity II: $\frac{r}{s} = \frac{7}{5}$

$$\frac{4r + 5s}{4r - 5s} = \frac{4\left(\frac{7}{5}\right)s + 5s}{4\left(\frac{7}{5}\right)s - 5s} = \frac{\left(\frac{28}{5}\right)s + 5s}{\left(\frac{28}{5}\right)s - 5s} = \frac{\frac{53s}{5}}{\frac{3s}{5}} = \frac{53}{3}$$

Quantity II > Quantity I

38. **Answer:** A

Solution

Quantity I: Ratio of shares of W, L, M, N = (24 × 3) : (10 × 5) : (35 × 4) : (21 × 3) = 72 : 50 : 140 : 63.

Let total rent be Rs. x.

Then W's share = Rs. $\frac{72x}{325}$

$\frac{72x}{325}$ = 1440 x = $\frac{1440 \times 325}{72} = 6500$

Quantity II: Ratio of investments of A, B and C = Ratio of their profits = 337.50 : 1125 : 637.50 = 9 : 30 : 17.

A's investment = Rs. $(14000 \times \frac{9}{56}) = Rs.2200$

B's investment = Rs. $(14000 \times \frac{30}{56}) = Rs.7500$

Hence, required difference = Rs. (7500 – 2250) = Rs. 5250

39. **Answer:** A

Solution

Quantity I:	**Quantity II:**
$72 = 9^2 - 9$	$68 = 4^3 + 4$
$90 = 10^2 - 10$	$130 = 5^3 + 5$
$156 = 13^2 - 13$	$222 = 6^3 + 6$
$182 = 14^2 - 14$	$350 = 7^3 + 7$

Quantity I: > Quantity II

40. **Answer:** A

Solution

Quantity I: The word 'vitriolic' contains 9 letters, namely 1v, 3i, 1t, 1r, 1o, 1l and 1c.

∴ Required number of ways = $\frac{9!}{1! \times 3! \times 1! \times 1! \times 1! \times 1! \times 1!}$ = 60480

Quantity II: Required number of ways

= $({}^8C_5 \times {}^{10}C_6)$

= $({}^8C_3 \times {}^{10}C_4)$

= $\frac{8 \times 7 \times 6}{3 \times 2 \times 1} \times \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1}$

= 11760

Quantity I > Quantity II

41. **Answer:** B

Solution

Quantity I: $\log_{10} 5 + \log_{10} (5x + 2) = \log_{10} (3x + 14) + \log_{10} 5$

$\therefore \log_{10} (5x + 2) = \log_{10} (3x + 14)$

$\therefore 5x + 2 = 3x + 14$

$2x = 12$

$x = 6$

Quantity II: $\log 2^{26} = 26 \log 2 = 26 \times 0.30103$

=7.82678

Its characteristic is 7. Hence, the number of digits in 2^{26} is 8.

42. **Answer:** A

Solution

Quantity I:

Total number of ways of selecting 8 students from 20 students = ${}^{20}C_5 = 15504$

Number of ways of selecting 2 girl and 3 boys = selecting 3 boys from 12 boys and 2 girls from 8 girls

⇒ Number of ways in which this can be done

= ${}^{12}C_3 \times {}^8C_2 = 6160$

⇒ Required probability = 6160/15504 = 0.397

Quantity II:

The probability of getting a Jack card = 4/52

The probability of getting a spade card = 13/52

The spade card contains already a jack card, therefore required probability is,

4/52 + 13/52 – 1/52 = 16/52 = 4/13 = 0.307

Quantity I > Quantity II

43. **Answer:** D

Solution

X is multiplied by a three digit number and the result is also a three digit number.

∴ X has to be less than 4 i.e. X= 1, 2 or 3

However, X #1 since 1 x XYX = XYX and each letter represents a different alphabet. In this case, L = X and M= Y. This violates the given condition.

∴ X= 2 or 3

Consider X = 2:

$\frac{2Y2 \times 2}{4(2Y)4}$

∴ L=4 and M=2Y

Since there is no carry, M and 2Y are single digit numbers.

Y < 5 i.e. Y = 1,2, 3 or 4.

Consider X = 3

$\frac{3Y3 \times 3}{9(2Y)9}$

∴ L = 9 and M=3Y

Since there is no carry, M and 3Y are single digit numbers.

Y < 3 i.e. Y=1 or 2.

Thus, X and Y can be equal or unequal. Hence, the relationship between X and Y cannot be established.

44. **Answer:** B

Solution

Quantity I:

Ajay and Vijay can do the whole work together $= \frac{1}{\frac{1}{24}+\frac{1}{15}} = \frac{1}{\frac{5+8}{120}}$

$= \frac{120}{13}$ days

Quantity II:

Ajay's 1 day work = $\frac{1}{24}$

Ajay's 4 day work = $\frac{4}{24} = \frac{1}{6}$

Ajay's 4 day work =1 – $\frac{1}{6} = \frac{5}{6}$

Ajay and Vijay can do the remaining work,

$= \frac{\frac{5}{6}}{\frac{1}{24}+\frac{1}{15}}$

$= \frac{\frac{5}{6}}{\frac{13}{120}}$

$= \frac{100}{13}$ days

Total time = $\frac{100}{13} + 4 = \frac{152}{13}$

Quantity I < Quantity II

Hence option B.

45. **Answer:** B

Solution

Quantity I:

${}^{3}C_2 + {}^{4}C_2 / {}^{12}C_2 = \frac{3+6}{66} = \frac{9}{66}$

Quantity II:

${}^{5}C_2 + {}^{3}C_2 / {}^{12}C_2 = \frac{10+3}{66} = \frac{13}{66}$

Hence, Quantity I < Quantity II

46. **Answer:** B

Solution

Quantity I:

Let the radius of circle A be r.

Therefore, circumference of the circle A = 2πr

And diameter = 2r

Thus, 2r + 2πr = 232

Or, r (1 + π) = 116

∴ r = $\frac{116}{1+\frac{22}{7}} = \frac{116 \times 7}{29}$ = 28 cm

∴ Quantity I will be 56 cm

Now, radius of the circle B = 28 – 14 = 14 cm

Quantity II:

Circumference of the circle B

= 2 × $\frac{22}{7}$ × 14 = 88 cm = Quantity II

Hence, Quantity I < Quantity II

47. **Answer:** B

Solution

Quantity I:

A + B complete in 8 days

B + C complete in 12 days

A + C complete in 16 days

LCM = 48

Therefore,

Efficiency of A + B = $\frac{48}{8}$ = 6

Efficiency of B + C = $\frac{48}{12}$ = 4

Efficiency of A + C = $\frac{48}{16}$ = 3

= 2 × (A + B + C) = 6 + 4 +3 = 13

= A + B + C = 6.5

So, efficiency of A =(A + B + C) – (B + C) = 6.5 -4 = 2.5

Hence, A = $\frac{48}{2.5}$ = 19.2 days

A takes 19.2 days

Quantity II:

A + B completes in 24 days

For 8 days, $\frac{8}{24} = \frac{1}{3}$ work is completed

Now, $\frac{2}{3}$ remaining work is completed by B in 10 days

So, B completes in 15 days

While A completes in $\frac{1}{24} - \frac{1}{15} = \frac{1}{40}$

A completes in 40 days

Hence, Quantity I < Quantity II

48. **Answer:** A

Solution

Quantity I:

Let the cost of fruits be Rs 16, Rs 10 and Rs 8 for mango, guava and banana respectively.

The ratio of sales of the three types of fruits is (1 x 3) : (3 x 1) : (6 x 1) = 3: 3: 6 = 1: 1: 2

Hence net profit percentage

$= \frac{(1 \times 16)+(1 \times 10)+ (2 \times 8)}{(1+1+2)}\% = \frac{42}{4}\% = 10.5\%$

Quantity II:

Let cost of fruits be Rs 12 ,Rs 8 and Rs 10 for mango , guava and banana respectively.

The ratio of sales of the three types of fruits is (2 x 4) : (3 x 2) : (5 x 1) = 3: 3: 6 = 8: 6: 5

Hence net profit percentage

$= \frac{(8 \times 12)+(6 \times 8)+ (5 \times 10)}{(8+6+5)}\% = \frac{194}{19}\% = 10.2\%$

Hence, Quantity I > Quantity II

Hence option A

49. **Answer:** B

Solution

Quantity I:

Let the cost price of the first article is Rs x and the cost price of the second article is Rs y . The total sale price of these two articles is Rs 39,000 and there is no loss or gain so the total cost price of the article will become Rs 39,000

Therefore, x + y = 20,000

X × 84% + Y × 110% = x + y

By solving, x : y = 5 : 8

Reqd. difference = $39000 \times \frac{(8-5)}{(8+5)}$ = Rs 9,000

Quantity II:

Let the cost price of the cow = Rs x and the cost price of one goat is Rs y

Then, according to the question, 5x + 6y = 3000 ⋯ (1)

4x + 3y = 30000 – 12000 = 18000 ⋯ (2)

Multiplying by 2 in equation (2), we get

8x + 6y = 36000

By solving equation (1) and (3)

x = cost price of one cow = Rs 2000 and y = cost price of one goat = 20000/6

The cost price of five cow = 5 x 2000 = 10000

Hence, Quantity I < Quantity II

Hence, option B is correct

50. **Answer:** B

Solution

Let the efficiency of Q = 4x then the efficiency of P = 125% of 4x = 5x

When they work together then the total units of work done by them in 40 days = (4x + 5x) × 40 = 360x units

Quantity I:

P's original efficiency = 5x

20% 0f 5x = $20 \times \frac{5x}{100}$ = 1x

Two sixth of the work = $\frac{2}{6} \times 360x = 2 \times 60x$

The number of days , it will complete at 20% of his efficiency = $\frac{2 \times 60x}{x}$ = 120 days

Quantity II:

Q's original efficiency = 4x

25% of 4x = $25 \times \frac{4}{100} = x$

Half of the work = $\frac{360x}{2}$ = 180x

The number of days Q will take to do half of the work at 25% of his efficiency = $\frac{180x}{x}$ = 180 days

Therefore, Quantity I < Quantity II

Hence, option B is correct